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ABSTRACT

This document is the last of three volumes of the proceedings of an international seminar which focused on student misconceptions in science and mathematics as well as teaching strategies intended to deal with those misconceptions. An introduction to the volume explains the format of the conference and the organization of the sessions. It stresses the attention given to constructivism by the participants, as well as some of the problems associated with defining misconceptions and alternative frameworks. The remainder of the document contains 62 of the papers presented at the seminar, arranged in alphabetical order by senior author. These papers focused mainly on physics, mathematics, and chemistry. In addition, the document contains a list of the conference participants. (TW)

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Proceedings of The

Second International Seminar

MISCONCEPTIONS AND EDUCATIONAL STRATEGIES IN SCIENCE AND MATHEMATICS

July 26 - 29, 1987

CORNELL UNIVERSITY, ITHACA, NY, USA

Volume III

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SE 048 927

PROCEEDINGS CONTENT BY VOLUME

VOLUME I: Overview of the Seminar; Epistemology; Research Methodologies; Metacognitive Strategies; Use of Computers; Roster of Participants

VOLUME II: Overview of the Seminar; Teacher Education; Teaching Strategies; Biology; Elementary Science; Roster of Participants

VOLUME III: Overview of the Seminar; Physics; Mathematics; Chemistry; Roster of Participants

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Second International Seminar
Misconceptions and Educational Strategies in Science and Mathematics
July 26-29, 1987

Introduction

Our first seminar, held in 1983, showed that there was strong international interest in the general topic of student misconceptions in science and mathematics (see Helm and Novak, 1983). Advance announcements for our second seminar were more widely circulated, but the fact that over three times as many papers (177) were presented and more than three times as many participants (367) enrolled from 26 countries was clear indication of the great interest currently evidenced in the field. The proceedings are being printed in three volumes to accommodate all papers submitted. A roster of participants is included in each volume.

The format for the seminar followed the pattern of our first seminar: a wine and cheese informal reception on Sunday evening; morning and afternoon sessions for paper presentations and discussions; late afternoon plenary sessions to discuss "Issues of the Day"; and unscheduled evenings. There was the frustration for most participants of choosing between seven or eight simultaneous sessions, but papers were grouped by topics in an attempt to preserve some homogeneity of interests in each group. Papers are presented in the Proceedings in broad general categories similar to the groupings used in the seminar program, and in alphabetical order by senior author.

Meetings of the Psychology and Mathematics Education group were scheduled in Montreal, Canada just preceding our seminar and this facilitated participation by a number of math educators who might otherwise not have attended. In both our first seminar and again in 1987, there was a strong feeling that researchers in science education and in math education can benefit by greater interaction. Although parallel sessions devoted to science or math education research limited some of this interaction, plenary sessions and informal meetings offered some opportunities for much needed cross-disciplinary dialogue. There was a general consensus that many of the issues and problems were common to both science and math education. In some areas of research, math education appears to more advanced than science education (e.g., concern for epistemology as it relates to instruction) and in other areas the reverse is true (e.g., the use of metacognitive tools to facilitate understanding). In the plenary session on physics and chemistry, similar concerns were evidenced in communication between sciences.

There remains the problem of definition of misconceptions, alternative frameworks, or whatever we choose to call these commonly observed patterns in faculty understanding evidenced in students, teachers and textbooks. There were more papers presented in this second seminar on how to deal with misconceptions than in the first; however, there was still heavy representation of papers dealing with the kind, number and tenacity of misconceptions and probably too few dealing with educational strategies to mollify or remove the deleterious effects of misconceptions or to limit teacher or text initiation of misconceptions.

More emphasis was evidenced on the importance of epistemology to improvement of science and math education. In general, there was strong endorsement of "constructivist" epistemology both for clarifying the nature of knowledge and knowledge production and as an underpinning for lesson planning and pedagogical practices. Of course, there was debate on the value of constructivist ideas and even some questioning of constructivist epistemology in contrast to empirical/positivist views on the nature of knowledge and knowing. A number of participants observed that we promulgate constructivist thinking for students, but too often we conduct teacher education programs that seek to give teachers fixed truths and methodologies, rather than recognize their need to reconceptualize subject matter and pedagogical strategies as they engage in the slow process of conceptual change.

Although concern for teacher education was better represented by papers in this seminar than in our first, there remained a common perception that new ideas and methodologies to improve teacher education, and much more field-based research in teacher education, are badly needed. As we launch this year at Cornell University a new science and mathematics teacher education program, with new faculty, we were especially sensitive to the concerns expressed. They represent an important challenge to us as we move ahead in the design, evaluation and analysis of our new teacher education initiatives.

In our closing plenary session, Ron Hoz expressed concern for the limited representation of papers dealing with the psychology of learning as it relates to science and mathematics education. This concern appears to be warranted in view of the fact that most psychologists interested in human learning have abandoned bankrupt ideas and methodologies of behavioral psychologists (e.g., B.F. Skinner), and are now developing and refining strategies for study of cognitive learning (e.g., James Greeno). The early work of Jean Piaget, George Kelly, David Ausubel and other cognitive psychologists is now entering the mainstream of the psychology of learning. These works have important relevance to the study of teaching and learning as related to misconceptions. A note of caution, however. Most of the behaviorist psychologists turned cognitive psychologists still operate methodologically as positivists. They hold constructivists views of learning (i.e., that learner's must construct their own new meanings based on their prior knowledge), but they adhere to rigidly positivist research strategies and often recommend teaching practices that ignore the teacher as a key player in constructivist-oriented teaching/learning. The "constructivist convert" psychologists were conspicuously absent from our participant roster. What is the message here?

There were more papers dealing with metacognitive tools. This may reflect in part the rising national concern with helping students "learn how to learn." Almost every issue of the journal of the Association for Supervision and Curriculum Development (*Educational Leadership*) has articles on this topic extolling the merits of efforts to help students acquire "thinking skills." Another word of caution: a backlash is already developing in the American public that schools are so busy with numerous activities to teach "thinking skills" that too little subject matter is being taught! My own view is that most of the "thinking skill" programs lack solid underpinning in both the psychology of cognitive learning and in constructivist epistemology. They are too often an end in themselves, rather than a means to facilitate learning and thinking that places responsibility on the learner for constructing their meanings *about subject matter*. Concept mapping and Vee diagramming are two metacognitive tools that have had demonstrated success in this respect, as reported by a number of papers in Volume I of these Proceedings. From our perspective, we should like to see much more research done on the use of metacognitive tools to help teachers help students modify their misconceptions and form more valid and powerful conceptual frameworks.

In the mathematics groups in particular, but also in some of the science sessions, there was concern expressed regarding the importance of "procedural knowledge" as contrasted with "conceptual knowledge." Students often learn an algorithm or procedure for solving "textbook" problems but cannot transfer this skill to novel problem settings or across disciplines. They fail to understand the concepts that apply to the problems. The contrast between "procedural" and "conceptual" knowledge is, in my view, an artificial distinction. In our work with sports education, dance, physics, math and many other fields, we have never observed a procedure that could not be well represented with a concept/propositional hierarchy in a concept map. The limitation we see is that both strategies for problem solving and understanding basic disciplinary ideas derive from the conceptual opaqueness of most school instruction. Mathematics, voice and dance instruction are particularly bad cases of conceptually opaque teaching. Metacognitive tools such as concept mapping can reduce some of the dilemma evidenced in concern for procedural versus conceptual learning to the need for more research and practice to help teachers help students see more clearly the conceptual/propositional frameworks that underlie meaningful learning and transferability of knowledge.

The role of the computer in science and mathematics education is emerging more prominently. Several sessions dealt with papers/discussion on the use of the computer as an educative tool, and numerous other sessions had one or more papers that reported on studies that involve computers in some way. The rapidly increasing power and stable cost of microcomputers, together with better and easier authoring systems, are changing significantly the application of computers in science and mathematics education. In many cases, the computer is not a substitute for class instruction but rather a tool for extending learning in class to novel problem solving or simulation constructions. The use of the computer to provide directly large amounts of raw data, or to permit access to large data banks, makes possible problem solving activities that border on original research, thus providing opportunities for creative problem generation and problem solving by students in ways that offer an experience paralleling creative work of scientists or mathematicians. The emerging use of video disc with computers and the emerging technologies for monitoring laboratory experiments should provide exciting new opportunities for science and mathematics instruction and also for the use of metacognitive tools. We expect to see much more activity reported in this area of future seminars of our group.

It is interesting to note in passing that while video tape was often recognized as a powerful tool in research on teaching/learning and for teacher education, not one paper reported on the use of TV as a primarily teaching vehicle.

The much heralded power of television as an instructional vehicle in the 1950's has not materialized. What will be the fate of computer aided instruction or interactive video instruction in 20-30 years?

On occasion, especially in sessions dealing with teaching and teacher education, it was observed that the school and classroom are complex social settings. We know much too little about how social factors facilitate or inhibit acquiring or modifying and correcting misconceptions, or indeed any other learning. There is a need for an enormous increase in studies dealing with the school/teacher/learner sociology as it relates to misconceptions research. We need to learn more about what sociologists, anthropologists and linguists are learning about how people communicate or fail to communicate positive ideas and feelings. It is my hope that our next international seminar on misconceptions will reflect more knowledge and awareness of these fields.

There remains much work to be done. And yet there are reasons for optimism. We are learning more about why students fail to learn and how to help teachers help students learn better. I believe the science of education is building a solid theory/research base, and positive results in improvement of educational practices are already emerging. The next decade should bear fruit in tangible improvement of science and mathematics teaching.

Reference

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TABLE OF CONTENTS

Volume III

Papers in this volume are arranged alphabetically by first author and emphasize the following subject areas: OVERVIEW OF THE SEMINAR; PHYSICS; MATHEMATICS; CHEMISTRY; ROSTER OF PARTICIPANTS.

Artigue, M. & Viennot, L.	Some Aspects of Students' Conceptions and Difficulties About Differentials	1
Baker, D.C.	Some Perspectives on the Teaching of Mathematics and Natural Sciences in an Engineering Curriculum	9
Barrow, L.H.	Magnet Concepts and Elementary Students' Misconceptions	17
Bouwens, R.E.A.	Misconceptions Among Pupils Regarding Geometrical Optics	23
Brown, D.E. & Clement, J.	Misconceptions Concerning Newton's Law of Action and Reaction: The Underestimated Importance of the Third Law	39
Buchweitz, B. & Moreira, M.A.	Misconceptions in Physics: Research Findings Among Brazilian Students	54
Cachapuz, A.F. & Martins, I.P.	High School Students' Ideas About Energy of Chemical Reactions	60
Carter, C. & Bodner, G.	How Student Conceptions of the Nature of Chemistry and Mathematics Influence Problem Solving	69
Clement, J.	Overcoming Students' Misconceptions in Physics: The Role of Anchoring Intuitions and Analogical Validity	84
Clements, M.A. & Del Campo, G.	Fractional Understanding of Fractions: Variations in Children's Understanding of Fractional Concepts, Across Embodiments (Grades 2 Through 5)	98
Danusso, L. & Dupre', F.	Student Representations of Simple Electric Circuits	111
Dufresne, R., Gerace, W., Hardiman, P.T., & Mestre, J.	Hierarchically Structured Problem Solving in Elementary Mechanics: Guiding Novices' Problem Analysis	116
Ellerton, N.F. & Ellerton, H.D.	Mathematics and Chemistry Problems Created by Students	131
Faucher, G.	Pragmatical Conceptions in the Atomic Domain	137
Fischbein, E., Stavy, R. & Ma-Naim, H.	The Psychological Structure of Naive Impetus Conceptions	143
Fuchs, H.U.	Do We Feel Forces?	152
Fuchs, H.U.	Thermodynamics: A "Misconceived" Theory	160
Ganiel, U. & Eylon, B-S.	Electrostatics and Electrodynamics - The Missing Link in Students' Conceptions	168
Goldberg, F.M.	Using an Interactive Videodisc as a Tool for Investigating and Facilitating Student Understanding in Geometrical Optics	180
Gorodetsky, M. & Gussarsky, E.	The Roles of Students and Teachers in Misconceptualization of Aspects in "Chemical Equilibrium"	187
Hardiman, P.T., Dufresne, R. & Mestre, J.	Physics Novices' Judgments of Solution Similarity: When Are They Based on Principles?	194
Harel, G. & Hoz, R.	Declarative and Procedural Knowledge and Isomorphism of Speed Problems	203
Harel, G., Smith, D., & Behr, M.	Aggregate Versus Individual Element Interpretations to Facilitate Part-Whole Representations	211
Hativa, N.	Strategies for Self-Regulated Learning from Computerized Practice in Arithmetic	216
Heller, P.	Use of Core Propositions in Solving Current Electricity Problems	225
Henderson, D.W.	Informal Geometry is the True Geometry	236

Hershkowitz, R.	The Acquisition of Concepts and Misconceptions in Basic Geometry - Or When "A Little Learning is a Dangerous Thing"	238
Ivowi, U.M.O. & Oludotun, J.S.O.	An Investigation of Sources of Misconceptions in Physics	252
Jordaan, A.S.	Aspects of the Understanding and Teaching of the Laws of Science	258
Jung, W.	Understanding Students' Understandings: The Case of Elementary Optics	268
Kenealy, P.	A Syntactic Source of a Common "Misconception" About Acceleration	278
Kärrqvist, C.	Pupils Are Able	293
Krajcik, J.S. & Lunetta, V.N.	A Research Strategy for the Dynamic Study of Students' Concepts Using Computer Simulations	297
Levin, I. & Gardosh, R.	The Development of the Concept of Speed: A Case of Different Frames of Reference	302
Lightman, A.P., Miller, J.D. & Leadbeater, B.J.	Contemporary Cosmological Beliefs	309
Linder, C.J.	Tertiary Physics: A Case Study in Students' Conceptions of Sound	322
Maloney, D.P.	Rule Usage on Related Tasks: Patterns of Consistency	335
Mermelstein, E.	Clarification of Conservation of Liquid Quantity and Liquid Volume	350
Moreira, M.A.	Concept Mapping as a Possible Strategy to Detect and to Deal with Misconceptions in Physics	352
Oppenheim, J.R.	Patterns of Mathematical Misconceptions as Revealed on Tests of Algebra Readiness	361
Otero, J.	Comprehension Monitoring in Learning from Scientific Text	370
Peck, D.M.	Children Derive Meaning from Solving Problems About Physical Materials	376
Placek, W.A.	Preconceived Knowledge of Certain Newtonian Concepts Among Gifted and Non-Gifted Eleventh Grade Physics Students	386
Rafel, J. & Mans, C.	Alternative Frameworks About the Learning of Changes of State of Aggregation of Matter: Sorting of Answers into Models	392
Renström, L.	Pupils Conception of Matter. A Phenomenographic Approach	400
Ringnes, V.	Misconceptions in Environmental Chemistry Among Norwegian Students	415
Sadler, P.M.	Misconceptions in Astronomy	422
Schmittau, J.	Relating Concepts to Problem Solving in the Determination of Meaning in Mathematics	426
Schultz, K., Murray, T., Clement, J. & Brown, D.	Overcoming Misconceptions with a Computer-Based Tutor	434
Schuster, D.G.	Understanding Scientific Derivations: A Task Analysis and Constructivist Learning Strategy	448
Selden, A. & Selden, J.	Errors and Misconceptions in College Level Theorem Proving	457
Stein, J.S.	Raising Laboratory Learning to a Conceptual Level: Does MBL Help?	471
Steinberg, M.S.	Transient Electrical Processes as Resources for Causal Reasoning	480
Tan, S.K.	A Conceptualisation of Students' Meaning of Understanding of Physics in the High School Classroom	491
Thijs, G.D.	Conceptions of Force and Movement, Intuitive Ideas of Pupils in Zimbabwe in Comparison with Findings from Other Countries	501
Tobias, S.	In Class Methods for Getting at Student Misconceptions	514

Touger, J.S., Dufresne, R., Gerace, W. & Mestre, J.	**Hierarchical Organization of Knowledge and Coherent Explanation in the Domain of Elementary Mechanics**	517
van Zee, E.H. & McDermott, L.C.	**Investigation of Student Difficulties with Graphical Representations in Physics**	531
Veldhuis, G.H.	**A Variable for Use in the Study of the Categorization of Physics Problems and Associated Expert-Like Behavior**	540
Vinner, S. & Zur, S.	**Some Aspects of Geometry as a Deductive System in High School Students**	551
von Rhöneck, C. & Grob, K.	**Representation and Problem-Solving in Basic Electricity. Predictors for Successful Learning**	564
Wesley, W.G.	**Toward a Cognitive Physics Course**	578

SOME ASPECTS OF STUDENTS' CONCEPTIONS AND DIFFICULTIES ABOUT DIFFERENTIALS

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Laurence VIENNOT-LDPES- UNIVERSITE PARIS VII

I - INTRODUCTION

The research reported here about differentials is an interdisciplinary research led by mathematicians and physicists. We took this interdisciplinary point of view for the following reasons : at the beginning of higher education, failures are frequently observed in teaching this notion, especially in physics and, moreover, a discrepancy quickly appears between concerns of physicists and mathematicians.

Indeed, to mathematicians, the object to which differentials refer is primarily the linear map tangent to a given function at a given point (if it exists) ; to physicists, differentials evoke, roughly speaking, "little bits of something", which allow easier calculus.

Not only the object, but also the value ascribed to it seem to be different in mathematics and in physics. At first sight, and this point of view might need to be refined, exactness is linked to mathematics while approximation, with sometimes a pejorative connotation, is linked to physics.

The different steps in the evolution of this notion and its teaching, in particular since the end of last century, have been documented (see M.Artigue, L.Viennot and al. [4]). This part of the research is not reported here. Let us only note that mathematical papers as well as official instructions, comments about curricula and textbooks clearly show the relatively recent appearance of the present mathematical definition and its difficult penetration into elementary teachings :

- Differentiability was introduced by O.Stolz in 1887 and the differential was defined as a linear function, for the first time, by M.Frechet in 1911, in the framework of the development of functional analysis (see table 1 below). In fact, these definitions have rehabilited the old idea of approximation which had prevailed at the beginning but had then been left to one side, for reason of a lack of rigor.

> "Une fonction $f(x,y,z,t)$ admet une différentielle à mon sens au point (x_0, y_0, z_0, t_0) s'il existe une fonction linéaire et homogène des accroissements, soit : $A\Delta x+B\Delta y+C\Delta z+D\Delta t$, qui ne diffère de l'accroissement f de la fonction à partir de la valeur $f(x_0,y_0,z_0,t_0)$ que d'un infiniment petit par rapport à l'écart Δ des points $(x_0,..,t_0)$ et $(x_0+\Delta x,...,t_0+\Delta t)$. La différentielle alors est par définition : $df=A\Delta x+B\Delta y+C\Delta z+D\Delta t$."

TABLE 1 : DEFINITION OF DIFFERENTIAL BY M.FRECHET (1911)

- It is only since the sixties that, in french universities, differentials have been introduced according to this modern point of view, in mathematics courses for beginners. Before differentials were reduced to the role of algorithmic tools for Calculus. In physics, it is often still the case even at the present time.

A question arises here : as far as teaching is concerned, do we observe a real progression from an exclusively algorithmic or pragmatic use towards a deeper understanding of the notion itself?

* In mathematics, a look at exercises most frequently proposed to students may leave some doubts : the algorithmic use of the notion seems preponderant and even concerns about differentiability turn out to be simply an opportunity for applying mechanically such and such theorem, without a real insight into the notion.

* In physics, such a progession is hardly necessary. From the beginning, differentials are a tool more than an object. The algorithmic content of differentials is of considerable utility, which leaves little room for subtilties about legitimity of such a successful calculus.

As for approximation, it is supposed to be a natural and well founded attitude for physicists so that they are often unaware of wether they use it or not.

These first indications prompted us to investigate students' and to a lesser extent teachers' reasonings concerning differentials. This has been carried out by various methods : class-room observations, interviews, questionnaires. Here we will only present results obtained with this last technique. And for sake of brevity, only the most salient features will be reported.

They have been organized along three lines of analysis, suggested by our historical investigations:

- differentials : what for? an inquiring about the use of the notion,
- approximation and rigor in results and reasonings,
- status of differential elements : do they refer to functions (linear and tangent) or to "little bits" of given physical quantities ?

II - DIFFERENTIALS : WHAT FOR ?

• In mathematics, 85 students, in their third year at University were presented with questions, some of which are given in table II. The results, broadly speaking, show a discrepancy between, on the one hand what the students declare important (firstly, the definition of the notion and the links between continuity, derivability and differentiability......) and, on the other hand, what they show to be important to them, when solving problems (algorithmic procedures, computation of partial derivatives....)

The first aspect is prevalent for instance in replies to question 1d :

"If you had to explain what a differential is to a first year student, what important points would you stress ?"

- 33 mention the links between derivability, differentiability, continuity, existence of partial derivatives.....,
- 11 mention algorithmic procedures,
- 10, the idea of local approximation,
- 5, the fact that differentials are linear functions,
- and 2, difficulties about notations.

But in question 2, where function f was explicitely given as an expansion with a linear part and a remainder, 87% of students do not recognize it as such and jump towards the computation of partial derivatives in order to decide wether the function is differentiable.

Q1 : If you had to explain what a differential is to a first year student : 1a) what definition would you give ? 1b) what notations would you introduce ? 1c) what examples would you use ? 1d) what important points would you stress ?

Q2 : Is the function f | $R^2 \to R$ defined by :

$$f(x,y)=2x+4y+y^3[\sqrt{1-\cos x}+y]$$

differentiable at the point (0,0) ? Justify your answer !

Q3 : g and h are C^2 maps from R^2 to R and f is defined as follows :

$$f(x,y)=g(y,h(x,y))$$

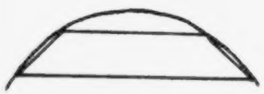
Show that f is a C^2 map and compute $\partial^2 f/\partial x\partial y$.

Q4 : By using differentials, is it possible to justify the following well known formulas of error calculus ?

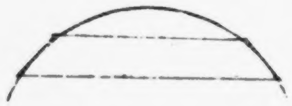
$$\Delta uv/uv = \Delta u/u + \Delta v/v \qquad \Delta(u/v)/(u/v) = \Delta u/u + \Delta v/v$$

If so say how, if not say why not.

Q5 : To find the volume of a sphere, physicists cut it into elementary slices and approximate each slice by a small right cylinder. Why cylinders ? Is it possible to choose other approximations ? For instance, do the following approximations lead to the same result ?



Cones generated by chords



Cones generated by tangents

Q6 : The map f | $R^2 \to R$ is defined by : $f(x,y)=\exp(y^2+x).\sin xy$

- find the differential of f at the point (1,0) and give a geometrical interpretation of it
- deduce an approximate value of f(1.001,0.01).

TABLE II : SOME QUESTIONS FROM THE MATHEMATICS QUESTIONNAIRES

This indicates a prevalence of algorithmic procedures over answers relying on a real understanding of what is at stake. This result is confirmed by rates at which different questions are answered : 86% for question 3 (computation of partial derivatives of a composed fonction) contrasting with the extremely low rates (less than 10%) obtained when a justification is required (for instance, question 4 about well known formulas used for calculating errors in physics and question 5 about the computation of the volume of a sphere by "slicing").

• In physics, at least in France, students often see differentials appear suddenly at the beginning of their higher education. They do not know yet very well what they are, they just feel a kind of emergency : differentials are needed.
Our question was the following : are students able to analyse this need, and to say why they had been able to do many things before in physics without differentials ? The key word here is "non-linearity".

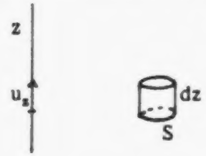
In the first questionnaire, proposed to 93 students in their first year at University (see table III), the beginning of a calculus about atmospheric pressure was given up to the expression :

$$dp = -\rho g dz$$

This relationship relies on a " slicing" of a column of air and on a balance of forces acting on each "element of volume" thus obtained.
This slicing is necessary because the relationship between a variation of pressure Δp and the corresponding variation of the altitude Δz is not linear: the specific mass ρ (and, strickly speaking, the value of g) is not a constant with respect to z.

The questions aimed at assessing understanding of this point. The results, summarized on table III show :

- a great conviction among students about the neccessity of cutting the column of atmosphere into small slices ($\approx$ 90%),
- a frequent ambiguous justification, where this necessity is linked with the fact that the (integral) function p(z) is not a constant (whereas the decisive point is wether the factor ρg is, or is not, a constant),
- finally a flagrant lack of understanding of what makes differentials useful : when it is proposed to replace air by water (with, this time, a constant specific mass - which was not recalled in the text), few students (15%) realize that slicing is no longer needed.



For a cylindric element of volume with base area : S et height : dz,

Force due to pressure on the bottom face : $Sp(z)\vec{u}_z$
Force due to pressure on the top face : $Sp(z+dz)(-\vec{u}_z)$
Weight : $\rho gSdz(-\vec{u}_z)$
Balance of forces : $Sp(z) - Sp(z+dz) - \rho gSdz = 0$
$dp = -\rho gdz$

Question : dz is supposed to be small
Is it necessary ?
If so, why ? because of p(z)
because of $\rho(z)$ or g(z)
Is it necessary with water ?

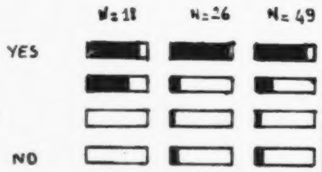


TABLE III : QUESTIONNAIRE ABOUT PRESSURE

When asked, directly this time, what makes differentials useful, students (N=100) in their first year at University are only, in their great majority able to cite examples or topics where differentials are often used (mechanics, thermodynamics, functions of several variables). Few of them reach the point of using the word "local", and still fewer mention "non-linearity".

All this suggests that in mathematics, as well as in physics, algorithmic uses of the notion have prevailed over, if not hidden, other preoccupations concerning justification and meaning of computations or formulas and also, the essential aspect of differentials : a mastering of approximations. This is confirmed by what follows.

III - APPROXIMATION AND RIGOR

The first results quoted above suggest that the idea of approximation is either overshadowed by the predominance of exercises calling for algorithmic procedures or associated with a kind of fatalism towards loose or obscure reasonings. The following results support this preliminary conclusion.

• In mathematics, questions on the control of approximations or the rigor of reasonings are frequently left aside by students (see section II), as if they were percieved as a breach of contract.

When answered, these questions are often poorly solved : for instance, only a quarter of replies actually given in question 6 mention the tangent plane and only a third, in question 5, propose acceptable justifications.

Moreover :

- more than half of the Taylor expansions with remainders elaborated by students in the different questions are erroneous and with a quasi unanimity mistakes concern the remainders
- in question 2, only a few students (2 out of 11) manage to prove that the non linear part of the given function is actually of order more than one.

For most of students, it seems as if rigor in reasonings and formulas would boil down to write an "ε" or an " o" at the end of expressions.

• In physics two kinds of results are available :

- first, a question was proposed to 100 students in their first year at University about the integral :

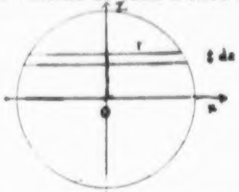
$$F = \int_0^H \text{pressure}(z).d(\text{surface}(z))$$

obtained at the end of a computation, concerning a force acting on a dam. The question was : "Is the result of this integration (strictly) exact or not ?". Hardly half of students answer a "yes" without ambiguity. A third say "no" or a "yes if...." with justifications ranging from a very common : "so long as dz is as small as possible" up to an ambiguous : "if the integral has its limit value" (negative answers that obviously do not rely on legitimate preoccupations about the adequacy of the given mathematical model !)

- second, students (65 in their first year, 26 in their second year) have been invited to criticize the proof given in table IV. Most of them, obviously ill at ease, owerflow their comments with incantations such as : "so long as dz is as small as possible". The favorite proposal for a better rigor is to use spherical or cylindrical coordinates (up to 30% of students) ! Less than 20% propose the classical justification using upper and lower estimates of the volume of each slice by that of right cylinders and less than 10% evoke the fact that the neglected term is of second order.

To find the volume of a sphere of radius R, the sphere is cut into elementary slices of thickness dz (as in the drawing).

The volume of such a slice at height z is :



$$dV = \pi r^2 dz = \pi(R^2-z^2)dz$$

So the volume of the sphere is :

$$V = \int_{-R}^{+R} \pi\ (R^2-z^2)dz = 4/3\pi R^3$$

Do you think this computation could be made more rigorous ? If you think so, say how ?

TABLE IV : COMPUTATION OF THE VOLUME OF A SPHERE

These few results lead to a paradoxal conclusion :

Approximation, though a constitutive element of differentials, appears detached from them in students' ideas :

- *students do not see this aspect as preponderant,*
- *they do not feel obliged, or they do not know how to control this aspect and its implications in the rigor of proofs.*

At the best, they check the differentiability of functions with powerful and simple criteria in mathematics (for instance, the fact that if f is a function C^1, *it is differentiable) and rely, in physics, on the well known fact that "it works".*

IV - DIFFERENTIALS : FUNCTIONS OR LITTLE PIECES OF....

The third part of this report deals with the status of differentials in students' mind, ranging from "small quantities" to a functional aspect.

The question was : to which extent do students understand a d(...) as a linear function, as opposed to a small part of some physical quantity.

• In mathematics, the results reported in section II show that the functional status of differentials mainly appears on a declarative level : 51% of students define the differential as a linear map, in question 1a, while less than a third propose differentials in the form of functions, in answers to questions 2 and 6 (see table II).

• In physics, the prevailing point of view is the following : "To integrate, it is essential not to think about what dl represents, but to procede mechanically, otherwise we are done for." (first year at University), "(This method) .....I work with it mechanically, without any idea of what a differential is. It is safer. I am sure not to make a mistake."

Our goal was to evaluate to what extent this point of view was, on the one hand general, on the other hand operational. In connection with various questinnaires, we have collected a list of comments about the meaning of dx, dl and such differential elements.

Two extreme tendances appear :

- in one of them, the differential element has lost any meaning except that it indicates the variable of integration (in some way, this is not far from a mathematical attitude : in measure theory, a notation such as $\int f$ is commonly used and in $\int f(x)dx$, x is presented as a mude variable) :

"I don't feel any need for a representation in integration" - "dx is not real" - "immaterial, abstract, a pure concept" - "the length is fictitious" - "in fact, it does not matter at all, when integrating dl becomes a variable of integration",

- in the other, the differential element has a material content which may exclude other meanings :

"dl is a small length" - "a little bit of wire" - "a little piece dl of the wire" - "one surveys all the possible dz, therefore all the subdivided parts of the sphere".

Of course, between these two extremes, we find all the possible comments such as :

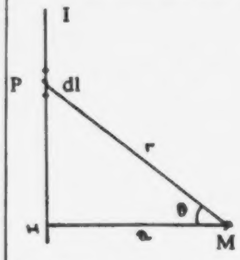
"dz is the limit of Δz when $\Delta z \to 0$" - "dl : element infinitely small" - "non measurable" - elementary variation" - "one cannot find anything smaller" - "it means ultra-simple".

To complete this rough classification, a questionnaire was specially written to explore possible links between the meaning students ascribe to differential elements and the way they work out questions where differentials are involved. Once more, the beginning of a calculus was proposed. It dealt with the magnetic field produced by a rectilinear infinite wire where a constant current flows and was led up to the expression of the contribution :

$$dB = \mu_0 Idl\cos\theta/4\pi r^2$$

of an element dl of the wire (see table V).

The problem is to find the magnetic field created by a current flowing in a wire of infinite length, at a point M (as in the drawing below).



An element of wire dl, round a point P creates a field dB at M.

In this case, the vectorial addition leads to an algebraic one, with :

$$dB = \mu_0 Idl\cos\theta/4\pi r^2$$

Questions :

- can dB be considered as the differential of a function ? If so, a function of what variables ?
- same question for dl ?
- express dB with only one variable.

TABLE V : COMPUTATION OF A MAGNETIC FIELD

Note that to integrate, which was finally asked, it is necessary to expresss dB with only one variable and, for instance, write :

$$l = a\,tg\theta \quad \text{so } dl = a\,d\theta/\cos^2\theta$$

Table VI shows the results obtained with 44 first year students. These findings indicate a great perplexity about our questions, probably encountered for the first time (although the questionnaire was given at the end of the year).

N = 44	Yes	No	No answer
dB differential ?	52%	27%	21%
dl differential ?	32%	27%	41%

TABLE VI : DIFFERENTIALS AND FUNCTIONS

We regrouped comments, on the one hand, by students (30%) who made the classical mistake : "$dl = rd\theta$" and, on the other hand, by students (7%) giving the correct relationship :

- the former deny the status of a differential at least to one of the objects dB, dl and frequently give for dl one of the extreme meanings described above,
- the latter all qualify dB and dl as differentials with abundant functional specifications :

$B(\Theta)$ or $B(r)$ or $B(HP)$, $dl = d(HP)$, $HP(\Theta)$, $HP(r)$.....

This suggests at least that thinking in functional terms is not so dangerous, after all!.

So, as far as teaching is concerned, the mathematical and physical facets of the notion of differential therefore meet in this respect : in both cases, the functional aspect is, in fact, weak. In mathematics, it disappears under a flow of algorithmic procedures, all the more as it is not backed up by a geometrical vision. In physics, it is overshadowed especially by the idea of "contribution", "piling up" which may exclude the variational and therefore functional aspect, and also by this conviction : "the less we think, the safer".

When modelling physical situations, the shift from a representation in terms of contribution towards the variational point of view is often needed, but it is far from obvious to students.

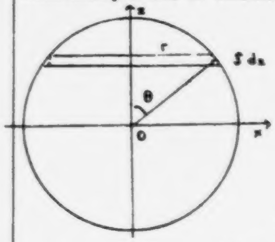
V - WHAT ABOUT TEACHERS ?

For differentials, it is hardly possible to speak of students' answers as simple manifestations of their spontaneous conceptions or preconceptions. What we observe is obviously, at least partly, an effect of what teachers said and did. Hence it would be useful to investigate not only official instructions and textbooks but also teachers' ideas. This has been done only in an exploratory way, teachers being always reluctant to be themselves the object of an investigation.

A questionnaire (see table VII) has been designed especially for them. It sets out a false proof for the computation of the area of a sphere which is, seemingly, a paraphrase of the one given in table IV about the volume of a sphere : the same cutting up into slices and the same assimilation of slices to cylinders occur !

But this time, it is not possible to make such an assimilation as the neglected term is of first order. In other words, the ratio of the lateral area of the slice to the lateral area of the cylinder does not tend to 1, when the thickness of the slice tends to 0. A factor $1/\cos\theta$ is there, which may take values dramatically farfrom 1.

To find the volume of a sphere of radius R, the sphere is cut into elementary slices of thickness dz, as in the drawwing below.



The volume of such a slice at height z is assimilated to that of a right cylinder of the same thickness dz and of base area :

$\pi r^2(z)$ (as shown in the drawing)

So $dV = \pi r^2 dz = \pi(R^2 - z^2)dz$

and the volume of the sphere is :

$$V = \int_{-R}^{+R} \pi(R^2 - z^2)dz = 4/3\pi R^3$$

If the same procedure is used to find the area of a sphere, the following expression is obtained for the area of an elementary slice of thickness dz, at height z :

$$dS = 2\pi r dz = 2\pi\sqrt{R^2 - z^2}\, dz$$

and therefore the area of the sphere is given by the integral :

$$S = \int_{-R}^{+R} 2\pi\sqrt{R^2 - z^2}\, dz = \int_{0}^{\pi} 2\pi R^2 \sin^2\theta d\theta = \pi^2 R^2$$

Could you explain why the same method leads to a correct value in the first case (volume) and to a false value in the second one (area) ?

TABLE VII : VOLUME AND AREA OF A SPHERE

13 physics pre-teachers, in their last year of education, were given this question. Only 4 were able to give a satisfactory answer (actually neglected term of first order plus, in one case, an interesting geometrical analogy) ; 5 answered by giving a correct computation without any more comment and 4 did not know what to answer.

For all of them, the question was obviously of a new type, if not unfair. Yet, it seems to us that it is precisely this kind of questions which could help students give some interest to concerns as legitimity, accuracy of computations, status of differentials.

Other results support this one but let us simply quote this comment by one of our pre-teachers :

"May be it is just by chance that it works (computation of the volume). Indeed the relationship $dV=S(z)dz$ is not true. It could be the reason why the other computation about the area of a sphere does not work."

This may be considered as purely anecdotal. But it could also be linked to the following fact : in physics texbooks, differentials are introduced, at least in France, in an incoherent way as they are nearly universally presented as a tool for obtaining approximate values of quantities and then nearly exclusively used in order to obtain differential equations or integrals leading to exact values, whitout a word of explanation or, in the best cases, with a mere undetailed reference to "first order computations".

VI - CONCLUSION

In this investigation we have observed students' difficulties widely connected with those of teachers. It could be argued that they just reflect this good old conflict between rigor in mathematics and effectiveness in physics, the last one seemingly going with looseness in reasonings.

We have been led to put some shades on this dichotomy and to oppone rather in mathematics as well as in physics, algorithmic procedures to concerns about the status and role of differentials, about legitimity of computations and rigor of proofs. In both disciplines, a kind of consensus encourages and develops algorithmic procedures linked to differentials at the expense of more conceptual aspects. But it must be acknowlegded that the proportion of each component, algorithmic or conceptual to be put in teaching is far from obvious. Moreover, several didactical researchs have shown that first year students have difficulties with the mathematical techniques of approximation : absolute values, inequalities, reasonings by sufficient conditions.... So one may think that teachers take refuge in algorithmic procedures as they are easier to teach successfully.

In fact, the essential questions are :

- which goals do we set for our teaching ?
- at which kind of efficiency do we aim ?

We think the three lines of analysis presented above and the corresponding questionnaires could contribute to specifying better some reasonable teaching goals about differentials :

- make explicit and salient the types of situations where differentials are needed,
- put approximation at the center of the notion and lead students to master this aspect,
- give a functional content to the notion and help students to reconcile the "contribution view" with the variational one.

We propose these questionnaires as tools to make teachers sensitive to the failures of the present teaching and to these possible goals. In a more technical perspective, we are now elaborating a classification of criteria in order to get validation in the simplest possible way for the types of situations most commonly used. This also, could hepl teachers in any attempt to improve students' mastery of rigor in this domain.

A last remark : the kind of goals we suggest here for teaching, obviously, does not uniquely concern differentials. But this is beyond the scope of this paper.

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SOME PERSPECTIVES ON THE TEACHING OF MATHEMATICS AND NATURAL SCIENCES IN AN ENGINEERING CURRICULUM

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1. INTRODUCTION

At the last conference on "Misconceptions in Science and Mathematics" [1] many authors identified specific problem areas within various courses in which misconceptions arise. Engineering, in this case specifically electrical and electronic engineering, has not been spared this experience. Recently two examples relating to signal processing [2] and electromagnetics [3] received attention in this regard.

Apart from the specific problems cited in the last two references, both of these fields rely on mathematical skills which are usually established by way of prerequisites. Electromagnetics is generally regarded as one of the most difficult courses in the engineering curriculum [3] and relies on both the mathematics as well as the physics prerequisite courses as a basis for further development.

Unfortunately it is at the level of interaction between the prerequisite basic science and mathematics courses and the engineering curricula that another misconception arises as far as many students are concerned. The specific problem relates to the value and relationship of these prerequisite courses to the aspiring engineers' future academic and professional career.

The purpose of this paper is to draw attention to this important interface between engineering and mathematics and the natural sciences as regards the education of engineers. Hopefully this aspect will merit further study by various educators and lead to continual improvement in the development of the mathematical and analytical skills of engineers.

Before proceeding further it is instructive to see how the Accreditation Board for Engineering and Technology (ABET) for example, views mathematics and basic sciences in relation to courses which are subject to review and accreditation.

2. MATHEMATICS AND BASIC SCIENCES IN ENGINEERING CURRICULA

In its 1985 Annual Report [4] ABET details the criteria with which engineering programs in the USA must comply in order to qualify for accreditation. (South Africa has a similar requirement administered by the South African Council for Professional Engineers. Since the ABET information is readily available to most potential readers of this contribution it is used instead).

It is stated in [4] that "Engineering is that profession in which a knowledge of the mathematical and natural sciences gained by study, experience and practice is applied with judgment to develop ways to utilize, economically, the materials and forces of nature for the benefit of mankind." Implicit in this statement is the importance of mathematics and the natural sciences coupled with insight and experience for engineering curricula.

In the case of mathematics, for example, it is expected that studies will go "beyond trigonometry and emphasize mathematical concepts and principles rather than computation." The scope of these studies includes differential and integral calculus and differential equations with additional work in one or more of probability and statistics, linear algebra, numerical analysis and advanced calculus.

The purpose of studies in the basic sciences is described as "to acquire fundamental knowledge about nature and its phenomena, including quantitative expressions." Usually these studies will include both general chemistry and general physics with at least a two-semester sequence of study (or its equivalent) in either.

In addition various engineering curricula may require a further course or courses in the life sciences or earth sciences, or advanced courses in physics or chemistry to meet the requirements of specific engineering curricula.

A four-year course of study in electrical, electronic or computer engineering thus typically requires the equivalent of one year or more of an appropriate combination of mathematics and the basic sciences or a minimum of 16 semester credit hours for each of mathematics and the basic sciences.

In addition students are also required to take one year or 32 semester credit hours in so-called engineering sciences. These sciences "have their roots in mathematics and basic sciences, but carry knowledge further toward creative application. These studies provide a bridge between mathematics/basic sciences and engineering practice." The subjects include, amongst others, mechanics, thermodynamics, electrical and electronic circuits, etc. The key here is the question of creative application and this determines the ultimate course content. Usually (at US Universities) at least one course will be outside the major discipline area.

There can thus be no doubt of the importance of mathematics and the basic sciences for engineering curricula. It is thus imperative that the students must be made aware of this. Further courses within the various engineering curricula must be carefully structured to integrate the required mathematics and basic science material in order to form a coherent learning experience. Unfortunately this is not always the case.

3. STUDENTS' ATTITUDES AND PERSPECTIVES

What follows describes the observations of the author and a number of his colleagues in the Department of Electronics and Computer Engineering at the University of Pretoria over a number of years.

The faculty are left with the distinct impression that most engineering students view getting a degree as the major objective of attending a university. The acquisition of an education as such often appears to be a secondary consideration. In this sense they appear to differ little from their US counterparts as reported in Time [5] after the release of the report "College: The Undergraduate Experience in America" [6]. To quote the words of a colleague with many years of teaching experience "The student is the only consumer who wants the least for his money." While this highlights an attitude which appears to be quite common in our and other societies, there are other factors which need to be considered before accepting this statement at face-value.

The teaching staff have indicated concern regarding the high failure rate amongst students, the lack of motivation, the unwillingness to undertake selfstudy or to use the library facilities, and the apparent desire to reduce the work load.

Perhaps our system of education and we, the teaching staff, must bear responsibility for some of the factors which give rise to the symptoms above. These factors include:

A. high work load - In order to compare workload at the

University of Pretoria with typical US norms an attempt was made to convert the course units to equivalent US semester credit hours. The assumptions made were:

(i) Fourteen weeks per semester.

(ii) Students are expected to spend twice the duration of lectures in selfstudy and preparatory work.

(iii) Because practicals are incorporated in the courses and students have to prepare for the practicals and write reports on these, at least the same amount of time as is spent in the practical is assumed for this preparation.

(iv) The total student hours (lectures plus practicals and selfstudy) was divided by 40 to approximate the equivalent US semester credit hours. Over a four year course of study with two semesters per year this totals some 174 semester credit hours at this University compared with the ABET requirement of 134 for an equivalent engineering curriculum. An important difference is that there is not the same requirement for a half year social studies equivalent as at US universities.

B. The number of courses range from a low of 5 in the second semester of the 1st (freshman) year in the existing curriculum to a high of 8 in the 2nd semester of the 2nd (sophomore) and both semesters of the 3rd (junior) year. The number of semester credits per semester, however, stays very nearly constant over the 4 years of study. The students' progress is evaluated by means of two tests in each course and an examination in each course after the end of the semester. From the time that tests start to a few weeks before the end of the semester the students write roughly two tests per week. This has the undesirable effect that the students' efforts are usually geared solely to the passing of the tests and not to a cohesive learning experience. This is clearly evident in the poor class attendance on days when tests are written, the failure to do homework and the lack of interaction between students and lecturers in class.

C. Because of the number of separate courses the curriculum is fragmented and the overall view and ultimate objective of the course is obscured. This in turn leads the students to view each course as a rounded module rather than a building block which is an integral part of a structured curriculum.

D. The students experience considerable frustration in switching between several subjects during an evening while preparing homework, consolidating the day's lectures or preparing for the next day's work.

E. An unexpected factor was found during the development of a new curriculum in electronics and computer engineering at this university. This was the often erroneous assumption by lecturers that topics which recurred during various courses had been adequately treated in some earlier course which was regarded as a prerequisite. The students often did not acquire sufficient insight into these topics as a result of this assumption. The topics include, for example, Thevenin's and Norton's theorems [9], the application of complex numbers, phasors, functions of complex variables, conformal mapping and calculus of residues, Fourier series and integrals, convolution, etc.

Given the above set of circumstances it is hardly surprising that the students do not adequately perceive the importance of mathematics and the basic sciences for engineering curricula or appreciate that these ultimately provide them with the computational and analytical tools for engineering.

4. SPECIFIC EXAMPLES

Solomon [7] discusses the question of "thinking in two worlds of knowledge" as observed with grade 9 students. In this discussion the author drew attention to the way in which the students are able to view the same concept in two distinct domains of knowledge, the life-world and the science domain.

A similar situation appears to arise in the case of mathematical concepts as taught as a prerequisite for, for example, electromagnetics, and the same concept as applied in the analysis of electromagnetic problems. The students often visualise a mathematics domain and an engineering domain in which the same basic concepts are somehow perceived to belong to different courses and not really to be related. As an example one could consider vector products or various applications of the "del"-operator. In mathematics the application of the divergence or curl operations seems to be regarded as a somewhat abstract operation with little practical benefit. In electromagnetics, however, they are associated with source terms describing charge and current distributions which lead to estimates of the electric and magnetic fields, the latter through the device of the magnetic vector potential. Harrington [8], for example, discusses the relationship between these mathematical concepts and physical sources. This illustrates, for the engineer at any rate, the importance of being able to relate mathematical representations to physical reality.

Another example relates to the use of surface and volume integrals. In mathematics these are usually treated abstractly as double and triple integrals over two or three variables. In electromagnetics, however, it is frequently required to apply these abstract principles in the evaluation of total charge distribution using the divergence theorem. In this case, for example, the theorem relates the integral of the normal component of the electric flux density over a closed surface to the integral of the divergence of the flux vector field throughout the volume enclosed by the closed surface. The students generally appear to have little problem solving the double and triple integrals once they are set up. However, on the whole they do not visualise the surface and volume increments if an expression is given for the electric-flux vector and they are required to solve both sides of the divergence theorem for a specific geometry.

The students are thus often able to grasp and apply the principles as taught in the mathematical courses correctly; after all they have passed the prerequisite course. However, something goes adrift in their perception of the application of these same principles in the engineering courses. It is as though the prerequisite courses are regarded as "over" and not needed in future work.

Examples from other courses abound. The above two cases should, however, suffice to illustrate the gap which generally exists in the students' ability to grasp the mathematical concepts as taught in the mathematics courses and to apply them to a physical or analytical problem in the engineering courses.

A final point which is of considerable importance in engineering education should be raised. Many students seem to feel that they must convert mathematical formulations to a numerical form as soon as possible. Consequently they tend to lose sight of analytical aspects of a problem and do not realise, for example, that the variable to which they have assigned a numerical value in fact apears in an integral equation. Once this habit has developed it can be difficult to break. It seriously impedes the students' ability to gain analytical perspective on a problem as well as their ability to assess what may be a physically reasonable answer or quantity. This problem is illustrated by an example from simplified array theory.

In the example a number of antenna elements are arrayed with a spacing and relative phasing ($\lambda/4$ and 90° or $\lambda/2$ and 0 or 180° respectively) which in fact permits the student to analyse the problem logically by considering summation of fields for the axial and broadside cases. Without too much effort the student should be able to sketch a reasonable approximation to the polar diagram of the array. The student is, however, required to derive the necessary formula for the array factor and to calculate and plot the polar diagram. Often the calculated results bear no resemblance to those which could be derived using inspection, basic physics and a little logic. This inability on the part of many students to check their efforts by simple physical reasoning in cases where this can be done is a cause for concern. Possibly because of the preoccupation with answers and the ease of getting one using a pocket calculator (even if it is incorrect) students do not easily develop a feel for magnitudes and what consitutes a realistic answer.

5. PROPOSED SOLUTION

From comments in the IEEE Education Society Newsletter [9 - 13] and also in [14] it is clear that the engineering community is deeply concerned with the state of education in the basic science and, in particular, mathematics and the way that courses in these fields are integrated into the engineering curricula. Van Valkenburg [12] quotes from the "The Calculus Tutoring Book" [15] that:

"Mathematicians and consumers of mathematics (such as engineers) seem to disagree as to what mathematics actually is. To a mathematician, it is important to distinguish between rigor and intuition. To an engineer, intuitive thinking, geometric reasoning and physical deductions are all valid if they illuminate a problem, and a formal proof is often unnecessary or counterproductive."

As far as mathematics is concerned Van Valkenburg [10] poses two questions which relate to the successful integration of such courses in engineering curricula. The first is:

"Could we negotiate with our mathematical colleagues to provide more nearly what we want,"

and the second,

"Are we really sure that we know what we want?"

The Department of Electronics and Computer Engineering at the University of Pretoria has recently completed a major review of its curriculum. This was done bearing in mind the student attitudes alluded to earlier as well as possible shortcomings of the old curriculum. A serious attempt was made to rationalise and consolidate the material to be taught and to reduce the total number of courses per semester to five throughout the four years of study. As before, appropriate practical work is incorporated in individual courses. The role and responsibility of the faculty in teaching the proposed courses received careful consideration to ensure teaching excellence as far as is possible.

The problem of concepts which occur and are used in several subject areas was resolved during the process of consolidating the course material. Careful liason between lecturers as regards course content identified these concepts. It was decided to develop these concepts fully and adequately the first time they were encountered. One of the obvious ways in which this could be done is by means of worked examples which are covered in considerable detail. Concepts are only firmly established once the students have grasped them, not when the lecturer has explained them. Subsequent use should for most students require only a very short review by the lecturer to re-establish the topic and a small amount of selfstudy to refresh the student's memory.

A survey of the requirements of basic physics topics as prerequisites for specific courses showed that a two semester sequence such as that given in most popular first-year physics texts was regarded as adequate. It was decided that in later engineering courses the connection to previous physics topics could best be made by a more careful discussion of these topics when they occurred. Several examples such as oscillatory motion, electro- and magnetostatics as prerequisites for later courses on electromagnetic theory, geometric and physical optics and propagation of light for courses in electro-optics; kinematics, linear and rotational dynamics as prerequisites in control theory courses; etc., spring to mind. It was also felt that aspects of modern physics should be taught when appropriate in courses such as semiconductor electronics. It is fortunate that this engineering department has many lecturers well qualified to do this.

In the case of mathematics, the department received the whole-hearted support of the Department of Mathematics and Applied Mathematics. The various mathematical topics needed for prerequisites in the engineering courses were identified by the engineering lecturers. The Department of Mathematics and Applied Mathematics used this information as the framework for the design of courses to meet the particular needs of the Department of Electronics and Computer Engineering at the University of Pretoria. In view of the fact that the mathematics department offers service courses for all engineering departments, it investigated the needs of those departments as well. Eventually a consolidated set of courses meeting the needs of the college of engineering as regards prerequisite courses in mathematics, was proposed. One small problem arose and this related to the preference for specific high-level computer languages by various departments. This was in turn resolved by the Mathematics Department's being prepared to offer two parallel courses in which the teaching material is slanted to the use of specific computer languages.

Indicative of the level of co-operation was the willingness of the mathematics department to use, where possible, examples appropriate to the various fields of engineering. It would be unwarranted interference on the part of the various departments of engineering to prescribe the textbooks to be used in the mathematics course. However, the mathematics department has decided to investigate the use of appropriate textbooks with more of an engineering flavour in the prerequisite courses.

This process of streamlining the Department of Electronics and Computer Engineering's curricular requirements has resulted in a drastic reduction of the number of semester credits required from 174 to about 140. It is believed that this restructured course will give rise to greater motivation of both sutdents and staff and ultimately produce a better "product", namely an adequately trained graduate engineer.

6. CONCLUSIONS

This short communication has addressed the problems which engineering students have in appreciating the relevance of the basic science and mathematics prerequisite courses in respect of their engineering education. It is at this stage based on observations and discussions which many of the lecturers have had with students rather than on an evaluation of opinion surveys. Much valuable insight has been gained from the junior members of staff who are able to establish a rapport with the students more readily than the older, more senior members. An attempt has been made to put the problem in perspective as regards work load and a number of other factors.

A possible solution requires that the teaching staff critically examine the curricular content on a regular and ongoing basis with a view to the final objective and how this objective can best be achieved. An important factor here is the requirement that the course structure be seen as an integrated unit in which treatment of recurring topics under the assumption that these have already been covered in some earlier course, does not occur. A serious effort must be made to treat such topics fully and adequately when first encountered.

It is obvious that the faculty must at all times remain aware of their roles and responsibilities as teachers and continue to strive to improve the quality of teaching.

Of major importance is a close liaison with the basic science and mathematics departments offering the service prerequisite courses which are regarded as essential for engineering education. It is hoped that this paper will stimulate educators in the basic sciences, mathematics and engineering to pay more attention to the critical interface between their respective disciplines. If this is done it is believed that much can and will be achieved in making the attainment of a degree in engineering a rich and rewarding learning experience rather than the obtaining of a piece of paper which almost guarantees a good job. This can only be to the benefit of the student, the teacher and ultimately society as a whole.

ACKNOWLEDGEMENTS

The author wishes to acknowledge the many fruitful discussions he has had with colleagues in the Department of Electronics and Computer Engineering at the University of Pretoria. Their concern with the continual improvement of engineering education and definition of attainable objectives and standards sets an excellent example.

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MAGNET CONCEPTS AND ELEMENTARY STUDENTS' MISCONCEPTIONS

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> "Well, a they's [SIC] one things that's inside it. It's like a type of gravity and it makes the magnet pull." Andy, grade 2.

The above response to why a magnet works from an eight year old boy who had studied magnets the previous year illustrates that just teaching about a topic will not result in the child understanding the concepts. Previous studies have reported elementary students misconceptions about weather (Stepans and Kuehn, 1985) and circulatory systems (Armaudin and Mintzes, 1986). It has been well documented that students often have misconceptions about various science concepts prior to instruction (Eaton, Anderson, and Smith, 1983) but at the First International Conference on Misconceptions (Helm and Novak, 1983) there were only two papers focusing upon misconceptions at the elementary level.

The emphasis with conceptual change teaching strategy is requiring the reorganization of a child's existing knowledge and the construction of new knowledge (Posner, Strike, Hewson, and Hertzog, 1982). At the community college level, Hynd, and Alvermann (1985) reported that a refutation text (contrasts correct ideas with incorrect ones) was an effective way of getting students to change their prior misconceptions about the principles of motion. However, elementary teachers are rarely aware of students' misconceptions. Smith and Neale (1987) reported on an indepth inservice training for 10 K-3 teachers on the concepts of light and shadows. This training component was similar to Apelman (1984).

A literature search was conducted to identify science education publications focusing upon studying magnetism at the elementary school level. Two major sources (Gargliano, 1981 and Henriques and Arnold, 1985) were found. Concepts 7 and 8 added a science/technology/society focus. The concept list was validated by a panel of three science educators and two physists. Figure 1 is the listing of eight magnet concepts for a K-6 continuim.

A group of 28 K-3 teachers interviewed a total of 78 K-6 students. In this pilot study, there was no attempt to randomly select subjects; therefore, there was considerable variation between grade levels, gender, and previous amount of studying about magnets. The purpose of the assignments was to provide the K-3 teachers with an awareness of misconceptions students have within their cognitive structure.

Results

Qualitative

All students identified that their family utilized magnets to "stick them on our refrigerator to hold all our papers up" (Andy, grade 2). A few students mentioned cabinet and refrigerator doors, compass, flashlight (sticks to refrigerator), pick up nails/pins, dryer door, clean aquarium from the inside by moving the magnet on the outside, and can opener as to what a magnet was used for in their home.

Regardless of whether they had studied about magnets, all 78 students were aware of magnets.

In response to the question to explain how magnets work, there was considerable variation in the student's responses. The most frequent response was "I don't know". Students who had not studied about magnets responded "chemicals in them makes them stick" (Dana, grade 6) while Solm (grade 5) "...they are magnetized. It is confusing that they don't stick on (teeth) braces and they are black." Even though Solm had not formally studied about magnets he had played with different ceramic magnets.

From students who had studied about magnets the responses were diverse. "It's like a type of gravity and it makes the magnet pull." (Andy, grade 2). "Gravity pulls things together" (Della, grade 6). "There are two poles - north and south. If the north side hits a north side of another magnet it won't work. It has to be a north side would [sic] to hit the south side of another pole to stick." (Andy, grade 4). "Magnets just stick together when they're close together, the energies make them go together." (Lee, grade 6). "They are just magic." (Mary, grade 4). "Electrons in one, protons in another, they attract." (Peter, grade 6). Nick, grade 5, had a very detailed response: "Well, there's several theories and nobody knows. One theory is that there's little like small magnets like molecules. One seeks north and one seeks south but, and when a magnet that doesn't work, the north and south are facing all messy. If you stroke a magnet among it, then it will get all of them facing north and south... ." "There are wires in them and when put with another one it will magnify it" (Kit, grade 2).

Quantitive

Tables 1-4 contains the frequency for females and males who had and had not studied about magnets. A total of 43 K-6 students had studied about magnets while 35 had not studied about magnets. It is interesting to note that 16 of the 35 who had not studied about magnets were in grades 4-6. Only concepts 3 and 8 had more than 25% of the K-6 students respond accurately. Less than 10% of the students had understood concepts 1 and 6.

Comparing the female and male responses for those who had studied about magnets, males were superior on 5 of the 9 concepts, females on one concept, and there was less than a 5% variation for two concepts. In contrasts to students who had not studied about magnets, males were superior on only one concept, females on two concepts, and there was less than a 5% variation for five concepts. Comparing studied with not studied group, males who had studied were superior on 7 of the 9 concepts with the first five being greater than 5%. The not studied males exceeded studied group on concept 8 by more than 10%. For females, the studied group exceeded the not studied group on four of the concepts with three being greater than 5%. Three of the concepts had the not studied group exceeding the studied group with two being greater than 5% difference.

Specifically for concept 1, of the seven individuals of the studied group who had misconceptions; six were males. Regarding concept 2, 20 of the studied group and 21 of the not studied group had misconceptions. Concept 3 had 12 individuals of the studied group who had misconceptions 10 were males. The two individuals who were aware of electromagnets were both sixth graders. Concept 8 had 5 individuals of the studied group with misconceptions, 4 being females.

Discussion

The results of this pilot study raises more questions to be researched. There is a need for a systematic study of elementary students naive conceptions regarding magentism, influence of mental structures as a result of instruction, teachers personal knowledge about magnetism, gender variation, textbook orientation and utilization, etc. This is a viable science education topic to research since magnets are a common topic in all elementary science textbook series.

Comparing the studied with the not studied group; there appears to be a change in understanding of the concepts. However, the majority are less than a 30% change. Surprisingly, three concepts were higher for the not studied group. For example, for concept 2 for females, it is possible that their personal experiences in the home such as can openers, etc. influenced their response. Concept 4 had only one female who answered correctly. Both males and females of the not studied group had greater knowledge

concerning concept 8 than the studied group. It appears that home experiences with magnets are being ignored in textbooks and/or teaching. Further more, it appears that what is taught in school is more theoretical rather than the practical. Therefore, there could be a view by elementary students that magnet instruction lacked personal relevance (Pope and Gilbert, 1983).

Concept 1 concerned poles of a magnet. The majority of the misconceptions were that the poles are only on the ends of a magnet. Typically horseshoe magnets were mentioned. Textbooks have ignored ceramic magnets because the poles are typically at other surfaces. Consequently, textbook and instruction about traditional magnets are different from practical home experiences. Textbooks should define poles as the area of the magnet where there is the strongest attraction/repelling.

Regarding concept 3, elementary students who had misconceptions tended to be unaware or recall repelling. It appears that if they did have "hands-on" experiences that it was only with one magnet. Consequently, it would be much easier to observe attraction than repelling.

Concepts 4 and 5 only had males demonstrating comprehension. It appears that these concepts are sex biased or taught in a sex-biased orientation. Concept 6 is considered to be the most advanced of the list. The two individuals who demonstrated competency were both sixth graders. Before students would be able to understand this concept, they must have studied electricity (i.e., batteries and bulbs). Concept 7 was the only concept in which females outscored males for both studied and not studied groups. This could be a sex-biased concept which has more of an orientation of gross features rather than its active mode (magnets in action).

This study provides additional support to Osborne and Cosgrove's (1985) conclusion that elementary students conceptions are quite different from scientists. Dykstra (1986) raised concern that there are occasions when younger students can not comprehend the scientific concepts. Wandersee (1986) and others have reported that the history of science provides insights to help science teachers understand misconceptions of their students. Treagust (1985) diagonostic text model is applicable for older students but not elementary students. Stepans (1985) identified that introducing a concept too early was the major cause for student's inability to understand a concept. Researchers should not ignore the massive Piagetian studies while pursuing misconceptions research. By providing students with a variety of situations involving magnets, students will be able to expand their conceptions.

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Code Number	Concepts
1	Magnets always have two poles where they exert the greatest force.
2	Iron materials are attracted to magnets.
3	When magnets are brought together, unlike poles attract and like poles repel.
4	Magnet has a force field which goes through things.
5	Compass points to magnetic north.
6	Moving electric current creates a magnetic field/ electromagnet.
7	Magnets come in a variety of sizes and shapes.
8	Magnets have a variety of uses.

Figure 1
Magnet concepts expected from K-6 students.

Table 1
Female Responses Who Studied Magnets

Grade	Concepts 1	2	3	4	5	6	7	8
Kindergarten								
Understood	0	0	0	0	0	0	0	0
Misconception	0	0	0	0	0	0	0	0
Did Not Mention	0	0	0	0	0	0	0	0
First Grade								
Understood	0	0	0	0	0	0	0	0
Misconception	0	1	0	0	0	0	0	0
Did Not Mention	1	0	1	1	1	1	1	1
Second Grade								
Understood	0	1	0	0	0	0	1	2
Misconception	0	0	0	0	0	0	0	0
Did Not Mention	3	2	3	3	3	3	2	1
Third Grade								
Understood	0	0	0	0	0	0	0	2
Misconception	0	2	0	0	1	0	0	0
Did Not Mention	2	0	2	2	1	2	2	0
Fourth Grade								
Understood	1	1	2	0	0	0	0	0
Misconception	0	1	2	0	0	0	0	4
Did Not Mention	3	2	0	4	4	4	4	0
Fifth Grade								
Understood	0	0	1	0	0	0	1	2
Misconception	0	2	0	0	0	0	0	0
Did Not Mention	2	0	1	2	2	2	1	0
Sixth Grade								
Understood	0	0	1	0	0	1	1	2
Misconception	1	2	0	0	0	0	0	0
Did Not Mention	2	1	2	3	3	2	2	1
Total								
Understood	1	2	4	0	0	1	3	8
Misconception	1	9	2	0	1	0	0	4
Did Not Mention	13	5	9	15	14	14	12	3

Table 2
Male Responses Who Studied Magnets

Grade	Concepts 1	2	3	4	5	6	7	8
Kindergarten								
Understood	0	0	0	0	0	0	0	1
Misconception	0	1	0	0	0	0	0	0
Did Not Mention	1	0	1	1	1	1	1	0
First Grade								
Understood	0	1	0	0	0	0	0	0
Misconception	0	0	1	0	0	0	1	1
Did Not Mention	2	1	1	2	2	2	1	1
Second Grade								
Understood	1	3	2	3	0	0	2	8
Misconception	0	6	6	0	0	0	0	0
Did Not Mention	8	0	1	6	9	9	7	1
Third Grade								
Understood	0	0	0	0	0	0	0	0
Misconception	0	1	1	0	0	0	0	0
Did Not Mention	1	0	0	1	1	1	1	1
Fourth Grade								
Understood	1	2	3	2	0	0	0	3
Misconception	1	0	0	0	0	0	0	0
Did Not Mention	2	2	1	2	4	4	4	1
Fifth Grade								
Understood	0	1	3	1	0	0	0	4
Misconception	2	1	1	0	0	0	0	1
Did Not Mention	3	3	1	4	5	5	5	0
Sixth Grade								
Understood	0	1	2	0	4	1	1	5
Misconception	3	3	1	1	0	0	0	0
Did Not Mention	3	2	3	5	2	5	5	1
Total								
Understood	2	8	10	6	4	1	3	21
Misconception	6	12	10	1	0	0	1	2
Did Not Mention	20	9	9	21	24	27	24	5

Table 3
Female Responses Who Had Not Studied Magnets

Grade	Concepts							
	1	2	3	4	5	6	7	8
Kindergarten								
Understood	0	0	0	0	0	0	0	1
Misconception	0	0	0	0	0	0	0	0
Did Not Mention	1	1	1	1	1	1	1	0
First Grade								
Understood	0	1	0	0	0	0	0	4
Misconception	0	1	0	0	0	0	0	0
Did Not Mention	4	2	4	4	4	4	4	0
Second Grade								
Understood	0	1	0	0	0	0	1	0
Misconception	0	2	0	0	0	0	0	1
Did Not Mention	4	1	4	4	4	4	3	3
Third Grade								
Understood	0	0	0	0	0	0	0	2
Misconception	0	2	1	0	0	0	0	0
Did Not Mention	2	0	1	2	2	2	2	0
Fourth Grade								
Understood	0	0	0	0	0	0	0	4
Misconception	0	4	1	0	0	0	0	0
Did Not Mention	4	0	3	4	4	4	4	0
Fifth Grade								
Understood	0	1	0	1	0	0	2	3
Misconception	0	1	0	0	0	0	0	0
Did Not Mention	3	1	3	2	3	3	1	0
Sixth Grade								
Understood	0	1	0	0	0	0	1	3
Misconception	0	2	0	0	0	0	0	0
Did Not Mention	3	0	3	3	3	3	2	0
Total								
Understood	0	4	0	1	0	0	4	17
Misconception	0	12	2	0	0	0	0	1
Did Not Mention	21	5	19	20	21	21	17	3

Table 4
Male Responses Who Had Not Studied Magnets

Grade	Concepts							
	1	2	3	4	5	6	7	8
Kindergarten								
Understood	0	0	0	0	0	0	0	1
Misconception	0	1	0	0	0	0	0	1
Did Not Mention	2	1	2	2	2	2	2	0
First Grade								
Understood	0	0	0	0	0	0	0	3
Misconception	0	2	0	0	0	0	0	0
Did Not Mention	3	1	3	3	3	3	3	0
Second Grade								
Understood	0	0	0	0	0	0	0	3
Misconception	1	2	0	0	0	0	1	0
Did Not Mention	2	1	3	3	3	3	2	0
Third Grade								
Understood	0	1	0	0	0	0	1	3
Misconception	0	3	0	0	0	0	1	0
Did Not Mention	4	0	4	4	4	4	2	1
Fourth Grade								
Understood	0	0	0	0	0	0	0	0
Misconception	0	0	0	0	0	0	0	0
Did Not Mention	0	0	0	0	0	0	0	0
Fifth Grade								
Understood	0	0	0	0	0	0	0	2
Misconception	1	1	2	0	0	0	0	0
Did Not Mention	1	1	0	2	2	2	2	0
Sixth Grade								
Understood	0	0	0	0	0	0	0	0
Misconception	0	0	0	0	0	0	0	0
Did Not Mention	0	0	0	0	0	0	0	0
Total								
Understood	0	1	0	0	0	0	1	12
Misconception	2	9	2	0	0	0	2	1
Did Not Mention	12	4	12	14	14	14	11	1

MISCONCEPTIONS AMONG PUPILS REGARDING GEOMETRICAL OPTICS

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1. AIM AND BACKGROUND OF THE RESEARCH

During the last decade Dutch physics education undergoes changes tending to put the theoretical subjects more in practical situations. As a result of this a committee (WEN) was brought into existence,that should revise the examination programs in Dutch high schools and bring it into line with the general trend of thinking.

In one of its reports the committee brings upon a conceptual program, in which to each subject one or more contexts are added, meanwhile reducing the number of subjects in the syllabus. Especially the time, that has to be spent on the subject of optics was reduced considerably. For example , combinations of two or more lenses were deleted, which excludes traditional applications such as the microscope and the telescope.

This strong reduction of the number of subjects in optics is regrettable, because on the other hand optics has gained many new applications, such as those in the communication field (glass fibre techniques), information technology (optical computers), audiovisual equipment (video long-player and compact disc-player) and the numerous applications for the laser (medical systems).

Due to the way that optics has been taught during the last few decades, tradition is slowly bleeding it to death: the mathematical construction of ray-tracing, Snell's law and the lens formula are so highly exalted in optics that they threaten to isolate pupils from its practical applications. Until now, too strong an emphasis on mathematical abstraction has brought about a disregard of everyday optics, thereby, reducing the general relevancy of the subject which, apparently, justified WEN partially eliminating it from the examination program.

Apparently so, because the new applications mentioned above show us clearly that optics is still a branch of physics that is important and it is not to be treated lightly. Indeed looking at the traditional curriculum, mathematical descriptions should be emphasized less or at least treated as the basis of physical phenomena and technological applications.

Research into optics education at the Eindhoven University of Technology in the Facultyof Technical Physics and Department of Physics Education is intended to contribute to improving the situation:

-by investigating the misconceptions of pupils regarding geometrical optics in pre-university schools and general high schools.

-by writing a review syllabus for geometrical optics.

-by developing a curriculum for thematic education based on the optical system of the compact disc-player, in which wave optics will play a major role.

Misconceptions can be seen as a part of the pupils' initial learning process at the beginning of their education. Thus, when defining the starting point of the new curricula, they must be given as much importance as the examination program gets at the end. Although investigating misconceptions can be regarded as pre-research for the development of both the review syllabus and the thematic curriculum, their connection with the review syllabus is even closer, because it is intended to be a strategy for attacking misconceptions: based on the idea, that teaching fundamental principles and the more abstract subjects, at the same time disposing of misconceptions that still exist according to the investigation should not be done when pupils contact the subject for the first time, but afterwards when the first phenomenologic description has sunk into their minds.

The review syllabus and the thematic curriculum together should cover the examination subjects as fully as possible, so that teachers can replace the traditional optics curriculum entirely without being distressed by the idea that they will have to select or add subjects from other curricula.

II. THE INVESTIGATION OF MISCONCEPTIONS

A. INTRODUCTION.

The flow diagram of the investigation is shown in Figure 1. After the pilot-study in September and October 1986 (Section IIB) which had the purpose of assembling as many known misunderstandings as possible, a list of misconceptions was constructed (IIC) in November. This list was the starting point for a questionnaire, (IID) which was developed in December 1986 and January 1987. In February this questionnaire was tested with a sample of pupils, who were asked to give their answers verbally. After revising the questionnaire, it was given to a larger group in March. Data has been put into the computer and will be analyzed by SPSS; the first results are coming out now (May 1987).

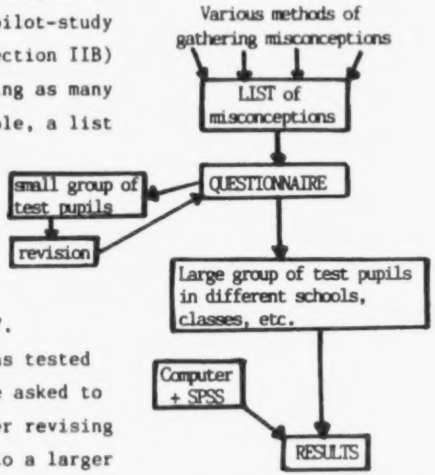

Various methods of gathering misconceptions
LIST of misconceptions
QUESTIONNAIRE
small group of test pupils
revision
Large group of test pupils in different schools, classes, etc.
Computer + SPSS
RESULTS

Figure 1.: flow-diagram of the investigation.

B. PILOT-STUDY.

In the preparatory phase of the investigation, it was most important to construct a list of as many misconceptions as possible so as to get an outline of the research field. At a later stage a selection from this list could produce theoretical variables for the pupil's questionnaire.

In brief, four methods were used to gather the misconceptions:
1/ by taking stock of those from previous investigations, in particular the work of Licht, Wubbels and Walravensteyn in The Netherlands, Andersson and Kärrqvist in Sweden, Larosa in Italy, Stead and Osborne in Australia, Jung and Wickihalter in Germany, Guesne in France, Salomon, Watts, Goldberg and McDermott in the United States of America.

2/ by interviewing pupils of pre-university schools using very open-ended questions or asking them to give their own descriptions and explanations of everyday optical phenomena in order to indicate the nature of their conceptions and misconceptions. The interviews were recorded on tape and analyzed afterwards.
3/ by taking a critical look at the historical development of optics. The pupils can be expected to have misunderstandings about subjects, that also caused blocks to development for the physicists, producing the light theory. The wave-particle-duality could be mentioned here as a stereotype, because it took physicists many years to create any clarity on this issue, so pupils would also have difficulty in understanding this subject.
4/ by reflecting on the experience gained from optics lessons in classrooms. 'Stupid' questions of pupils often give rise to thinking over the teaching of the subject and often the 'stupid' question would turn out to be a consequence of an underlying misconception, whose existence was not even taken into account by the teacher.

C. LIST OF MISCONCEPTIONS.

The overall outcome of the various methods for gathering misconceptions was scheduled into a list that consisted of around seventy misunderstandings comprising six categories:
1/ Notions about the *nature and the properties of light*; for example:
-light is a purely static phenomena: like air it fills up space, if there is much of it , it is bright, otherwise it is dark.
-light is identified with its source (lamp or sun) or with its effect (a spot of light on a wall).
It is not surprising that pupils are confused about the nature of light, because even history contains an infinite range of hypotheses, falsifications and theories about this subject, starting with Aristoteles' ideas and continuing up to the wave-particle-duality of this century.
2/ *Rectilinearity and dynamic properties of light*; for example:
-the distance, light can travel is limited by the extent of its visible effect (only a few meters unless it is a very bright light source like the sun).
-light travels at an infinite speed (a room is illuminated entirely

at the same time as when the light source is switched on), or has no speed at all (it is present somewhere or not).

-light can bend round a corner, hence a room with only a small window is illuminated entirely and not just the narrow strip in front of the window itself.

Andersson in Sweden and Larosa in Italy made further investigations into these topics. In history, it was Newton who became first aware of the finite speed of light and it was not until 1854 that Foucault was able to measure its velocity accurately.

3/ *Interaction of light with matter or objects:* for example:

-light rays can be seen from a distance if they are strong enough, even if they are not scattered by a cloud or some other object.

-confusion about the difference between specular and diffuse reflection.

-light 'contains' warmth and transfers it to any object, that it hits.

In particular, Guesne looked at the interaction between light and matter. Also in the preliminary interview, pupils turned out to be rather confused about the difference between specular and diffuse reflection, as well as, about the fact rays cannot be seen unless they are scattered by dust particles or vapour droplets.

4/ The concept of *vision.*

The dominating misconception in pupils' minds is the decoupling between light and vision: although they know that light near an object is the minimum condition for seeing it, they do not think it necessary for light rays coming from the object to enter the eye. Most of the investigators previously mentioned inquired into the concept of vision, often reporting things similar to this decoupling concept. In the preliminary interviews too a significant confirmation was observed: a girl pretended that she could see a fluorescent lamp burning many hundreds of meters away although the distance should be too large to travel along for a light ray coming out from the lamp.

5/ *Colour*, for example:

-objects can be seen only when they have a colour different from the background.

-all colours together form a range of light intensities from black to white.

-colour is purely a property of an object, not of the light itself.

-light passing through coloured glass, is 'painted' by that glass; therefore, it must be effected by some sort of pigment.

Andersson, Larosa and Jung studied pupils' concepts of colours in a more searching inquiry.

Most of the pupils' misconceptions about colours relate to their inability to distinguish physical colour properties in the environment from physiological properties of the colour perception by the brain. Five-year-old children can recognize colours and associate them with words that their parents taught them, but they cannot distinguish the sensation produced by colours in the mind from purely physical characteristics of the light itself.

6/ *Formation of an image,* for example:

-the location of an image behind a plane mirror.

-the understanding of a virtual image.

It will be the experience of every physics teacher that image formation is an abstract subject and it is very difficult to explain. It cannot be emphasized enough that the quintessence of image formation lies in the fact that every light ray that eminates from a light point and strikes an optical instrument (mirror or lens) will pass through the same image point. Hence most pupils think in terms of a 'one-to-one journey' from light point to image point by only one ray and do not see the particular thing that happens when many rays join at exactly the same point. This idea of the 'travelling' image without firstly being separated and then recollected seems to be a rather persistent one and it appears in many different situations. Pupils having this idea will answer incorrectly to the traditional question: 'what happens to an image if half of the lens is covered by a dark material ?' In fact, they make the same mistake if they describe the slide projector as just a linear enlarger. A quite extreme appearance of this misconception is the idea that in the middle between an observer and an object there has to exist an image half the object size according to the laws of perspective (see Figure 2).

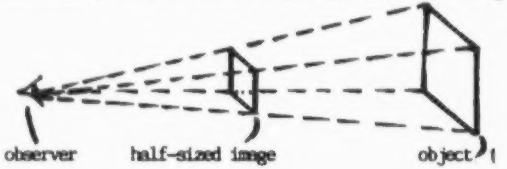


Figure 2.: Illustration of the misconception of the 'travelling image'.

D. DEVELOPMENT OF THE QUESTIONNAIRE.

1. Considerations about methodological aspects.

The list of misconceptions, described in the previous section, was the basis for developing the pupils' questionnaire. In particular, misconceptions that could be tested easily in a written examination and could be expected to occur among pre-university high school pupils were selected and presented in one or more statements of the questionnaire. For further considerations regarding the contents see section 2. A translated version of the questionnaire is included as an appendix of this paper. (Note: Available from author)

Firstly, some choices had to be made concerning the methods of research: for example, should it be a verbal or a written investigation. Many advantages and disadvantages had to be taken into account. Interviewing is an arduous method of investigation but, on the other hand, it can provide a complete picture of the pre- and misconceptions in a pupil's mind; using their own words and explanations of optical problems, pupils can freely describe their ideas without too much interference from interviewers. One problem of the interview method was trying to analyze the results systematically. It is difficult to categorise the rather haphazard remarks of pupils and to evaluate them statistically. So a report of an interview method of investigation on this subject can be little more than a collection of remarks made by certain pupils.

For a purely practical reason, the much less laborious written method of investigation is easy to use for a large group of pupils; also, it brings about the possibility to make a more scientific report from a statistical viewpoint.

Once it was decided to use a written questionnaire, the next issue was whether the questions should be open-ended or not. In a certain sense, open-ended questions have similar advantages and disadvantages to an interview: once again, it allows the pupils to express their ideas freely but, on the other hand, it is difficult to assess all their free expressions. Besides, with open questions, language difficulties interfere with the real misconceptions. Pupils who are not able to express their conceptions are easily accused of having misconceptions. This problem does not occur and data is more reliable with closed questions, i.e. multiple-choice questions or statements, where the pupils have to decide which is right and which is wrong.

Finally, the choice was made to use a questionnaire consisting of 42 statements which the pupils had to decide which was right and which was wrong on a five-point scale with the following meanings:

1. You definitely think that it is right.
2. You think that it is right.
3. You do not know or you are not sure it is right on every occasion
4. You think that it is wrong.
5. You definitely think that it is wrong.

There is much to be said for the central meaning of the scale. Strictly speaking there is a big difference between 'not knowing' and 'thinking that a statement is not right on every occasion', this difference seems to be too subtle to be recognized at the abstraction level of pupils. In the interviews, pupils were often searching for arguments to make a decision in a rather arbitrary way; so that, 'not exactly knowing' is similar to 'not being able to recognize and distinguish situations in a proper way', while in fact, these two meanings of the central point of the scale are in practice much closer to each other than when it is considered purely methodologically.

2. Considerations regarding the contents.

The 42 items can be divided into five main subjects, analogous to the six categories of the list of misconceptions, excluding 'colour'.

For each subject, two statements were made in a purely theoretical way and the pupils were confronted with 10 'theoretical' items, before they had to deal with some practical items later in the questionnaire. In the 32 'practical' items the pupils were faced with problem situations, illustrated by photos and drawings. This division into two sorts of items took into account that there might be a discrepancy between pupil's theoretical knowledge of a subject and his ability to use it in practice. For example, a pupil remembering from the optics course that light always propagates rectilinearly might be confused when asked why a room with just a small window will be illuminated completely and not just the strip in front of the window.

The 42 items can be grouped into the following five categories:

1/ Nature and properties of light: 1,2,11,12,13 and to some extent, 24,25,26,27 and 28. Pupils are mainly faced with the problem: is 'light' the same as the light source (1,2,13) or is it the effect of it (1,12) or can it be identified with something between the source and its effect (1,11,25) ?

2/ Rectilinearity and dynamics of light: 3,5,14,15,16,17,19 and 20. In the theoretical items, it was stated that light always propagates rectilinearly (3) for an unlimited distance (5) when there are no impeding objects. Item 14 handled the fully illuminated room with a small window; while, 15 showed a lightning flash suggesting a non-rectilinear light propagation. In 16 and 17, a connection was made between rectilinear light propagation and the origin of shadows. In 19 the distance that light travels was compared in a light and dark environment and in question 20 it was compared with bright or dim light sources.

3/ Interaction with matter or surfaces: 4,9,18,33,34,35,36,37,38 and 39. Items 4 and 18 concerned the visibility of light rays in a completely clear environment, whilst, the others denounced any difference between specular and diffuse reflection.

4/ The vision concept: 6,7,22,23 and to a certain extent 24,25, 26, 27 and 28. In these items the major misunderstanding of sight is tackled: the decoupling concept. Pupils are asked if it is necessary for light rays coming from an object to enter the eye to make the object visible. They are asked directly (6,7) and in problem situations (22 to 28).

5/ Formation of an image: 8,10,21,29,30,31,32,37,40,41 and 42. The rather diverse aspects of image formation are brought together in the context of the plane mirror. Some of the statements were about the location of the image (8,30,31,32 and 37) and how the mirror or the object have to be illuminated (10,40 and 41). Later, there are two items about lenses: the parallel beam of a lighthouse (21) and the location of the real image from a picture in a slide-projector in comparison to a plane mirror (29). Finally, there is an item about the traditional question: is it best to hold a plane mirror further away from you in order to see more of yourself in it ? (42).

3. Theoretical variables to be measured.

Five other characteristics were added to the 42 answers from each pupil in order to form a complete set of rough variables:

-the pupil's school;
-the pupil's year (3rd, 4th and 5th year of the pre-university high school or 4th year of the higher general secondary school).
-the pupil's gender;
-the pupil's age;
-the optics course curriculum, used by the pupil.

This rough data was used for statistical analysis by the SPSS-computer-program.

The aim of this analysis was to obtain values for the following theoretical values:

1/ The actual number of specific misunderstandings among the pupils related to the 42 items in the test-questionnaire; deduced from the mean score and its standard deviation for each item.

2/ The correlation between the certainty of a pupil and the number of misconceptions; certainty being measured by comparing the number of 1's and 5's to 2's,3's and 4's.

3/ The number of misconceptions in a more general sense in each subject category of the questionnaire, deduced from a set of the mean scores in each subject category.

4/ The difference between the theoretical knowledge of a pupil and the way, he used this knowledge in problem situations, which is fairly a common issue in education. A value for this variable was deduced by comparing the theoretical items (1-10) with the more problematic ones (11-42), both in general and for each subject category separately.

5/ The existence of certain alternative frameworks in optics. An essential question of this research was: Do pupils use more or less coherent alternative sets of arguments in their answers or do they reason in a purely arbitrary way ?

It is evident that pupils on their own abstraction level will not use theories as consistently as scientists do: the need to include many different phenomena into one comprehensive principle is expected too much of 16 year-old pupils. However, their desire for uniformity could increase with age and it would be interesting to learn the extent, that an answer to one item implies something in another, because this might have been evident for a scientist, but not for a pupil.

Frameworks might exist within a category of items, but it is also possible that items of different categories correlate better than the average, based on similarities between items less obvious to scientists than to pupils. Therefore, a factor analysis has to be made of all the items together; blocks of items that significantly correlate better than average could give rise to an awareness of alternative frameworks, that pupils use for solving optical problems.

6/ Significant differences between the values of the above mentioned five theoretical variables in seperated groups, which were distinguished to the following criteria:

-school;

-year;

-gender;

-age;

-curriculum.

With respect to the last aspect, a problem could rise: in almost every school, one curriculum was used for a number of years, so, differences between the curricula might interfere with those affected by differences in school characteristics.

Information about these differences can be gathered by doing T-tests on the specific data from each group.

III. RESULTS.

A. CHARACTERISTICS OF THE TEST-GROUP

Ultimately, the questionnaire was tested with 639 pupils in five different schools. Two of these schools were situated in Eindhoven (260.000 inhabitants; school no.1 had 152 pupils and no.2 had 56 pupils), one in a suburb of Eindhoven (school no.5 with 260 pupils) and two were in the extreme south of The Netherlands which is a more rural area of the country (school no.3 with 70 pupils and no.4 with 101 pupils).

The pupils came from four different years:

-third year pre-university high school (14/15 year-old: 56 pupils),

-fourth year general high school (15/18 year-old: 191 pupils),

-fourth year pre-university high school (15/17 year-old: 201 pupils),

-fifth year pre-university high school (15/18 year-old: 191 pupils).

Except for the first group (third year), the pupils have already made a choice of 7 (6 in general high school) out of the 12 available subjects in high schools, so, this test-group was likely to be more interested in the sciences than the average school-children. Unfortunately, this group was not divided equally into girls and boys, 68% of the test-group were boys (433) and only 32% were girls (206).

Each school used a different curriculum, varying from a theoretical one (Middelink) to a practical one (PLON: physics curriculum development project).

The distribution of the pupils' ages are shown in Figure 3. Further information about the composition of the testgroup can be found in the Appendix.

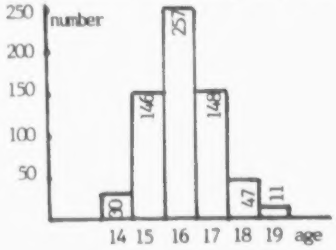


Figure 3.: Age-distribution of the test-group. Mean: 16.11 ± 1.04

B. NUMBER OF SPECIFIC MISUNDERSTANDINGS.

For the mean scores of the whole group, distinguished to school, year age and gender, see Table 1. in the Appendix.

In the lower part of the table, values for the 'deviation' (DEV) and the 'certainty' (CER) can be found (for the whole group and for separate sub-groups), where:

-deviation is a measure for the number of wrong answers of a pupil and is defined as follows:

$$\text{DEV} := \sum_{i=1}^{42} (a_i - 1) + \sum_{j=1}^{42} (5 - a_j)$$

with a_i and a_j being the answers to the i^{th} or j^{th} items;
i runs over right statementnumbers, j over wrong ones.

-certainty is a measure for how often a pupil gives answers 1's or 5's in comparison with 2's, 3's and 4's and is defined as follows:

$$\text{CER} := \sum_{k=1}^{42} |a_k - 3|$$

with: a_k the answer to the k^{th} item;
k runs over all statementnumbers.

In Table 2 of the Appendix the distributions of the answers for each item are given in percentages, as follows:

R (ight): the percentage of 1's or 2's for a right statement or 4's and 5's for a wrong statement.
M (iddle): the percentage of 3's for each statement.
W (rong): the percentage of 4's and 5's for a right statement or 1's and 2's for wrong statements.

On average, according to this division 64% of the 639 x 42 answers were right, 26% were wrong and 10% scored in the middle class. T-tests on the mean scores made it clear that the theoretical items were answered significantly ($p<0.001$) better than the problem situations (items 11-42). See the figures in Table 3. on the next page.

Table 3.:Percentages of answers in classes R,M and W, divided for theoretical items (1-10) and problem situations (11-42).

%	Total	Theoretical	Problems
Right	64	79	60
Middle	10	8	10
Wrong	26	13	30

The number of misunderstandings for each item will be regarded in a more rigorous way in section C. of this chapter, where the subject categories of the misconception list will be treated consecutively.

Since this item does not fit in well with one of the subject categories, the only thing we want to mention here is the large number of wrong answers to statement 42 (69%): the traditional problem if it is better to hold a little mirror further away in order to see more of yourself in it. When there was any time left after the reply-forms were collected (pupils used 30-40 minutes to answer the questionnaire), a discussion was initiated about this item. Although the geometrical explanation of this problem is rather complicated, it was amusing to learn that many pupils were convinced that 'holding it further away' must be successful, simply because they did it in practice everyday themselves. Even after doing experiments in the class-room some pupils maintained their opinion and got irritated when the discussion went on.

C. NUMBER OF MISUNDERSTANDINGS IN SUBJECT CATEGORIES OF THE MISCONCEPTION LIST.

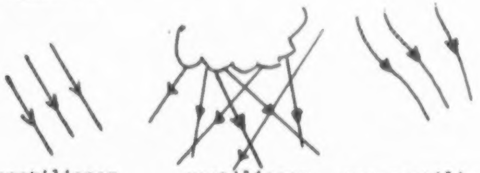
1. Nature and Properties of Light

Neither from the theoretical (1,2) nor from the problem situations (11,12 and 13) can the conclusion from Guesne for 12-15 year-old pupils be extended for 16-18 year-old pupils, that they should identify light only with its source or its effect. Most of them were aware of the presence of light in the space between a light source and a wall or a picture (87%). In the interviews every pupil, who was asked for, argued this by something like:'if you hold something in between, you see that it is illuminated too !'

2. Rectilinearity of light.

The theoretical statement about the rectilinearity of light propagation (3) was answered correctly by 76% of the pupils and roughly the same number understood the problem situation in which non-rectilinear light propagation was suggested to illuminate a room with only a small window (14). The item with the lightning flash was more confusing: 34% thought that a lightning-flash (15) was an example of non-rectilinear light propagation, not realising that a flash is not a light ray at all.

The coupling between rectilinear light propagation and its possibility to form shadows of an object seemed to be very obscure for the pupils. 64% Of them stated that shadow formation on a cloudy day was not possible because the clouds absorbed too much of the(16) light. In fact, in a light intensity a hundred times lower than the daylight intensity on a cloudy day, shadows can easily be formed when the light is coming in from one direction. Pupils have problems to distinguish this last condition - one-directional light in contradiction with diffuse light - from rectilinear light propagation. See figure 4.



rectilinear light, coming from one direction: shadows can be formed.

rectilinear, but diffuse light: no shadows will be formed.

non-rectilinear light-propagation.

Figure 4.: illustration of the differences between non-rectilinear and diffuse light.

Consequently, 14% of the pupils thinks that the reason why no shadows will be formed on a cloudy day must be non-rectilinear propagation of light when it has passed the clouds.

3. Dynamics of light.

The item about the limitation of the distance, light travels along in empty space (5) scores the most wrong answers among the theoretical questions (24%). Even more pupils (58%) agree with the statement that light emanating from the sun will travel further than that of a light bulb (20). This shows us clearly that pupils - regarding these questions - do not think in terms of fundamental light dynamics, but more in terms of light intensities and its effect: 'is it still visible at that distance ?'

From this point of view it becomes clear that these questions strongly correlate with those about the visibility of light rays in situations without mist droplets or dust particles. However, in the interviews the absorption of light in a 'foggy' medium was mentioned far more often by the pupils to cause a decrease of light intensity than the general decrease of light intensities of spherical expanding waves with the square of the distance to the source.

4. Interaction with matter and surfaces.

About 80% of the pupils know that light rays are not visible unless they are scattered by mist droplets or dust particles. In the problem situation (18) gave more right answers to this subject (81%) than in the theoretical situation (4: 72%), but this can be caused by the fact, that in the figure belonging to statement 18 the presence of a foggy corona round the cloud was strongly suggested in the picture.

The other issue in this category brought about more problems. Particularly, the difference between diffuse and specular reflection seemed to be rather obscure for the pupils.

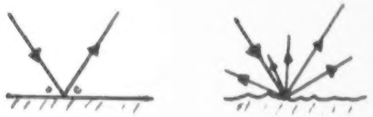
The mean score of right answers to statements 9 and 33-39 was 35%. In fact, there is a methodological problem in the questions 9 and 33: what do pupils mean with 'better reflection' ? It can be interpreted in two ways:

-a surface reflects better when it reflects more of the light, that hits the surface; physicists should say: the surface has a high reflection-coefficient.

-a surface reflects better when it reflects light rays according to the law of reflection, i.e. it reflects more specular like a mirror.

However, in spite of the ambiguity of these questions, the pupils' lack of ability to distinguish diffuse and specular reflection becomes evident in the statements 34-36 and 38,39. 60% of the pupils think that a mirror only reflects light at places where the light spot can be seen (34), not aware of the fact that specular reflection causes image formation: so, the whole mirror reflects light, but only the spot where the light is seen reflects it in the direction of the eye. There are only a few pupils who realise that specular reflection is a minimum condition for forming an image. Consequently,46% of the pupils thought that you could see yourself in a white sheet of paper, if it should reflect all the light (35). In the items 38 and 39, a very 'clean' situation, testing the difference between diffuse and specular reflection, was represented. 55% thinks that an observer in front of a mirror can see a light spot at a mirror where a slanting light beam hits it, while it is evident from the diagram that definitely no light is reflected in the direction of the observer.

In the interviews some pupils were asked about their knowledge of specular and diffuse reflection. Most of them remembered two drawings of their physics course; see Figure 5.


reflection by a mirror

diffuse reflection

Figure 5.: Illustration of the differences between diffuse and specular reflection as it is presented in most of the optics curricula.

In most curricula something like these drawings explains the origins of the two sorts of reflection, but they seldom take into consideration the important consequences of the differences between them; for example: most objects are visible because they reflect light in a diffuse way.

5. *The concept of vision.*

In the theoretical questions 6 and 7 it is stated, that light rays from an object have to enter the eye of the observer when he is able to see it, while it is not necessary that light rays coming from the light source hit him directly. 82% of the pupils seemed to have a good understanding of this vision concept.

In the interview they were encountered with the situation of Figure 6, showing a light source, an object and an observer. From the six light paths in the diagram, they had to choose the ones, that were necessary to make the object visible for the observer. All pupils chose the right ones:I and III.

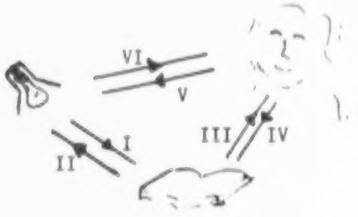


Figure 6.: Diagram of the interview situation.

It is quite surprising that - in spite of the pupils' good theoretical knowledge about the concept of vision - they fail to use this knowledge in the problem situations 22 and 26. Although in 22 a photographer has taken a bright picture of a building at night only 61% thought it necessary that light comes over to the photographer. In 26 an observer sees lit headlights of a car approaching him, but only 58% of the pupils thought that light rays emanating from the car really should reach the observer. In these situations the pupils were confused by the darkness of the meadow in 22 and the dark part of the road between car and observer in 26.

These misunderstandings typically belong to the decoupling concept as described in section IIB4. Besides, in these situations pupils seemed to identify the presence of light only with its effect: 'there is light when the meadow or the road is illuminated.'

D. FACTOR ANALYSIS: SEARCHING PUPILS' ALTERNATIVE FRAMEWORKS.

1. General considerations.

As we have seen in section IIC6. a factor analysis for the 42 items could show us alternative frameworks, that pupils use in their reasoning in geometrical optics. The issue in which we are interested here is whether pupils use more or less coherent sets of arguments in solving the problems or do they use arguments for each item separately and in an arbitrary way ? Secondly, if such frameworks exist, is it possible to compare them with the consistent theories that scientists use or must we really call them 'alternative' ?

As a basis for the factor analysis we have to take a look at the correlation matrix of the 42 items. The correlation matrix is represented in a visual form in Table 4 in the Appendix. A relatively strong correlation between the items i and j is represented as a block in the diagram; the area of the block is a measure for the value of the correlation.

A first conclusion is that interitem correlations are rather low (the average of the absolute values is 0.08). Although this gives a first hint about a low consistency in the reasoning of the pupils, it is difficult to make sharp conclusions out of this figure. That is because there is no objective criterion that gives us an objective critical value, above which a correlation should be regarded as important. It is troublesome to compare the absolute values of the correlations with other investigations: the apparently theoretical connections between items can easily be overrated and, besides, the sequence of the questions is important as well, since, an item out of another subject category between two items of the same category can confuse the relations.

The best way to find a critical value is to start at a high level and look how many items correlate. After that, pairs of items with lower correlations can be taken into account. Decreasing the critical value more and more bundles of items come into appearance; when these blocks become spread out over the whole matrix, the critical value is underrated. For this investigation a critical value for the correlations was 0.15: correlations below this value were blurred out over the whole matrix, correlations above 0.15 bundled more or less.

2. Correlations between items in the subject category of the misconception list.

Looking accurately to bundles of high correlations in the matrix brings us to the following conclusions:

-In the category of nature and properties of light (1,2,11,12,13) high correlations were found between 11,12 and 13. This is not a surprising result, since these three questions are about the same problem situation. Further correlations were low: a connection between 1,2 and 11,12,13 was hardly seen by the pupils.

-Rectilinearity and dynamics of light: 3,5,14,15,17,19,20. Correlations were found between pairs of items, but not throughout the whole group:

- -14 and 15, the problem situations about rectilinearity correlate strongly with each other but not with the theoretical item: 3.
- -16 and 17 about forming shadows correlate strongly, but hardly any connection is laid with rectilinearity: 3,14 and 15.
- -5,19 and 20 is a separate sub-group about the dynamics of light. They were correlated with each other, but also with the items of the category: 'vision'.

-Interaction with matter and surfaces 4,9,18,33-39.
Like the previous one, this category splits up in little groups:

- -4 and 18 correlate strongly. This is not surprising: the theoretical item and the problem situation are related closely.
- -9,33 and 35 form a separate category, which we could call: questions about the reflection coefficient of a white sheet of paper or a mirror.
- -36,37,38 and 39 is a group of questions about the difference between specular and diffuse reflection. They correlate more than average with each other, but not with the items 9,33 and 35. To a certain extent, this can be due to the ambiguity of the items 9 and 33 (see section IIIC4).

-The concept of vision 6,7,22,23,24,25,26,27,28.
The items 6 and 7 were correlated, but not with the group 22-28. In the group 22-28 very strong correlations were found (up to 0.88), but 24-28 are items in the same problem situation. The separation between the items 6 and 7 on one hand and the group 22-28 on the other shows us clearly, that the decoupling concept of vision is latent, not when it is tested in theoretical questions, but more in problem situations. The transfer of the theoretical knowledge in practice seems to be very low for this subject.

-Formation of an image 8,10,21,29,30,31,32,37,40,41,42.
Hardly any correlations were found in this category. Although pupils show us in theoretical item 8 their knowledge of the location of the image of an object at a plane mirror, they are hardly able to use this knowledge in problem situations like that of the boy who looks at the blackboard via a mirror (30) and the situation of the photographer who wants to take a picture of himself in a mirror (31 and 32). The connection with other items about image formation, such as the situation of the lighthouse (21) and the slide projector (29) was even less.

3. Alternative frameworks.

The previous conclusions were all about interitem correlations within categories. Now we can look at strong correlations between items in different categories in order to search for alternative frameworks.

The first group of inter-category correlations is that between dynamics of light (5,19,20) with the items about visibility of light rays in an environment with mist droplets or dust particles (4,18). We already saw in section IIIC3., that when a pupil is asked about the distance light can travel, he looks in first instance at the light intensities and if it is still observable and not at the fundamental dynamic properties of unlimited light propagation. Combining this with the idea, that the most important factor for decreasing the light intensity in a medium is absorption instead of the general decrease with the square of the distance to the light source for a spherical propagating wave, it becomes obvious that the above mentioned categories are correlated. When we accept the pupils' misunderstanding of the items 5,19 and 20 as an ambiguity on their abstraction level, we have to admit, that the connection pupils have made here is quite a legal one:i.e. when light rays are visible in a 'foggy' environment, some of the light must be scattered in the direction of the observer, and so the light intensity of the beam decreases in forwards direction.

A second framework can be found when we look at the correlations between the items 4,5,7,11,12,13,20,22-28. All these items have something about the presence of light at a certain distance and with the vision concept. As we have seen in section IIIC5., the decoupling concept of vision is a rather persistent one. Although pupils do not see clearly that light rays of a visible object have to enter the eye on every occasion, they do realise, that the presence of light in the environment of the object is a minimum condition for seeing it.

This idea together with the decoupling concept forms the basis for an alternative framework, causing correlations between the above mentioned items. This framework is analogous with the idea, that light is a purely static phenomena: it is present in a limited part of the space and in that part objects are visible.

E. T-TESTS: DIFFERENCES BETWEEN GROUPS.

In this section we shall restrict the review about differences between groups to some general conclusions about the average scores of the groups. It is not very useful to look as detailed as done in the previous section at each subject category separately, because quantitatively, the figures are very similar to each other (there are only a few items that show significant differences for separate groups) and qualitatively, the same conclusions will arise for each group.

When we look at Table 1 in the Appendix, we can see that there are differences in the deviation score (measure for the total number of misunderstandings, definition see section IIIB.) for different schools. In Table V on the next page the T-values are given for these differences.

Table 5: T-values for differences in 'deviation' for schools

school no.	DEVIATION	Number	T-values 2	3	4	5
1	55.1 ± 16	152	1.6	3.0	1.2	1.0
2	58.9 ± 15	56	-	1.0	0.6	1.1
3	61.7 ± 14	70	-	-	1.9	2.7
4	57.5 ± 14	101	-	-	-	0.5
5	56.6 ± 14	260	-	-	-	-

t	0.5	1.0	1.5	2.0	2.5	3.0
p	0.5	0.2	0.07	0.02	0.006	0.002

From Table 5 we can see that differences between schools in the deviation-score are not highly significant. The differences between the certainty-scores are not significant at all. As we saw in section IID3. it is difficult to get clear why there are differences between schools. In fact, school characteristics, teacher characteristics and the curriculum used for the optics course will interfere.

When we look at different years of the high school and to the two types of high schools, we can observe highly significant differences in the deviation-scores. See Table I in the Appendix and table 6 below.

Table 6: T-values for differences in 'deviation' for years.

Type	DEVIATION	Number	2	3	4
1. Pre-univ HS 3rd y.	61.7 ± 15	56	0.0	2.0	4.7
2. General HS 4th y.	61.8 ± 13	191	-	3.4	7.7
3. Pre-univ HS 4th y.	57.2 ± 14	201	-	-	7.1
4. Pre-iniv HS 5th y.	51.2 ± 14	191	-	-	-

t	2.0	3.0	4.0
p	0.02	0.002	0.0001

The number of misunderstandings in the 4th year of the general high school was about the same as in the 3rd year of the pre-university high school. The number of misunderstandings in the pre-university high schools increases rapidly in the upper classes.

Because of the highly significant differences mentioned above the factor analysis was done for the 5th year of the pre-university high school and the 4th year of the general high school were done separately. No qualitative differences came out: the same subcategories and the same bundles of correlations were formed. However there was a significant difference in the average of the absolute values of the correlations: 0.11 for the pre-university and 0.07 for the general high school group (t=4.2; p=0.0001). With some precaution it can be said that the pre-university pupils use more consistent frameworks than the general high school pupils.

Comparing the average scores of boys and girls, we see that the deviation-scores as well as the certainty-scores differ significantly.

Table 7 : differences in 'deviation' and 'certainty' between boys and girls.

	Boys (433)	Girls(206)	t	p
Deviation	55.5 ± 15	60.7 ± 13	4.4	< 0.0001
Certainty	61.7 ± 12	56.1 ± 12	5.5	< 0.0001

We have to treat these results with some caution, because the deviation and certainty - as they are defined in IIIB. - are mathematically dependent. By recoding the rough data into a three-point-scale, we can construct an independent scale for deviation. The differences between girls and boys decreased by recoding the data, but were still significant (t=3.9).

As we look at the scores for deviation and certainty for different ages, we can find no significant differences. This result emphasizes the fact, that the difference between separate years in the pre-university high schools are caused by educational effects and not by a generally higher abstraction level of older pupils.

IV. CONCLUSIONS AND DISCUSSION

To get some information about the reliability and the validity of the investigation the reliability-coefficient α was calculated and found to be 0.76 . After selecting and leaving out the ambiguous items (such as 9,21 and 33) this value increased to 0.79 .

It is rather difficult to summarize the results, presented in the previous chapter in a few conclusions, but here are the most important ones:

Misunderstandings about geometrical optics among 15-18 year-old general high school and pre-university high school pupils were mainly found in the following subject categories:

-vision; in particular the decoupling concept: roughly one third of the pupils do not think it necessary for light rays to enter the eye of an observer to make an object visible for him.

-interaction with surfaces: particularly the differences between diffuse and specular reflection. Although most of the pupils have a theoretical knowledge about the origins of this difference, they do not understand the consequences of this in practice.

-image formation: most of the pupils know that the image of an object at a plane mirror is as far behind the mirror as the object is in front of it, but in problem situations they fail to apply this knowledge.

As already seen in the last two examples, there exists a discrepancy between the theoretical knowledge of a pupil and his ability to use it in problem situations. The theoretical items (1-10) were answered significantly better than the problem situations (11-42). It is possible that this difference is brought about by the fact that most of the theoretical items were recognizable for the pupils from their optics courses. Most of the problem situations were new, allthough only everyday phenomena were regarded.

The 'total number of misunderstandings' of a pupil appeared to depend on the pupil's school, the type of the high school (general or pre-university) and on the number of years he went through on the high school. However, it did not depend on the pupil's age; from this the conclusion can be drawn, that improvements in the understanding the geometrical optics are purely due to educational effects and not to the higher abstraction level of older pupils. Finally, significant differences in the number of misunderstandings and in the certainty, with which they filled up the reply-form, were found between boys and girls. After recoding the data to make the deviation-score and the certainty-score mathematical independent, differences became less, but still significant.

The most striking results of the factor analysis were:

-The inter-item correlations were relatively low (on the average 0.08). Although it is difficult to find a theoretical basis which provides an objective critical value for the correlations, these low values give us the idea, that the consistency of the pupils' reasoning in solving optical problems is easily overrated (by investigators as well as by teachers).

-Subject categories, in which correlations were expected to be stronger, split up in small groups of two of three items.

-Some correlations were found between groups of items of different categories. Except for two cases (a connection between the limitation of light propagation and absorption and one between the decoupling concept and light as a statical phenomena), these bundles of correlations were difficult to interpret as pupils' alternative frameworks.

-The correlation matrix showed a subdiagonal effect: i.e. a pair of items which were close together in the questionnaire correlated stronger than a pair, from which one is placed at the beginning of the questionnaire and the other at the end.

With some reserve, we want to formulate a conclusion, based on the four characteristics of the factor analysis mentioned above: theories used by pupils when they are solving problems in geometrical optics are not very consistent. With this, it is not stated that they reason in a purely arbitrary way, but their frameworks are not very persistent, short-living and mostly restricted to only a few items.

The desire for uniformity in a theory explaining as many physics phenomena as possible within one principle is far less developed in pupils' minds as teachers expect they are. On the one hand it is possible that their abstraction level is not yet high enough to see through the consistency of scientific theories, on the other hand it could be that there is too little emphasis on the methodological aspects of the development of theories in physics education. Should it not be the teachers' task to show their pupils, that uniformity of physical principles and their applicability in contexts as broad as possible is one of the most charming disciplines in science ?

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TABLE 1. Average score on each item, deviation of the 'good score' and certainty, separated for schools, years, gender and age.

Item	Mean Score	Schools					years				Gender		Age			
		1	2	3	4	5	1	2	3	4	M	F	1	2	3	4
	N=639	N=152	N=56	N=70	N=101	N=260	N=56	N=191	N=201	N=191	N=433	N=206	N=176	N=257	N=148	N=58
01	4.469	4.316	4.482	4.271	4.515	4.592	4.554	4.382	4.632	4.361	4.406	4.602	4.551	4.479	4.439	4.259
02	4.391	4.224	4.429	4.400	4.287	4.519	4.411	4.257	4.433	4.476	4.476	4.214	4.381	4.362	4.399	4.534
03	2.006	1.849	2.036	2.371	1.881	2.042	2.214	1.827	1.915	2.220	2.000	2.019	1.977	1.988	2.047	2.069
04	3.997	4.263	4.000	3.971	3.911	3.881	4.232	3.759	3.801	4.372	4.065	3.854	3.886	4.035	4.081	3.968
05	2.322	2.013	2.196	2.800	2.495	2.335	2.375	2.435	2.328	2.188	2.254	2.466	2.443	2.261	2.311	2.259
06	1.789	1.605	2.089	1.814	1.921	1.773	2.107	1.843	1.776	1.654	1.702	1.971	1.903	1.681	1.851	1.759
07	4.239	4.270	4.161	4.029	4.287	4.277	4.161	4.209	4.129	4.408	4.314	4.083	4.125	4.288	4.297	4.224
08	1.700	1.566	1.411	1.771	1.703	1.819	1.929	1.785	1.672	1.576	1.718	1.660	1.807	1.716	1.588	1.586
09	2.459	2.493	2.446	2.600	2.356	2.442	2.518	2.346	2.587	2.419	2.383	2.617	2.545	2.444	2.426	2.345
10	4.014	4.086	3.893	3.900	3.861	4.088	3.607	3.990	4.055	4.115	4.115	3.801	3.847	4.078	4.061	4.121
11	2.028	2.092	2.321	2.057	2.000	1.931	2.625	2.073	1.821	1.026	2.065	1.951	2.136	1.856	2.209	2.000
12	4.117	4.079	4.196	3.857	3.990	4.242	4.143	4.063	4.154	4.126	4.118	4.117	4.023	4.233	4.014	4.155
13	4.463	4.414	4.571	4.457	4.416	4.488	4.304	4.366	4.572	4.492	4.476	4.437	4.415	4.541	4.385	4.466
14	4.041	4.309	3.875	3.786	3.950	4.023	4.125	4.084	4.249	3.754	4.090	3.937	4.188	4.039	3.919	3.914
15	3.806	4.000	3.482	3.586	3.911	3.781	3.518	3.764	3.731	4.010	3.952	3.500	3.682	3.739	3.966	4.069
16	2.477	2.395	2.429	2.557	2.475	2.515	2.268	2.251	2.562	2.675	2.427	2.583	2.392	2.607	2.358	2.466
17	3.964	4.013	3.982	3.886	3.911	3.973	3.929	3.770	4.090	4.037	3.977	3.937	3.938	4.004	3.953	3.897
18	1.897	1.789	1.946	1.914	2.079	1.873	1.982	1.948	1.950	1.764	1.783	2.136	2.000	1.926	1.719	1.724
19	4.452	4.474	4.464	4.371	4.277	4.527	4.625	4.199	4.532	4.571	4.446	4.466	4.500	4.455	4.466	4.259
20	3.443	3.329	3.625	3.757	3.426	3.392	3.875	3.639	3.388	3.178	3.233	3.883	3.585	3.424	3.331	3.379
21	2.435	2.467	2.125	2.057	2.297	2.638	3.089	2.236	2.214	2.675	2.333	2.650	2.506	2.284	2.608	2.448
22	2.479	2.296	2.946	3.014	2.267	2.423	2.911	2.681	2.428	2.204	2.339	2.772	2.750	2.412	2.365	2.241
23	4.397	4.408	4.500	4.286	4.416	4.392	4.393	4.382	4.408	4.403	4.349	4.500	4.426	4.405	4.318	4.483
24	1.277	1.296	1.161	1.286	1.277	1.288	1.125	1.288	1.249	1.340	1.293	1.243	1.182	1.276	1.304	1.500
25	2.227	2.178	2.393	2.571	2.109	2.173	2.357	2.529	2.159	1.958	2.185	2.316	2.381	2.136	2.189	2.259
26	2.543	2.461	2.589	3.000	2.366	2.527	2.661	2.890	2.557	2.147	2.443	2.752	2.688	2.479	2.493	2.517
27	3.117	3.118	3.125	3.386	2.921	3.119	3.446	3.497	3.229	2.524	2.947	3.476	3.381	3.054	2.986	2.931
28	3.687	3.421	3.786	3.914	3.772	3.727	3.804	3.775	3.657	3.597	3.667	3.728	3.767	3.646	3.608	3.828
29	4.233	4.224	4.321	4.043	4.297	4.246	4.411	4.073	4.174	4.403	4.224	4.252	4.278	4.237	4.223	4.103
30	4.358	4.487	4.196	4.371	4.277	4.346	4.161	4.283	4.328	4.524	4.381	4.311	4.244	4.381	4.419	4.448
31	3.532	3.493	3.232	3.386	3.634	3.619	3.036	3.319	3.632	3.785	3.524	3.549	3.364	3.681	3.520	3.414
32	2.156	1.993	2.071	2.371	2.178	2.204	2.357	2.162	2.318	1.921	2.069	2.340	2.295	2.148	2.122	1.862
33	2.603	2.711	2.679	2.729	2.594	2.492	2.625	2.461	2.682	2.654	2.552	2.709	2.688	2.619	2.608	2.259
34	2.582	2.342	2.625	2.786	2.525	2.681	2.321	2.293	2.498	3.037	2.665	2.408	2.386	2.607	2.736	2.672
35	2.964	3.086	2.929	3.086	3.218	2.769	2.821	2.782	2.896	3.314	2.896	3.107	2.926	2.977	2.953	3.052
36	4.203	4.329	4.268	4.157	4.178	4.138	4.179	4.110	4.164	4.346	4.219	4.170	4.182	4.132	4.324	4.276
37	3.679	3.855	3.839	3.686	3.525	3.600	3.214	3.670	3.612	3.895	3.958	3.092	3.369	3.763	3.804	3.931
38	2.732	2.901	2.857	2.814	2.911	2.515	2.518	2.560	2.488	3.225	2.935	2.306	2.386	2.829	2.818	3.138
39	2.152	2.158	2.179	2.329	2.089	2.119	2.250	2.408	2.209	1.806	2.046	2.374	2.199	2.230	2.020	2.000
40	2.235	2.191	2.232	2.343	2.525	2.119	2.125	2.220	2.363	2.147	2.141	2.432	2.335	2.272	2.128	2.034
41	2.304	2.309	2.214	2.100	2.446	2.319	2.232	2.497	2.279	2.157	2.171	2.583	2.398	2.222	2.365	2.224
42	1.592	1.428	1.411	1.957	1.455	1.681	1.446	1.560	1.627	1.628	1.607	1.558	1.602	1.630	1.541	1.517
DEV.	57.18	55.14	58.93	61.74	57.55	56.62	61.70	61.84	57.18	51.19	55.49	60.72	60.07	56.14	56.13	55.67
CER.	59.87	63.28	60.43	58.23	57.21	59.22	60.16	58.77	60.24	60.48	61.65	56.11	59.47	60.28	59.06	59.62

years:
2: 4th year general highschool
1,3,4: 3rd,4th and 5th year pre-university highschool
N:Number of test persons

Gender:
M=male
F=female

Age:
1: 14/15 year old
2: 16 year old
3: 17 year old
4: 18/19 year old

$$DEV := \sum_{i=1}^{42}(a_i-1) + \sum_{j=1}^{42}(5-a_j)$$

$$CER := \sum_{k=1}^{42}|a_k-3|$$

with: a_i answer on item i
i runs over right, j over wrong, k over all items

TABLE 2.: Distribution of right and wrong answers for each item.

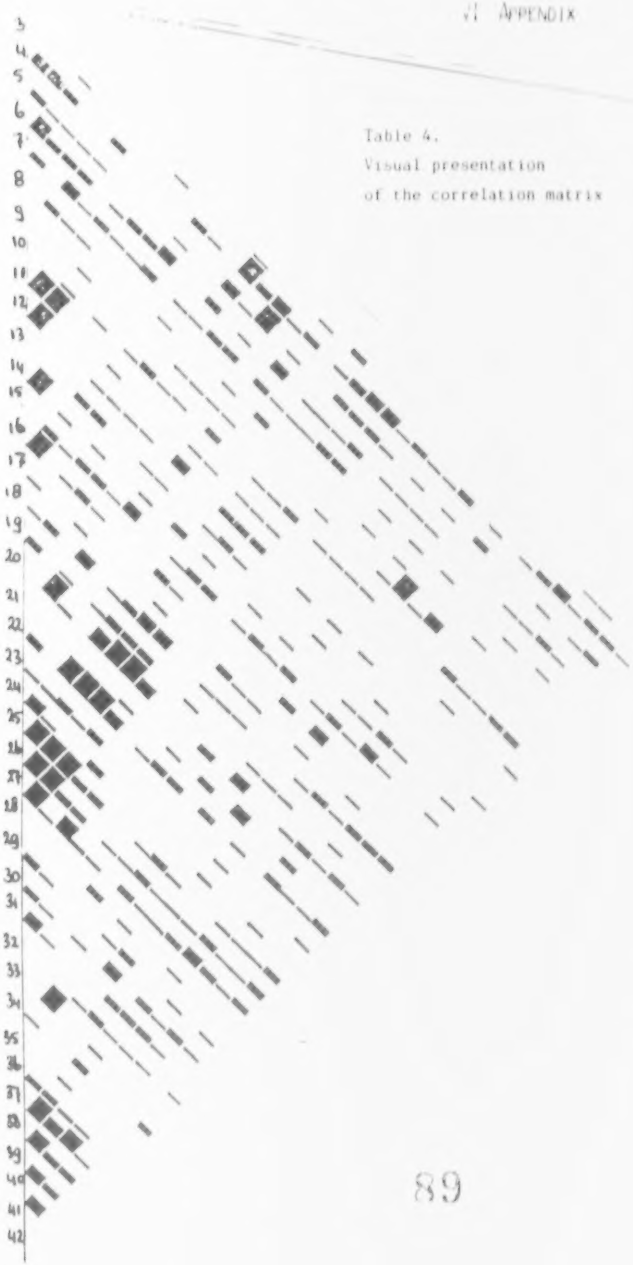
No.	ans	R	M	W
1	5	87	6	7
2	5	86	3	11
3	1	76	8	17
4	5	72	11	16
5	1	65	10	24
6	1	82	7	11
7	5	82	8	10
8	1	85	5	10
9	5	25	14	61
10	5	76	11	13
11	1	77	5	18
12	5	79	9	13
13	5	90	3	7
14	5	75	8	16
15	5	66	18	17
16	5	25	11	64
17	5	75	10	14
18	1	81	11	9
19	5	88	4	7
20	1	30	12	58
21	5	24	13	63
22	1	61	7	32
23	5	90	6	4
24	1	97	2	1
25	1	68	11	20
26	1	58	12	30
27	1	40	15	45
28	5	65	13	22
29	5	82	10	8
30	5	87	8	5
31	1	59	10	31
32	5	71	14	16
33	5	28	15	57
34	5	28	13	60
35	5	40	14	46
36	5	82	12	7
37	5	66	4	29
38	5	37	7	55
39	1	72	10	18
40	1	71	11	19
41	1	69	5	25
42	5	25	5	69

R: right: percentage of answers 1's and 2's if answer must be 1 or the percentage 4's and 5's if answer must be 5.

M: middle: percentage of answers 3.

W: wrong: percentage of answers 4's and 5's if answer must be 1 or the percentage 1's and 2's if answer must be 5.

Table 4.
Visual presentation
of the correlation matrix



MISCONCEPTIONS CONCERNING NEWTON'S LAW OF ACTION AND REACTION: THE UNDERESTIMATED IMPORTANCE OF THE THIRD LAW*

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Introduction

A number of studies conducted in recent years have demonstrated a wide range of beliefs about physical phenomena which students have apparently formed on their own without the benefit of formal instruction. Particularly well documented have been student beliefs which are in contradiction with the ideas of Newtonian mechanics. For example, many students hold the belief that there is a force on or in an object in the direction of the object's motion (Viennot 1979, Sjoberg and Lie 1981, Clement 1982) when in fact no force is necessary to keep an object moving at a constant velocity. Reviews of research on students' alternative conceptions in classical mechanics are provided by Driver and Erickson (1983), McDermott (1983), McCloskey (1983), and McDermott (1984).

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Most of the student errors that have been documented in Newtonian mechanics have been on questions designed to test conceptual understanding of Newton's first or second laws. Only a few (e.g. Maloney, 1984, Boyle and Maloney, 1986, and Terry and Jones, 1986) treat the third law. This emphasis on the first two of Newton's three laws is in keeping with the emphasis placed on the first two laws in textbooks. The ability to flexibly use the quantitative statement of the second law (the net force acting on an object is numerically equal to its mass times its acceleration) is arguably the most important ability a student can acquire for success in an introductory physics course. This preeminence of the quantitative statement of the second law in instruction is illustrated by the fact that the very popular PSSC high school textbook (Haber-Schaim, Dodge, & Walter, 1986) speaks of "Newton's Law" (meaning the quantitative statement of Newton's second law) rather than speaking in plural of Newton's laws.

By contrast, many textbooks treat the third law in passing, either simply mentioning it briefly as an unsupported statement of fact or as an addendum to the section covering conservation of momentum. The results of this study indicate that this type of treatment is insufficient to counter the misconceptions students hold about the third law. This might be a small concern if the third law is in fact only an insignificant piece of the Newtonian picture, but in this paper we argue that the third law should be treated as a much more significant part of an introductory physics course since it is important for developing the students' qualitative concept of force.

Method

A multiple choice diagnostic test was administered to four physics classes, two classes in each of two high schools. All of the questions (shown in the appendix) concerned the concept of force in various contexts, and the majority could be answered using a basic knowledge of Newton's third law. The test was administered at the beginning of the year and again after all instruction in mechanics had been completed in order to assess gains from instruction. In addition to answering the questions, students were asked to rate how confident they were in their answers. Teachers were not aware of the contents of the test.

At school A, both of the physics classes were taught at the same level, however, the intellectual level of the students was fairly high as indicated by an average SAT math score of 610. At school B, students can take a standard level course in physics or an honors level course. The data used here was gathered only from the standard level classes. For any analysis beyond simple percent correct, only school A has been used because of these differences. Also, the science curriculum in school A is more typical with students taking physics generally in their senior year following chemistry. However, in school B, students take physics generally in their sophomore or junior year preceding chemistry. Scores are reported only for students who took both the pre-course and post-course tests.[1]

Results of the Pre-test Diagnostic

As one purpose of this paper is to disseminate the results of the diagnostic, tables 1, 3, and 4 present the results of all the problems on the diagnostic. However, the starred questions are those which will be discussed further as these are the questions which could be answered with an understanding of Newton's third law. As can be seen in table 1, most of the questions on the pre-test were answered correctly by far less than half of the students, providing a

Table 1

PRETEST SCORES FOR BOTH SCHOOLS

	A (n=23)	B (n=27)	Both (n=50)
* Lamp	13%	56%	36%
* Stock Cars	13	22	18
* Stationary Boxes	9	11	10
* Office Chairs A	74	89	82
* Office Chairs B	13	19	14
* Mosquito	4	30	18
* Steel Blocks	17	33	26
* Bowler	0	0	0
* Magnets	35	48	42
* Pulling Block A	30	26	28
* Pulling Block B	30	19	24
* Handsprings	39	11	24
* Three Boxes	0	0	0
Suitcase	17	11	14
Crate A	78	70	74
Crate B	9	15	12
Book Pile	9	7	8

* Can be answered using Newton's Third Law

Table 2

AVERAGE CONFIDENCE SCORES OF TYPES OF PROBLEMS
(SCHOOL A: n = 23)

	Incorrect answers		Correct answers	
	Pretest	Posttest	Pretest	Posttest
Third law				
Continuous forces				
Static	1.68	1.92	1.84	2.33
Dynamic	1.79	1.90	1.46	1.98
Impulse forces				
Explosions	1.68	1.93	1.50	2.20
Collisions	1.85	2.09	2.40	2.48
Friction	1.92	2.05	1.93	2.13

Sample Confidence Scale

0	1	2	3
Just a blind guess	Not very confident	Fairly confident	I'm sure I'm right

Table 3

POSTTEST SCORES FOR BOTH SCHOOLS

	A (n=23)	B (n=27)	Both (n=50)
* Lamp	83%	89%	86%
* Stock Cars	26	15	20
* Stationary Boxes	30	33	32
* Office Chairs A	91	89	90
* Office Chairs B	44	19	30
* Mosquito	48	52	50
* Steel Blocks	48	48	48
* Bowler	4	4	4
* Magnets	48	33	40
* Pulling Block A	35	15	24
* Pulling Block B	35	15	24
* Handsprings	44	26	34
* Three Boxes	9	7	8
Suitcase	4	22	14
Crate A	83	89	86
Crate B	17	41	32
Book Pile	22	30	26

* Can be answered using Newton's Third Law

Table 4

PRE-POST GAINS FOR BOTH SCHOOLS

	A (n=23)	B (n=27)	Both (n=50)
* Lamp	70%	33%	50%
* Stock Cars	13	- 7	2
* Stationary Boxes	21	22	22
* Office Chairs A	17	0	8
* Office Chairs B	31	0	16
* Mosquito	44	22	32
* Steel Blocks	31	15	22
* Bowler	4	4	4
* Magnets	13	-15	- 2
* Pulling Block A	5	-11	- 4
* Pulling Block B	5	- 4	0
* Handsprings	5	15	10
* Three Boxes	9	7	8
Suitcase	-13	11	0
Crate A	5	19	12
Crate B	8	26	20
Book Pile	13	23	18

* Can be answered using Newton's Third Law

possible indication of widespread preconceptions.

An indication that these answers represent student beliefs (rather than simply incorrect guesses) comes from the students' ratings of their confidence in their answers. In addition to marking one of the multiple choice answers, students were asked to rate how confident they were in their answers on a scale from zero to three. Students used a rating of zero to indicate that they were just guessing, a rating of one to indicate that they were not guessing but were not very confident, a rating of two that they were fairly confident, and a rating of three that they were sure their answer was correct. Students could mark their confidence anywhere along this continuous scale. As can be seen in table 2, students' average ratings of their confidence in their incorrect answers on the pre-test indicate that their answers were not blind guesses, but were held with some conviction. This suggests that the wrong answers represent preconceptions rather than simply wrong guesses.

Results of the Post-test Diagnostic

From the results of the post-test diagnostic shown in table 3, the instruction was apparently not particularly effective in raising the students' scores, providing an indication that the preconceptions in the domains tested by these questions are quite resilient. The pre-post differences shown in table 4 are disappointingly low with a number of gains below 10% and even some losses. The average gain for the classes in the two schools was only 12.8%, raising the average score on the test from 25% to 38%.

Again the confidence scores shown in table 2 indicate that students were not simply guessing but answered with some conviction. As would be hoped, the average confidence score of those students answering correctly was higher after instruction in each of the areas. However, the average confidence score of those students answering *incorrectly* was also higher after instruction. This would seem to indicate that not only did most of the students have misconceptions after instruction, but the students answering incorrectly after instruction were more convinced their misconceived beliefs were correct than those answering incorrectly before instruction. Thus it seems clear from the low percentage of correct answers on the post questions and the students' high confidence in these answers that instruction was not effective in instilling a knowledge of Newton's third law.

The Importance of the Third Law

In this section we take a deeper look at the problems on the diagnostic which required a knowledge of Newton's third law and provide an interpretation of the results. We will argue that the low scores on the post-test may not indicate simply a failure to remember a verbal statement of the third law but rather may indicate a failure at a deeper conceptual level. Further, we submit that if students can gain a deep conceptual grasp of Newton's third law, they are in a much better position to answer both qualitative and quantitative questions involving forces.

Force as an interaction. Before proceeding to a discussion of students' conceptions of force, it is helpful to consider the Newtonian view. Warren (1979), in his book arguing for increased clarity in the presentation of the concept of force in introductory physics, states the third law as follows:

> Forces result from the interactions of bodies. The force exerted by body A upon body B is equal in magnitude, opposite in direction and in the same straight line as the force exerted by B upon A.

There are at least five ideas important in a careful consideration of the third law in classical mechanics which we further elaborate below:

1) A body cannot *experience* a force in isolation. There cannot be a force on a body A without a second body B to exert the force.

2) Closely related to the above point is the fact that A cannot *exert* a force in isolation. A cannot exert a force unless there is another body B to exert a force on A. We then say that A and B are *interacting*. (Thus, for example, it is incorrect to say that an astronaut punching empty space with his fist is exerting a force since there is nothing exerting a force back against his fist.) The attractive or repulsive force between two bodies arises as a result of the action of the two bodies *on each other* because they are either in contact or experience between them a force acting at a distance.[2]

3) At all moments of time the force A exerts on B is of exactly the same magnitude as the force B exerts on A.

4) An important implication of the above point is that neither force precedes the other force. Even though one body might be more "active" than the other body and thus might seem to initiate the interaction (e.g. a bowling ball striking a pin), the force body A exerts on body B is always simultaneous with the force B exerts on A.

5) In the interaction of A with B, the force A exerts on B is in a direction exactly opposite to the direction of the force which B exerts on A.

Thus, the third law can be seen to be much more involved than is implied by the simple epigrammatic version "for every action there is an equal and opposite reaction."

Force as an innate or acquired property. The concept of force as embodied in the third law and developed above does not seem to be the naive conception of force which most high school students hold. Minstrell and Stimpson (1986) have proposed that in addition to viewing forces as pushes or pulls, students treat force as a property of objects. We have also observed such student reasoning (e.g. Brown and Clement, 1987, Brown, 1987). In this view single objects "have" force as a result of qualities of the object which would make it seem "force-full."

Minstrell and Stimpson list such factors as an object's weight, motion, activity, or strength as important to students in determining an object's force. These factors fit well with Maloney's (1984) data. A student holding this view would consider a heavy, fast-moving, strong football player to have a great deal of "force-fullness" (our term) and would be able to exert more force on other people or objects "having" less force than they would be able to exert back. This is in direct contrast to the Newtonian concept in which a force does not exist except as arising from the interaction of two objects, the forces on each object being equivalent in magnitude.

This type of reasoning is illustrated by the following student reaction to the steel blocks problem. In this problem, a 200 pound steel block (block A) rests on top of a 40 pound steel block (block B), and the student is asked to compare the force A exerts on B with the force B exerts on A. The student expressed a belief that since the heavier block "had" more force, not only would it exert a greater force, it would push the lighter block into the ground.

> S: ...I think it [the 40 pound block B] exerts a force up, but I don't think it exerts enough to stop A [the 200 pound upper block] from pushing B into the ground. See, it just makes the thing slower. So say B only weighed one pound, then A would have 199 pounds more than B would, and so it would push it into the ground faster... (Brown and Clement, 1987, p. 18)

The data in this study are certainly consistent[3] with the view that many students adopt a concept of force as an innate or acquired property of objects rather than as arising from an interaction between two objects. Six problems in particular on the diagnostic would tend to draw out this conception of force. In each of these problems, there is a relatively unambiguous object which is stronger, faster, heavier, more acting as an agent of causation than the other object, or some combination of the above.[4] Students with a concept of force as an innate or acquired property of objects would be expected to answer that the heavier, faster, etc., object (the object which "has" more force) would exert the greater force, while the other object would exert either a lesser force or no force at all.

As an example consider the bowler problem in which the student is asked to compare the force a 16 pound bowling ball exerts on a 4 pound pin when the ball strikes the pin. In this case the student might consider the bowling ball to be clearly more "force-full" because it is moving, it is heavier, and it is more able to cause damage than the pin. The two answers (of the five possible incorrect answers) consistent with this view of force are that the bowling ball exerts a greater force than the pin or that the ball exerts a force while the pin exerts no force at all.

Table 5 shows the overall percentage of students giving answers consistent with this view of force for these six problems. Table 6 shows what percentage of incorrect

Table 5

PERCENTAGE OF ANSWERS CONSISTENT WITH
A CONCEPT OF FORCE AS AN INNATE PROPERTY OF OBJECTS
(SCHOOL A: n = 23)

	Pretest	Posttest
Stock Cars	61%	57%
Stationary Boxes	52	52
Office Chairs B	83	52
Steel Blocks	78	43
Bowler	100	95
Pulling Block A	57	55

Table 6

PERCENTAGE OF INCORRECT ANSWERS CONSISTENT WITH
A CONCEPT OF FORCE AS AN INNATE PROPERTY OF OBJECTS
(SCHOOL A: n = 23)

	Pretest	Posttest
Stock Cars	70%	77%
Stationary Boxes	57	74
Office Chairs B	91	91
Steel Blocks	94	83
Bowler	100	100
Pulling Block A	81	86

answers these particular incorrect answers represent. Thus, for example, table 5 shows that on the pre-test 61% of the students gave answers for the stock cars problem which were consistent with the view of force as an innate or acquired property, and table 6 shows that these particular incorrect answers represent 70% of all incorrect answers for this problem. On both the pre-test and the post-test, of the students answering incorrectly, in all cases the answers consistent with a concept of force as a property represented over 50% of the incorrect answers and for most problems was a much higher percentage.

Particularly striking are the results of the bowler problem. In this problem, students might tend to think the bowling ball "has" more force than the pin since it is both heavier and it is moving. This would translate into answering that the bowling ball exerts a greater force than the pin, which exerts a lesser force or no force at all. All incorrect answers for these students were of this type for this problem on both the pre-test and the post-test. Thus the data are consistent with the hypothesis that the great majority of students have a conception of force as a property of objects as the great majority gave answers consistent with this conception. Further, traditional instruction seems to have had a disappointingly low impact on this conception for these students.

Consequences of the view of force as a property. Heller and Reif (1984) present a study indicating what student performance could be if, among other things, they had a concept of force as an interaction between two objects. In this study, Heller and Reif compared problem solving performance under two different conditions, one of which (the model condition) explicitly guided students to view the forces in the problems as arising from the interaction of objects. Subjects guided by the model answered the quantitative problems correctly 90% of the time versus only 20% of the time for the control group, indicating that a conception of force as an interaction would aid problem solving if internalized by students.

Since the concept of force as an interaction between two objects is at the heart of the third law, it seems from these considerations that a careful and extended treatment of Newton's third law (also known as the law of action and reaction) may be quite important in an introductory physics course. If students acquire a deep understanding of the third law, they might be much less apt to have difficulty with both quantitative problems (as demonstrated by the Heller and Reif study) and qualitative problems such as those drawing out the "impetus" misconception (cf. McCloskey 1983) in which force is viewed as a *property* of a moving object causing it to move with constant velocity.

A simple solution? A possible solution to students' difficulties with the concept of force as an innate or acquired property of objects is to simply re-label their naive concept of force by calling it, for example, "momentum" or "kinetic energy" since both of these can be properties of an object and both do depend on the object's mass and speed. But this is not a satisfactory solution for at least two reasons: 1) momentum or kinetic energy do not *cause* motion (as students view force causing motion), they are simply properties of a moving object arising as a result of the motion of that object, and 2) momentum and kinetic energy vary with the frame of reference. If a student were to simply re-label his conception of force to be, for example, momentum, he might well ask how an object could have a lot of force (or strength or forcefulness) in one frame of reference and none from another perspective.

Conceptual change necessary. For the above reasons, re-labeling the student's naive concept of force is not a satisfactory solution to the problem of the naive view of force as a property and may lead to even greater confusion (how many times have students used the words "the force of momentum" in a physics class?). What is necessary is a modification of the concept itself. This modified conception of force should involve a deep understanding (rather than a mere memorization) of the third law, that is, a concept of force as an interaction between two objects rather than as an innate or acquired property of objects.

Conclusion

The results of this study indicate that high school students enter physics classes with preconceptions in the areas of Newton's third law. Evidence from a post-course test indicates that these preconceptions are persistent and difficult to overcome with traditional instructional techniques. The data from this study are consistent with the hypothesis that the persistence of preconceptions concerning the third law may result from students' general naive view of force as a *property* of single objects rather than as a relation between objects.

We have argued that the consequences of such a view of force extend well beyond problems explicitly treating the third law to all problems, both quantitative and qualitative, which deal with forces. This suggests that ideas concerning the third law, which makes explicit the relational quality of forces, may play a more important role than is ordinarily granted in teaching. Helping students develop a mature conception of force will undoubtedly involve an extended and multipronged approach, but we submit that innovative strategies for teaching the third law should comprise a significant part of the unit on forces and Newton's laws rather than receiving the cursory treatment which it has usually been afforded.

Notes

1) We are currently expanding the data base for this study and additional data should be available in the fall of 1987.

2) Of course even "contact" forces are considered electromagnetic forces acting at a distance on the atomic or molecular level. However, on the macroscopic level, the distinction between "contact forces" and "forces at a distance" provides a helpful dichotomy.

3) It is of course impossible to attribute a particular type of reasoning to students without clinical interview data simply based on students' choices on a multiple choice diagnostic. However, in preliminary clinical interviews and in prior studies (e.g. Brown and Clement, 1987), students have often been observed to maintain that the greater force in an interaction is due to one object's "having" more force. In particular, in collision problems students have been observed to maintain that a moving object has more force prior to a collision with a stationary object of the same weight and thus exerts the greater force during the collision.

4) Maloney (1984) describes a study designed to explore which of several of these factors are most important in determining students' answers to problems such as the pulling blocks problems. In particular he examines whether students view the mass of an object or whether the object acts as an agent of causation as more important in determining their answer to the question of which object exerts the greater force. In PULLING BLOCKS A presented here, block A is both heavier as well as being the agent of causation in the interaction, pulling block B to the left, whereas in PULLING BLOCKS B, A is heavier but it is not the agent of causation. Because of this conflict of influences in PULLING BLOCKS B, data is presented here only for PULLING BLOCKS A.

5) Problem described in Maloney (1984).

A special note of thanks to David Palmer and Jill Shimabukuro for their assistance in data analysis.

Appendix

This appendix contains the questions asked on the diagnostic. These are divided into two main categories: those questions which can be answered with a knowledge of Newton's third law, and those concerning friction. For all of the questions, students were asked to indicate their confidence using the scale below.

0	1	2	3
Just a blind guess	Not very confident	Fairly confident	I'm sure I'm right

Third Law Questions

Six answer format questions. The third law questions are divided into two categories. In the first category are those questions which used a format of six choices for the possible answers. These questions asked the student to consider the possible existence and relative magnitude of two forces from two different objects. For all of these problems except one the correct answer is choice number three, that both objects exert (or feel) a force and these forces are equal in magnitude. The one problem for which this is not the correct answer is the three boxes problem. For this problem the correct answer is that the lowest block (block C) exerts the greater force. The six possible choices were:

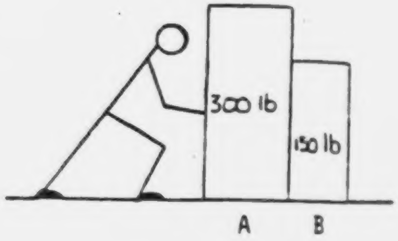
1) Both exert (or feel) a force, but the first exerts (or feels) a greater force.
2) Both exert (or feel) a force, but the second exerts (or feels) a greater force.
3) Both exert (or feel) a force, and these forces are of equal size.
4) Only the first exerts (or feels) a force.
5) Only the second exerts (or feels) a force.
6) Neither exerts (or feels) a force.

STOCK CARS

At a demolition derby, one stock car weighing 2000 lbs. runs head-on at 20 MPH into another identical stock car which is standing still. [The question then asks the student to compare the force experienced by each car.]

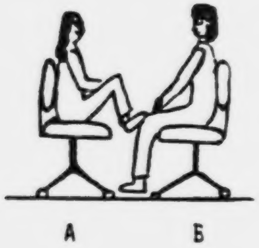
STATIONARY BOXES

A warehouse worker is strong enough to slide a large box A up against a smaller box B. He then tries to move both boxes at once as shown in the picture, but he is not strong enough and nothing moves. Think about whether A exerts a force on B and whether B exerts a force on A while he is pushing but unable to move them.



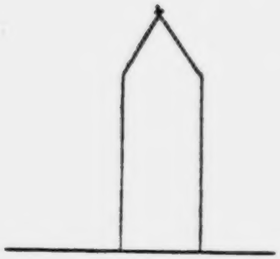
OFFICE CHAIRS B

Two students who both weigh 120 lbs. sit in identical rolling office chairs facing each other. Student A places his bare feet on student B's knees, as shown below. Think about whether A exerts a force on B and whether B exerts a force on A when A kicks outward.



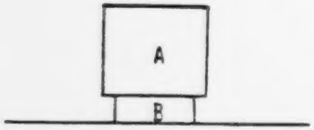
MOSQUITO

On a day with no wind, a mosquito lands on top of the Washington Monument. Think about whether the mosquito exerts a force on the monument and whether the monument exerts a force on the mosquito while it is resting there.



STEEL BLOCKS

A large steel block weighing 200 lbs. rests on a small steel block weighing 40 lbs. as shown below. Think about whether A exerts a force on B and whether B exerts a force on A.

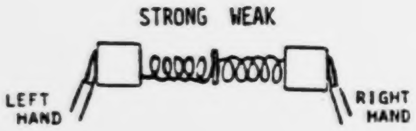


BOWLER

A bowling ball weighing 16 lbs. hits a bowling pin weighing 4 lbs. [The question asks the student to compare the force the ball exerts on the pin with the force the pin exerts on the ball.]

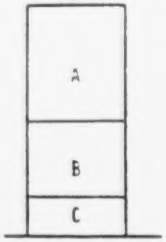
HANDSPRINGS

As shown in the diagram below, you are holding two springs, a strong, stiff one and a weak, soft one, between your hands. You move your hands a few inches closer together, compressing the springs a bit against each other. When you hold your hands still in this position with the springs somewhat compressed, which of the following choices is true? [The question asks the student to compare the force the left hand feels with the force the right hand feels.]



THREE BOXES

Three boxes are stacked on top of each other with the lightest on the bottom and the heaviest on the top. Think about whether the top and bottom blocks A and C exert a force on the middle block B. [This question concerns a non-example situation in that the forces to be compared are not equal. The correct answer is that block C exerts the greater force.]



Questions using a different format. The following questions, which can also be answered with a knowledge of Newton's third law, were asked in a format different from the six answer format above. Since there is not a standard format for the choices for these questions, the actual choices are presented with each problem.

LAMP

Jim buys a new floor lamp and leaves it standing in the corner of his room. Which of the following do you think is true?

1) The floor exerts an upward force on the floor lamp.
2) The floor does not exert an upward force on the floor lamp.

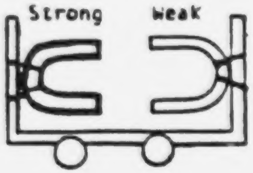
OFFICE CHAIRS A

Two students who both weigh 120 lbs. sit in identical rolling office chairs facing each other. Student A places his bare feet on student B's knees, as shown below. When student A kicks outward, B moves to the right. What happens to A? [See the diagram above for office chairs B]

1) A moves left (<----)
2) A moves right (---->)
3) A remains motionless

MAGNETS

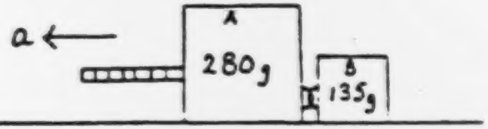
Two magnets are securely fastened to opposite sides of a cart, and aligned so as to repel each other, as shown in the diagram. The cart is sturdy so the repulsion between the magnets cannot break the cart sides. If one magnet is much stronger than the other, and we place the magnets as shown in the diagram so that they push away from each other, what will happen to the cart?



1) It will move left (<----)
2) It will move right (---->)
3) It will remain motionless

PULLING BLOCKS A[5]

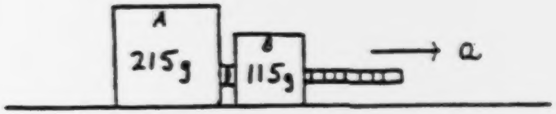
Two blocks are hooked together and pulled by a rope on a horizontal surface. The rope pulls the blocks so that they accelerate at a constant rate. Think about whether one block, A or B, is exerting a larger force on the other block, or whether the forces they exert on each other are equal. Which one below is true?



1) A exerts a larger force
2) B exerts a larger force
3) The forces are equal

PULLING BLOCKS B[5]

In the next case below two different blocks are being accelerated to the right. Which one below is true?



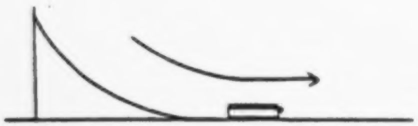
1) A exerts a larger force
2) B exerts a larger force
3) The forces are equal

Friction Questions

The following questions all concern friction. Although they cannot be answered simply with a knowledge of Newton's third law, they do deal with the directional aspect of force and are thus informative in a discussion of students' misconceptions of the third law. Again, because the types of answers are not standardized, the actual choices are included with the problems.

SUITCASE

A suitcase slides from a ramp onto the steel floor of the baggage area at an airport. While it is still sliding on the floor, which one of the following sentences explains why the suitcase stops?

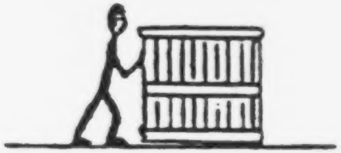


1) The floor pulls down on the suitcase, causing it to stop.
2) There is a frictional resistance to the motion of the suitcase, but it is not in any particular direction.
3) The floor does not exert a force on the suitcase which affects its motion, but the weight of the suitcase pushes down against the floor.
4) The floor exerts a force on the suitcase in the direction opposite to the suitcase's motion causing it to stop.
5) Other (explain).

CRATE A

A man tries to push a crate weighing 400 lbs, but he cannot move it. While he is pushing to the right, is friction one of the forces acting on the crate in this situation?

1) YES
2) NO



CRATE B

If you said YES above, which is larger, the force of the man pushing, or the force of friction on the crate?

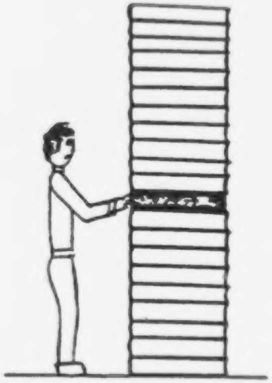
A) The friction force is larger
B) The force of the man pushing is larger
C) These forces are the same size

If you said NO above, think about whether the block exerts a force on the man while he is pushing.

D) The block exerts a force on the man
E) The block does not exert a force on the man, it's just in the way.
F) The block exerts a force on the man that is equal to his pushing force.
G) The block exerts a force on the man that is larger than his pushing force.
H) The block exerts a force on the man that is smaller than his pushing force.

BOOK PILE

Twenty large books are stacked in a pile in Roger's garage, and Roger wants to read the black one in the middle. He tries to pull it horizontally out of the pile without taking the books above it off, but can't move it. This is primarily because:



1) There is a frictional force exerted in a downward direction on the book from the one above it.
2) There are frictional forces acting horizontally on the book.
3) The book's inertia opposes Roger's pulling force.
4) Gravity pulls down on the book
5) Roger exerts the only force on the book, but the book is trapped because of the number of books on top of it.
6) Other (explain).

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MISCONCEPTIONS IN PHYSICS : RESEARCH FINDINGS AMONG BRAZILIAN STUDENTS.*

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Introduction

In recent years several studies concerning students misconceptions in physics have been carried out with students enrolled in introductory physics courses at the Federal University of Rio Grande do Sul (UFRGS), Brazil. These studies have been done or advised by a group of physics professors -- known as the physics education group -- of the Department of Physics of this University. This group has been working for the improvement of physics education both at local and regional levels since 1967; research in physics education, although in small scale and with quite limited resources, also began in 1967 at UFRGS.

Many reasons could be identified as important stimuli to lead the group to perform research studies related to students conceptions and alternative conceptions -- or misconceptions as they are usually called -- in physics. Of course, there is an international trend in this direction, but, certainly, one of the most relevant reasons to do this kind of research is that concepts and their relationships play an essential role in a science such as physics. Emphasis on the importance of concepts in the process of learning is shared by authors such as Gowin (1981) by claiming that "people think with concepts", Gagné (1977, p.185) in pointing out that "the acquisition of concepts is what makes learning possible", and Ausubel et al. (1978, p.88) by saying that "anyone who pauses long enough to give the problem some serious thought cannot escape the conclusion that we live in world of concepts rather than in world of objects, events, and situations". On the other hand, in the light of Ausubel's theory the most important single factor affecting students' learning is their existing knowledge prior to instruction. In this view, learning (conceptual change in learner's cognitive structure) involves the interaction of new knowledge with existing knowledge (conceptions and alternative conceptions).

This paper summarizes the research findings concerning students misconceptions already done at UFRGS. As it will be reported, these findings tend to show that the target population holds the same kinds of alternative conceptions held by similar student populations in other countries. This fact may be interpreted as additional evidence of external validity for research results previously found by other people in the area of misconceptions in physics, and this is the major value claimed by the authors for this paper.

At present, research studies in this area continue to be performed and are being extended to neighboring universities. Furthermore, they are progressing toward the development of educative materials and teaching strategies for a particular student population, using their previously identified knowledge basis.

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The studies

Alternative conceptions in electricity among college students have been investigated by Domínguez and Moreira (1985, 1986, 1987). The subjects of their study were twenty-six engineering students enrolled in an introductory electromagnetism course at UFRGS, in 1984. Clinical interviews were used to detect alternative conceptions held by the students concerning the concepts of electric field, electric current intensity, electric potential, and electric potential difference in simple electric circuits. In addition, the work examined the permanence of these alternative conceptions after instruction. The results showed that the clinical interviews proved to be a very useful technique for identifying certain students' alternative conceptions and that, in general, these existing conceptions remained almost unaltered after instruction (Moreira and Domínguez, 1987). Prior to instruction, most of students responses suggested the following misconceptions :

a)Electric Field: students do not consider electric field as a vector, confuse it with electric force, and believe that it is some kind of finite atmosphere surrounding an electric charge (Domínguez and Moreira, 1987).

b)Electric Current Intensity: students consider that a constant current "leaves" the terminal (either positive or negative) of a battery independent of the resistors existing in the circuit; it depends only on the battery. They also consider that this current is consumed or transformed into energy , in a series circuit, by every circuit component it finds on "its way". The electric current, thus, is not conserved, part of it is used or transformed each time it "crosses" a component. The "clashing currents" model (Shipstone, 1984), according to which the electric current leaves both terminals of a battery and is consumed within circuit elements, was also detected in at least one student (Moreira and Domínguez, 1986).

c)Electric Potential Difference: students emphasize electric current rather than electric potential difference in analysing simple electric circuits; they mix up electric potential difference and electric current, or consider that electric potential difference is a consequence of electric current and not its cause (Moreira and Dominguez, 1986).

d)Electric Potential: not a single student evoked a meaningful conception or alternative conception concerning this concept. It seems that prior to instruction students do not assign even functional meanings to this concept (Domínguez and Moreira, 1986).

After instruction , only five out of seventeen students changed their conceptual understanding meaningfully from existing alternative conceptions to scientific concepts. There was therefore a poor progress of the students in learning scientific concepts.

Another investigation (Silva, 1986; Silva and Moreira, 1986, 1987) carried out with nineteen physics and chemistry students was concerned primarily with the identification of students' alternative conceptions of temperature, heat, and internal energy before and after instruction. Data obtained from interviews made it possible to detect and organize categories of conceptions of temperature and heat, such as " the concepts of temperature and heat were confused", and "temperature is the amount of heat a body contains". The results also showed that some students' existing alternative conceptions tended to remain the same after they had been exposed to instruction on these subjects, e.g., "heat is a substance which can be stored within a body". On the other hand, most students had no single conception of internal energy, even after instruction.

Analysing data gathered from several thousands of students' answers to some multiple-choice items of the physics entrance examination of UFRGS during five years, Axt (1986) identified the existence of spontaneous conceptual structures, a kind of alternative "laws" of motion, which conflict with the formal Newton's laws of motion. Students' responses were found to offer evidence of most of the usual alternative conceptions in this area of physics, which were already widely reported (e.g., Gilbert and Watts, 1983), and can be summarized as follows :

a)If a body is moving there is a force acting on it in the direction of movement.

b)Constant motion requires a constant force.

c)If a body is not moving, there is no force acting on it.

A test having 15 multiple-choice items on physics was designed by Silveira et al.(1986) to detect whether or not a student possesses the Newtonian conceptions of force and motion. The five options provided for each item contained the scientific conception (correct option) and some known or suspected alternative conception. This test was administered to different groups of undergraduate students. Their scores were analysed by means of statistical tests in order to gather evidences of validity for this instrument. Data offered evidence of predominance of alternative conceptions among students. This study also involved an experiment carried out with 68 enginnering students distributed in two sections. These students were taught by being first presented to alternative conceptions as if they were scientifically accepted, then inconsistencies were progressively introduced to pave the way for the introduction of scientific conceptions. The aforementioned test was administered to both groups before and after instruction. Statistical analysis of data gathered at this time indicated that the score differences between posttest and pretest were statistically significant for the treatment group, which offered evidence of students' conceptual change.

A similar test is being developed to detect whether or not students have the notion of conservation of electric current. The point in constructing and validating paper and pencil tests to detect whether or not a student has a certain conception is that they might be used in classroom situations. So far, the best instrument to investigate students' previous knowledge, including conceptions and misconceptions, is the clinical interview; however, it is not adequate for classroom purposes since it is time consuming and requires expertise. But research data gathered through clinical interviews can be used as a basis for constructing tests which can be easily administered and evaluated. This is exactly what is being attempted at UFRGS with the development of these tests. Nevertheless, besides tests, concept mapping is also being tried as a possible strategy to detect and to deal with misconceptions (Moreira, 1987).

In addition to the development of instruments to detect alternative conceptions, new studies are being currently performed using clinical interviews in order to identify misconceptions in other areas of physics, as well as to gather adequate information to prepare instructional materials and teaching strategies which take into account specifically relevant aspects of students' prior knowledge.

One of these studies is being carried out to investigate students' understanding of optics, particularly conceptions related to reflection, refraction, interference, and diffraction. Preliminary data suggest, again, that students' conceptions and alternative conceptions are almost the same before and after instruction (Belle and Buchweitz, 1987). Another research (Gravina, 1987) already started (the pilot study was conducted last semester) in order to identify additional alternative conceptions in electricity and to test educative materials and teaching strategies specially designed to effect conceptual change from alternative to scientific conceptions, using student's prior knowledge as starting point.

Conclusion

This paper summarizes results of research studies carried out at UFRGS to investigate students conceptions and misconceptions -- or alternative conceptions -- in physics.

As it was said at the beginning, one of the basic reasons to do this kind of research is that students previous conceptions and misconceptions play a crucial role in subsequent learning. According to Ausubel's theory, for example, the most important single factor influencing students' learning is their existing prior knowledge. In the light of this theory, meaningful learning involves the interaction of new knowledge with existing knowledge (contextually accepted conceptions as well as alternative conceptions). To Ausubel, subordinate meaningful learning, or the so-called assimilation principle, is a process in which a new concept or proposition (a subsumer) is assimilated under a more inclusive concept or idea already existing in the cognitive structure. The product of such interaction is $\underline{A'a'}$. The use of superscripts in this interactional product suggests that not only the new information $\underline{a}$ but also the existing subsumer $\underline{A}$ are modified in the assimilation process. However, when the new information is perceived as a mere example, as just another instance, of the subsumer it changes very little. This is called derivative subsumption. On the other hand, when the new information is perceived as some sort of extension of the subsumer, as related but not implicit in it, the process is called correlative subsumption and implies a more substantial modification of the subsumer.

Ausubel suggests that assimilation probably facilitates retention, and in order to explain how new information recently acquired can be recalled separately from the anchoring idea he proposes that during a certain period of time the interactional product $\underline{A'a'}$ is dissociable, that is :

$$A'a' \rightleftharpoons A' + a'$$

However, immediately after assimilation a second process -- which he calls obliterative subsumption -- starts : new information becomes spontaneous and progressively less dissociable from their anchoring ideas until they are no longer reproducible as individual entities. That is, the dissociability of $\underline{A'a'}$ decreases until it is reduced just to $\underline{A'}$,i.e., the modified subsumer.

Differently from meaningful learning, rote learning is defined as a process in which new information is arbitrarily and literally stored in cognitive structure with no interaction with existing subsumers.

These highlights of Ausubel's theory provide a framework for a possible interpretation of the results reported in this paper. This interpretation is presented here as a concluding remark :

When a misconception serves as subsumer for new learning, the final result of the process is the modified misconception, i.e., the misconception with some additional meanings. Nevertheless, since this modification might be quite small, this result in some cases is almost the original misconception. This would explain the stability of misconceptions found in the studies referred in this paper.

On the other hand, when students do not have subsumers to link the new concept or proposition, or do not perceive any relation between the existing subsumers (misconceptions or not) and the new information, they will rote learn this information assigning no meaning to it. This hypothesis would explain why concepts like electric potential and internal energy did not seen to have even functional meanings for the students who were the subjects of studies referred in this paper.

Of course, this is just a preliminary attempt of interpreting research findings on misconceptions in the light of Ausubel's theory. A more complete interpretation is being currently attempted by the authors.

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"HIGH SCHOOL STUDENTS' IDEAS ABOUT ENERGY OF CHEMICAL REACTIONS"

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1 - INTRODUCTION

This study describes an investigation of misconceptions held by Portuguese secondary students about the processes of energy changes associated with a chemical reaction.

In Portugal, the topic of chemical reactions is formally introduced in grade 8 (age 14). At this level the idea of chemical change is simply equated in terms of the separation/reorganization of atoms with no mention of the nature of the chemical bonds involved as this is only elaborated in grade 10; notions of exothermic/endothermic processes are introduced in the usual manner (heat given out/heat taken in). In grade 11 (age 17) the above aspects are refined and energy changes are now explained in terms of a formal model involving the ideas of bond breaking/bond forming and associated enthalpy changes. At this stage students become familiar with the notion of internal energy.

An important type of changes which are studied in the chemistry course are spontaneous endothermic reactions. These have been recognized to be conceptually difficult for secondary students (Johnstone et al. 1977). Part of the trouble probably derives from an inadequate analogy with mechanical systems (e.g. fall of bodies) where spontaneity is usually related with energy being evolved. However, little is known about students'misconceptions in this domain.

Until grade 11, Portuguese students are not formally taught the relationship between the processes of energy changes (energetic component) and the way the reaction takes place (structural component). Also, the topic of energy, namelly the principle of energy conservation, is only introduced in grade 9 in the physics course and with no links with the chemistry course, although the teacher in both courses is the same. Thus for beginning chemistry students (grades 8 and 9) one may reasonably predict the two above aspects interfering with proper learning. Further, inadequate representations elaborated in this domain in the earlier years may later become barriers to conceptual change as suggested by research on students' misconceptions in other content areas (see Driver and Erickson, 1983; Gilbert and Watts, 1983, for reviews). It follows that this is an important domain of enquiry.

2 - RESEARCH QUESTIONS

The study aims to investigate the following questions:

a) What conceptions are used by Portuguese secondary students to explain the energy changes associated with a spontaneous endothermic reaction, in particular: (i) whether structural and energetic aspects are articulated; (ii) how the principle of conservation of energy is used.

b) How different these conceptions differ between students from different grades?

c) Which possible reasons lie behind existing misconceptions?

3 - METHOD

A) The sample (N = 30) was formed by two instructional groups: 15 subjects from grade 9 (average age 15; 10 males) and 15 subjects from grade 11 (average age 17; 15 males). Subjects were randomly selected from an initial group of volunteers drawn from mix ability classes of a high school located in an urban area of Portugal. In each sub-group students had been exposed to the same chemistry course and also had the same teacher. All had already been taught the principle of conservation of energy.

B) The experimental task consisted in dissolving NH_4Cl (10g) in H_2O (50 cm^3)*. The average temperature fall was -12.9°C. All NH_4Cl was dissolved. The reaction took place in a 100 cm^3 insulated container (with thermometer and stirrer) so that thermal energy transfer with the environment was minimized. This design was intended to promote cognitive conflict involving the idea of energy conservation.

C) Students were interviewed individually during the summer of 1984. No time limit was set and the average time spent with each student was 40 min. Tipically the experiment was first demonstrated by the researcher. Subjects were then requested first to describe and then to explain what they had observed using a semi-structured interview format. Typical questions were: "What happens to the NH_4Cl?" or "How do you explain the temperature fall?". The interviews were tape recorded and later transcribed in written protocols. These were subsequently analysed to extract students' ideas following closely the method proposed by Erickson (1979). Ideas were organized under two content oriented categories: structural component category and energetic component category.

4 - RESULTS

Results obtained for each content category are presented first. Tentative relationships between the two content categories are then analysed.

4.1 - Structural component

Five basic ideas were used by students to explain the way the reaction took place (Table 1). The overall results may be rationalized in terms of a model of chemical change involving the attibute of the permanent nature of substance (Méheut et al. 1983). As the three last ideas closely correspond to misconceptions already referred to in the litterature though in other contexts (see Andersson, 1986 for a review) only details concerning the first two will be dealt with in the following sections.

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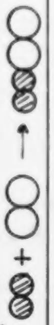
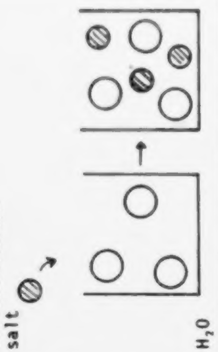
* As it involves deep structural changes with separation of ionic entities and not simply intermolecular changes (state of aggregation), this transformation is better acknowlegded as a chemical change (Gil, 1974).

Table 1 - Ideas used to explain the chemical change

Ideas	Main features		N(grades 9 + 11)%
	Process of chemical change	Conservation of reactant(s)	
Serial interaction	sequence model	no conservation	0 + 3 = 3 10%
Principal reactant	different status between the two reactants; only one has an active role	H_2O is conserved	1 + 2 = 3 10%
Conglomerate *	reactants simply connected fi: 	H_2O and NH_4Cl conserved	2 + 1 = 3 10%
Diffusion **	no interactions 	"	2 + 4 = 6 20%
Mixing ***	H_2O and NH_4Cl simply mixed; only macroscopic aspects; descriptive view.	"	6 + 3 = 9 30%
Uncodeable+Other	-	-	4 + 2 = 6 20%

For references see: (*) Shollum, 1982; (**) Cosgrove and Osborne (1981); (***) Andersson (1986).

Serial interaction

The processes of bond breaking/bond forming are not simultaneous, rather they take place one after the other. Firstly, the reactants are broken down into smaller structural units so that the initial substances ceasses to exist; secondly, new chemical species are formed:

Manuel (grade 11): "When water reacts with ammonium chloride both reactants are broken down into particles... then they combine to form new species."

There is no distinction between the nature of the structural units of the two reactants (NH_4Cl "molecules"). H_2O molecules would also suffer intramolecular ruptures:

Rui (grade 11): "Maybe the molecules of ammonium chloride all split apart... the bonds of both (the reactants)are broken down and then maybe the atoms coming from the water are gathered together with those coming from the ammonium chloride."

In spite of the erroneous character of these explanations, serial interaction is the concept which is closest to the acceptable answer according to the teachers point of view.

Principal reactant

There is an interaction between the two reactants but structural changes only take place in NH_4Cl; the water remains unchanged and only plays a passive role:

José (grade 11): "The bonds in the ammonium chloride are broken down and the water doesn't experience any change."

It is not clear whether the fragments formed are NH_4^+ / Cl^- ions or other entities: "... it's (the salt) breaking into smaller particles..." The notion of a principal reactant may presuppose that changes at microscopic level would only occur where changes are observed at macroscopic level, a view reflecting a perceptually guided model of chemical change. Thus it is highly predictable for heterogeneous systems in which one of the reactants is not directly perceived (combustions) or remains apparently unchanged (this study). Such a misconception is probably reinforced by language habits: textbooks often mention "the reaction of... with...", instead of "the reaction between... and...". Also relative order of the reactants represented in the reaction equation may be influential (the "principal reactant" would be the one written in the first place).

To sum up, the configuration of the results doesn't reveal any major differences between the two instructional groups thus suggesting the persistence of earlier misconceptions despite formal instruction. Ideas of chemical change involving some sort of interaction between the reactants seem to be difficult (30% of subjects) and over 70% of the students are probably guided by perceptual features in their understanding of the chemical change. No one conception is acceptable according to academic standards.

4.2 - ENERGETIC COMPONENT

For the students, four conceptions were used to explain the processes of energy changes (Table 2). These conceptions may be organized under two categories according to whether students did equate energy changes as the result of a process involving both reactants and energy changes as the result of a process involving both reactants and products (bilateral process) or simply the reactants or the products (unilateral process).

Table 2: Ideas used to explain energy changes

Ideas	Main features	N(grades 9+11) %
Sequential transfer	sequence model	0 + 2 = 2 6.7%
Absorption	temperature and energy not differentiated, hence energy was used; no energy conservation.	3 + 1 = 4 13.3%
Dissipation	temperature and energy not differentiated, hence energy was transferred to somewhere; energy conservation.	3 + 5 = 8 26.6%
I can't explain it	temperature and energy not differentiated; the energy is conserved, hence T should remain constant.	7 + 4 = 11 36.7%
Uncodeable + Other	-	2 + 3 = 5 16.7%

4.2.1 - Energy of the reaction is the result of a bilateral process

Sequential transfer

The energy associated with the chemical change results from a two step process: firstly, there is energy absorption (bond breaking of reactants); secondly, energy is evolved (bond forming of products). The net effect is endothermic.

Miguel (grade 11): "... the bonds (of reactants) break apart and there is absorption of energy. Then, the atoms which come from the water are linked with those coming from the ammonium chloride and energy is evolved... this (energy evolved) is smaller than the energy absorved... during the reaction there is energy consumption."

Consistently, the two students who used this idea also used a sequence model in 4.1 (Serial interaction). They do not seem to appreciate energy transformations (kinetic to potential) at the molecular level; the process would simply involve a transfer of the energy existing between the water molecules, "non-bonding" energy, to the bonds. The temperature of the water would depend on the amount of "non-bonding" energy and that would explain the temperature fall:

Miguel (grade 11): "There is energy scattered between the water molecules and it was absorved by the new bonds formed... I think that's it. The temperature is... how can I put it? Let us say, it results from the energy not used in the bonds. But the total energy inside (the container) is always the same".

For these students conservation of energy involves the balance of two terms, each corresponding to the same form of energy though functionally different. "Non-bonding" energy seems to have the status of a material substance and it's not excluded that the concepts of temperature and energy were not fully differentiated.

4.2.2 - Energy of the reaction is the result of a unilateral process

In this case the idea of a net effect was absent. A common trait to all answers was to identify temperature with energy (heat energy) a common misconception for junior and secondary students (Erickson, 1979; Tiberghien, 1984). Such a misconception facilitated cognitive conflict between ideas of energy conservation and the observation of the temperature fall. Each of the three conceptions belonging to this category corresponds to a different way used by the students of solving such a conflict. An important influence of perceptual aspects of the task on the students' views should not be excluded. They should, therefore, be regarded as tentative explanations.

Absorption

The energy decreases inside the container because of the temperature fall. Thus it was used during the process, probably absorbed by one or both the reactants:

Isabel (grade 9):

(student, S) : "... some energy was used.

(interviewer, I): Why?

S : The temperature decreases!

I : How was the energy used?

S : Well, the container is insulated and energy can't get through it, thus maybe the substance, that substance (ammonium chloride), kept the energy."

These students achieved cognitive reconciliation assuming that energy which is "absorved" doesn't matter when considering energy conservation. With a non-insulated system they would probably use the conception which is presented below.

Dissipation

As the temperature decreases the energy decreases hence energy is transferred to somewhere outside the solution. Two basic ways were suggested for such an energy transfer:

(i) energy occupies the small space existing between the cover of the container and the solution:

Neto (grade 9): "... The energy comes out (of the solution) to that small space... to that area (under the cover)... energy must have been released because the temperature is like that (decreases)".

(ii) The system is not insulated. Hence energy may be transferred to the environment, which becomes hotter:

Carlos (grade 11): "... The temperature (of the water) decreases because energy was released... the container isn't completely insulated. It (energy) comes out through these little holes in the cover (where the thermometer and the stirrer enter)".

The logic of this answer implies that endothermic reactions may be equated with a temperature increase of the surroundings It also reveals an inadequate understanding of thermal balance.

I can't explain it

Subjects consider that energy must be conserved but don't know how to equate it with the observation of the temperature fall (which is related with an overall energy decrease). Thus, in this case, the conceptual conflict was not solved.

Filipe (grade 11): "The energy remains the same inside the container. It can't come out because it's a thermal insulator... the temperature should also be the same! I am confused, the temperature decreases and the energy is constant... I don't see how it works!"

These students would probably use a "Dissipation" explanation in the case of a non-insulated system.

To sum up, similar ideas were generally used by students of different grades to explain energy changes associated with the reaction. As for the structural component (4.1) this result suggests that previous knowledge was probably an important factor as misconceptions seem to have persisted despite formal instruction.

The results also suggest that the principle of conservation of energy might have been accepted but not properly understood. Students in grade 11 weren't able to appreciate the conversion of kinetic to potential energy at molecular level and to equate temperature in terms of average kinetic energy of particles. Rather, they have used conceptually less demanding transfer models. As referred to in the Introduction, the conservation of energy was introduced in the physics course. It is not because students are able to properly conceptualize the conversion of kinetic to potential energy in a physics context (where it can be directly observed with the help of mechanical systems) that ideas about energy conversion will be properly used in a chemistry context where the conversion of one form of energy into another essentially lies beyond personal experiences and have to be equated in terms of internal energy. Teachers should be aware of such a crucial difference.

Over 90% of the students did not articulate energy changes with the way the reaction took place. Students appear to have conceptualizad these two related aspects as two independent bits of information. In particular: (i) the relation of a net effect of energy with structural aspects was absent; (ii) only part of the system, reactants or products, was considered. Except for students using a sequence model no coherent pattern emerged for ideas used by individual subjects to explain both the structural and the energetic component. Sequence models were also used by secondary students to explain how

electric current flows in a circuit (Shipstone, 1982) and, in our view, their use is consistent with a limited capacity memory model (Johnstone, 1980). In fact, understanding of energy changes associated with a chemical reaction implies a hollistic perception of the reacting system involving the coordination within and between two sub-systems, hence a high level of information. Use of a sequence model may be a way to lower memory overload.

5. CONCLUSIONS

The main goal of science education is to promote concept learning, for concepts are what we think with. Teachers and curriculum developers, although generally competent in their subjects may not be familiar with students' common misconceptions. Improvements in student learning can be brought about if they are more knowledgeable about the nature of students' misconceptions and how to plan instruction so as to promote conceptual change.

The results of this investigation suggest that the topic of chemical reactions in secondary chemistry curriculum in Portugal should be carefully redesigned. First, the timing of introduction of certain key ideas (e.g. energy conservation) and its coordination between different subject areas should be under close scrutiny. The process of transferring new information from one context to another is not automatic. Such a transfer, if not properly achieved, may lead to inadequate conceptualizations which may persist despite formal instruction. Secondly, one has to question the role of some standard experiments used to introduce essential ideas of thermodynamics in schools. For example, widely used non-insulated systems don't seem to be of great help to promote conceptual change in the misconception "temperature = energy (heat energy)". Thirdly, the language used to introduce chemical concepts may be responsible for the development/reinforcement of inadequate ideas as the meanings students ascribe to science facts and principles may not coincide with those intended by the teacher. Moreover, both teacher and student may be unaware of this (possibly the case of the notion of "principal reactant").

Of course the above proposals are not intended to solve all the problems faced by students in the understanding of spontaneous endothermic processes. This will remain a difficult concept to master until they can appreciate the direction in which energy changes take place. It should be noted that recent proposals have been made to teach a simplified version of the second principle of thermodynamics even to 14/15 year old students (Solomon, 1982). We hope that all these proposals will contribute to a more successfull science teaching effectiveness in such an important area of chemistry.

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HOW STUDENT CONCEPTIONS OF THE NATURE OF CHEMISTRY AND MATHEMATICS INFLUENCE PROBLEM SOLVING

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Students beliefs about the nature of chemistry and mathematics and about learning and problem solving in these disciplines establish a psychological context for problem solving. The focus of this study is the role beliefs serve in limiting strategy selection during problem-solving and in determining the conceptions students form during their chemistry and mathematics courses.

Our results show evidence that beliefs influence (1) selection of algorithms; (2) the degree to which students rely upon algorithms; (3) willingness to examine concepts and to attempt alternate solutions when solving problems; (4) decisions as to when a problem is solved; (5) the degree of evaluation (6) confidence in one's solution; (7) perceptions of what tasks and problems are "fair" or solvable; and (8) basic approaches to learning and studying chemistry, such as determining the appropriateness of relational or instrumental approaches.

The Role of Beliefs in Science and Mathematics

Recent research in mathematics education has examined student beliefs about the nature of mathematics and learning in mathematics, as well as the relationship of these beliefs to both learning and problem solving. Research on the developmental processes of adults, such as that by Perry (1968), Buerke (1981), and Belenky, Clinchy, Goldberger & Tarule (1985) has also emphasized the importance of epistemology to learning within disciplines such as mathematics and science.

This paper will describe the results of two recently-completed studies of beliefs and how these studies relate to misconceptions. One study was an investigation of the relationship between beliefs and problem solving in general chemistry (Carter, 1987). The other was a study of math-anxious individuals in a course designed to help them overcome their anxiety and become successful with mathematics by changing their mathematical beliefs (Yackel and Carter, 1987).

Beliefs

Students have many beliefs about learning in chemistry and mathematics and about the nature of problem-solving. These beliefs are central to what our students learn because they "establish the psychological context" for work in science and mathematics (Schoenfeld, 1985). Beliefs determine the nature of the strategies our students use in studying and learning chemistry or mathematics, and in solving chemistry or mathematics problems (Frank, 1985; Carter, 1987).

Schoenfeld (1985), proposed a framework for problem-solving that incorporates 4 factors: resources, heuristics, control strategies, and belief systems. The first category, resources, contains the individual's knowledge of mathematics that can be used in the problem, and includes concepts, intuition, and algorithms. Heuristics refer to generalized strategies and techniques, such as drawing figures, working related problems, working backwards, trial and error, etc. Control strategies include meta-cognitive stages or acts such as planning, monitoring and decision-making. The fourth, overarching, factor is belief systems, or the individual's world view of the discipline. Beliefs determine what knowledge or problem strategies are seen as relevant in a given context. Students who use formal knowledge or a given strategy in one context will not necessarily use that knowledge or strategy in another context, because they do not believe it is useful or applicable.

Many students believe that chemical or mathematical knowledge is handed down by authorities and that they are not capable of understanding or discovering this knowledge. Thus, if they forget a formula or an algorithm, they are stuck. They can't derive an equation or develop a problem-solving strategy on their own. They must accept the algorithms we present them at face value, without trying to understand these ideas or relate them to other ideas. Skemp (1978) calls such beliefs "instrumental". Students who have instrumental beliefs view knowledge as a series of rules without reason. These beliefs drive the student's approach to learning.

> Students who have constructed instrumental beliefs about mathematics expect that future classroom mathematics experiences will fit these beliefs. They intend to rely on an authority as the source of knowledge, they expect to solve tasks by employing formal procedures that have been explicitly taught, they expect to identify superficial cues when they read word problem statements, and so forth.
> (Cobb, 1986)

Types of Beliefs

We found students to have beliefs about the following:

1. The nature of chemistry or mathematics.
2. Their ability to do chemistry or mathematics.
3. How (and why) chemistry and mathematics should be taught.
4. The role of the teacher.
5. The role of the student.
6. How one should study and learn chemistry or mathematics.
7. How one goes about solving problems.
8. The definition of a problem.
9. Strategies appropriate when solving a particular problem.

These beliefs influence:

1. The student's selection of algorithms.
2. The degree to which the student relies upon algorithms.
3. The student's willingness to examine concepts and attempt alternate solutions when solving problems.
4. The student's decision as to when a problem is solved.
5. The degree to which the student evaluates the solution.
6. The degree to which the student is confident of the solution.
7. The student's perception of what tasks and problems are "fair" or solvable.
8. The student's basic approaches to learning and studying chemistry or mathematics.

To illustrate the central role of beliefs, this paper will use excerpts from interviews with two students (Lynne and C. J.) enrolled in a college general chemistry course. These students were in the mainstream course for science and engineering majors at Purdue. Lynne and C. J. are illustrative of two epistemologies and two approaches to learning chemistry. Although this paper focuses on individual students the model for the influence of beliefs drawn from these individuals is transferable to all students interviewed. This paper will then use excerpts from interviews with one student (John) in the math anxiety study to illustrate that changing beliefs about the nature of a discipline can create a psychological context in which students can learn relationally and begin to overcome their misconceptions.

Instrumental Understanding and Received Knowledge

Lynne's beliefs about chemistry could be described as instrumental. To her, chemistry was many loosely-connected or totally unconnected rules and procedures. Chemistry, was not a subject of which she could make sense. Her view of learning chemistry was one of total reliance on authorities such as texts and teachers.

> L: It's a subject that some things you aren't too sure about. I mean, no one is really definite about. You just have to believe that they're right. And, a lot of things are really abstract. You just have to kinda deal with it.
>
> C: How much of it don't you believe in?
>
> L: A lot of it. ... It's like, you really don't know. You just have to believe it. There's a lot of things you just have to believe. Like, half of it, I would say.

To Lynne, chemistry was not a science or a way of looking at the world. Instead it was a subject she learned in school, something she just had to accept at face value from some remote, nameless authority.

Lynne was totally dependent on these authorities when operating in a chemistry context. When involved in chemistry tasks (but not in non-chemistry tasks) her transcripts were full of the word "they." She talked about what "they" wanted her to do, the strategies "they" wanted her to use, and what "they" said about learning chemistry. This remote, externalized (Confrey, 1984) view of chemistry influenced what she did when solving problems. For example, after working unsuccessfully on a problem for a considerable period of time Lynne described what made the problem difficult.

> There's a lot of variables involved and it's like they want three different things. You know, what they could of done is break it down into three different problems so you get one answer. When they ask you a question about that answer you can keep bringing it down.

When Lynne was asked to break the problem down into the three steps "they" should have broken it into, she was shocked at the suggestion. After she was prodded, she unsuccessfully attempted to break the problem down into the steps. After bailing out of the situation she still felt that "they" should have broken it down for her.

Memorized solutions

Lynne's view of the role of authority in chemistry meant that she relied heavily upon memorized solutions and problem types that she had to be explicitly taught. She did not feel capable of solving a chemistry problem based on her own knowledge and reasoning processes. This influenced her general approach to solving a problem.

> Write out what they give you. Try and think of something else that is similar to the problem that would give you direction to work in. And think of some similar examples.
>
> Work like from the beginning of the problem. Work it as far as sentences go, and parts of sentences. And sometimes when you do that some things in the sentence that come first may not be done till like in the middle or something. But you have to read the problem and work it in bits and pieces.

When presented with a chemistry problem Lynne would read a phrase and immediately jot down an equation or try and do a calculation. She did not try and develop an overall representation for the problem, or even read the problem statement in its entirety. She just started plugging in what she could, phrase by phrase. (This is what she believed "they" expected her to do).

When outside of the chemistry context or on non-traditional chemistry tasks Lynne did not exhibit this extreme reliance on authority. After qualitatively reasoning through a non-traditional gas law task she was asked whether she developed qualitative representations for more traditional gas-law problems.

> No, because in gas laws you have the PV = nRT. And you don't really think about it. The first thing you gotta do is just get the problem done. Oh -- sometimes, if you want to check your answer and you're not too confident about it.

Her goal was to get the problem done, not to understand or learn from the problem-solving process. She did not even think about developing a representation, she just had to get an answer. The only time she used qualitative reasoning was when she wanted to check her answer. (Checking her answer was very rare on the interview problems.)

Received knowledge

While Lynne is what Skemp (1978) would call an instrumental learner, Belenky, et.al. (1986) would describe her as being at the epistemological level of "received knowledge" For "received knowers" words are central to the knowing process. Lynne's knowledge comes from the words of others, in this case, the authorities who write chemistry texts and teach chemistry classes. These authorities are the sources of truth. Belenky, et. al. would describe Lynne as equating "receiving, retaining, and returning the words of authorities with learning." (Or at least with learning chemistry).

Lynne was confused and frustrated when asked to solve problems instead of traditional textbook exercises. She expected to be explicitly taught any necessary strategies. She stored material in her head without trying to assimilate it. It was simply filed and reproduced on exams and quizzes. Applying the material or working novel problems on her own was unfair. Belenky et.al. (1986) describe the learning strategies of received knowers this way:

> These women either 'get' an idea right away or they do not get it at all. They don't really try to understand the idea. They have no notion, really, of

> understanding as a process taking place over time and demanding the exercise of reason. They do not evaluate the idea. They collect facts, but do not develop opinions. Facts are true; opinions don't count.

Students with instrumental beliefs or in the position of "received knowledge" such as Lynne tend to believe that scientific and mathematical knowledge are absolute. Learning is a process of internalizing absolute, disconnected bits of knowledge and procedures along with the greater surface cues that allow them to reproduce this knowledge when asked to do so by an authority.

Beliefs and contextual problem solving

Lynne's beliefs constrained the strategies she could use when solving problems. Problem solving in chemistry was merely a matter of reproducing the appropriate patterns that she had been repeatedly taught. However, she was free to use creativity and reasoning on problems that were in non-chemistry contexts, as illustrated by her performance on the following task:

> You are making homemade pizza. You have 1500 g of pizza dough, 105 slices of pepperoni weighing 300 g, 150 pieces of sausage weighing 450 g and 1200 g of cheese.
>
> How many complete pizzas can you make if each pizza contains 250 g of dough, 25 slices of pepperoni, 30 pieces of sausage, and 75 g of cheese?
>
> What will be the total weight of all the complete pizzas made?

This task was "concrete" to Lynne. She read the question and immediately developed a viable strategy. Even though she became confused at times, she was always able to look at her work, make sense of it, and recover from the confusion. She knew when she was confused, and persisted in working on the problem even when confused. She evaluated each step several times, recalculated and checked to make sure that her answers made sense. When the task was complete she was confident of her answer.

Immediately after completing the task, Lynne was presented with a typical limiting-reagent task.

> A mixture containing 100 g of hydrogen and 100 g of oxygen is sparked so that water is formed. How much water is formed?

Lynne's approach to this problem contrasted sharply with her approach to the pizza task. She immediately began working and came up with a snap answer of 150 grams. She just plugged in the numbers; she did not try to develop a qualitative relationship, because she had a numerical relationship. Since it was chemistry, something she "just had to believe," she did not attempt to make sense of the problem as she did with the pizza task. She had to "just get the problem done."

Her initial evaluation of the answer was only that it was "weird." She started to write an equation, but immediately decided that was "stupid." Unlike the previous task, she had no way of checking her answer. She did not make connections between grams and moles as she had with slices and grams. She did not persist in seeking alternate solutions while working this task.

Misconceptions

Lynne's transcripts were permeated with misconceptions. For example, she made no distinction between atoms and molecules. Acid "atoms" donated nuclear protons and elements changed their identity in acid/base reactions. Compounds could not react with other substances because "their shells were full" and they were stable. Lynne never attempted to make connections between the qualitative and quantitative representations of the concepts in chemistry or even connections between the same concept used in two different chemistry contexts. She became very frustrated when she was asked "why" or when given non-traditional problems because she saw no need to make conceptual relationships. Her knowledge was stored in thousands of individual little boxes. There were no connections among the boxes, because there was no need for connections.

Nothing in the chemistry course convinced Lynne that her conceptions were not viable. To her, the failures she experienced were failures of her memory, not of her conceptual network. Each specific instance was a new situation to learn. There was no need to connect the situations. If she made a mistake on a homework problem or on an exam question it was because she "forgot" something or had never seen a particular problem type, not because there was a problem with her understandings.

Lynne and other "received knowers" can be quite successful in courses that don't require real problem-solving or conceptual integration. For example, Lynne took the same chemistry course over the next semester. She was very successful this time in a course that was taught in a much more memory- and algorithm- oriented fashion.

Relational learning and constructed knowledge

Lynne's approach to chemistry can be contrasted to that of C. J. To C. J., chemistry was an extremely creative discipline. He disliked memorization, and constantly struggled to understand how and why. This understanding was chemistry. The teacher and text were not authorities. They were there only to "expand his mind" or occasionally explain difficult concepts and provide insight by asking questions. Practical applications of what he learned in chemistry were important because C. J. believed that he only understood a concept when he could apply it to a new situation.

Successfully completing an exercise was not sufficient for C. J. He found non-routine problems to be challenging and fun, not damaging to his ego. Non-routine tasks were tests of his relational approach to learning, which allowed him to examine his understanding of concepts and problem solving. They were not unfair situations or tests of memory.

Autonomous learning

Lynne's idea of a good teacher was someone who told her exactly what she needed to do, and tested her over exactly what she was told. C. J. had a different perspective. For C. J. good teachers were creative and provided examples that "give you like insight into different problems. If you know how to do one, you can apply that to different things." Teaching assistants

> shouldn't hand it down to you on platter or something, but they should be able to clear up what you don't understand from the professors or the book. But they shouldn't do your homework for you or anything like that. ... Its almost like teaching by asking questions.

C. J. would rather have someone explain concepts than work a problem for him. He controlled his own learning. Because C. J. viewed chemistry as something he could learn and do creatively, he did not have to rely heavily upon external sources or authorities for success. To succeed in the chemistry course: "The most important thing is like ME putting the time and effort to it."

Understanding

C. J. tried to focus on general relationships. His beliefs allowed him to explore problems and use multiple strategies. His focus was from the general to the specific. Unlike Lynne, who focused entirely on right answers as measures of understanding, C. J. believed that right answers and good grades did not necessarily imply understanding.

> ... like you do a problem and get the right answer, that's not really important. Just because you can do a problem, doesn't mean you know chemistry. You have to understand components involved in chemistry, like why an element reacts with another in a certain situation. I mean, just using the proper equation doesn't mean that you know it. I think understanding is the important part.

Understanding lead to emphasis on intuition and creativity for C. J. Understanding was part of his view of intuition.

> I like intuitive things - I hate memorizing - if you understand why something happens, you can usually work through it, whereas if you just memorize a formula, you forget the formula [and] you don't know what's coming. ... So I usually try to think of things that strengthen my understanding and not ways a professor wants you to do it.

Understanding was C. J.'s goal, not good grades. The important issue is whether or not a concept makes sense.

Because he believed that the relations among concepts are central to understanding chemistry, C. J. felt that it is reasonable for teachers to test for conceptual understanding. Thus, C. J. believed it was fair to put unfamiliar questions on exams, as long as students have the specific background information -- a position in strong opposition to Lynne's view.

While Lynne viewed creativity as alien to chemistry, for C. J. creativity was an essential part of the learning and problem-solving process. He believed that to explain chemical concepts, or even understand concepts, one has to be creative.

> I think you'd have to be creative in how you explain things - because chemistry is -- you have to be kind of creative to understand about atoms. Just to visualize an atom is kind of [a] creative thing in itself, because you're never going to see one. You just think about and draw on paper, but you're not going to actually look at one.

C. J. was not always encouraged by his teachers to be creative. He believed they often encouraged use of algorithms without understanding.

> I hate to really set down a set of rules and how you do something. A lot of ways I've done things is different from the

> ways other people have done them. In high school they don't like that. My calculus teacher liked that but none of my other teachers really liked it. I don't like when somebody says "this is the way to do something," because that's usually not true.

Instead of being frustrated or confused by multiple ways of working problems, C. J. wanted to be encouraged in creativity. He felt constrained by rules, unlike many of his classmates who believe that learning chemistry is learning rules and exceptions (Carter and Brickhouse, 1987). His self-confidence was strong enough that he was able to continue approaching chemistry creatively in spite of the disapproval of most of his high school teachers and the vastly different approach to chemistry of many of his peers.

For example, C. J. did not consider the following task a problem:

> How many pennyweight of P_4S_{10} (444 amu) are produced by the reaction of 0.50 pennyweight of S_8 (256 amu) with excess phosphorus?
>
> $$4\ P_4 \ +\ 5\ S_8 \longrightarrow 4\ P_4S_{10}$$

He decided that it was not necessary to know the definition of a pennyweight, because he knew the ratio. He quickly came up with an answer of which he was quite confident. He noted, however:

> Some of the people in my class would just die when they saw this. ... They're totally just sticking by the books and I more or less use the books to expand my mind. I use it to think in different ways and they use it to totally close their thinking off. ... Because of the book, they set their ideas as like rigid on how to do stuff, like that one.

He saw his classmates as being constrained by their rigid ideas based on algorithms from the textbook. He, on the other hand, saw the texts as a way to expand his way of looking at the world.

Problem solving

Because C. J. put a strong emphasis on what Skemp (1978) would call relational learning, his general approach to a problem differed greatly from Lynne's instrumental approach. Instead of blindly jumping in and doing something just to get something done or arrive at an answer, he took a different approach.

> I'd think about what the problem is made up of. About ... the different steps it would take to solve it and what I knew about those steps -- and how they related to the stuff in here. I'd look at the information in the problem and see what I knew about that and I'd figure everything out before I wrote anything down.

C. J. began by thinking about the problem as a whole, establishing subgoals, and evaluating how these subgoals relate to the problem. He looked at the information and examined the relationships involved before actually starting to work out the problem on paper. His process was therefore diametrically opposed to Lynne's, who started immediately writing phrases and symbols.

C. J. believed there were several ways to work a given problem. There is rarely a universal "best way" to work a problem.

> ... in all problems there's usually at least two ways - not all of them are equally good. But, in certain situations you've got methods that are better than others but actually one best method, it really depends on the person doing it.

This perspective influenced how C. J. spent time studying chemistry and doing homework. His approach was the antithesis of Lynne's. She spent tremendous amounts of time working the same problems over and over without reflecting on what she was doing. C. J. spent much of his time thinking about what he was doing and trying to understand. When solving a problem he did not just do something in order to get the problem done. He carefully considered what he was doing before he sat down to try anything.

Problems and exercises

C. J. distinguished between problems and "plug and chugs". Problems required creative thinking, not just a definition or a formula. A task he could solve easily, with the use of algorithms or match to a familiar problem type was not a problem. The status of a task as a problem or non-problem depended upon the problem solver.

Sources of chemical knowledge

Chemistry is both creative and created to C. J. He believed he constructed chemical knowledge himself. This was consistent with his view of chemistry as something created by humans.

> ... way back when I think that man got all of these pieces of evidence together and he formed, like he decided like an atom was. ... We've got to know how it behaves, how it reacts with things. We got to decide what it is and something that man has labeled. Like this variable X and man has decided what it is. ... Like it's just the name man has given something. An atom, and that's what started it all on the basis of something man assigned. And man has since created. A lot of the evidence man used in creating the atom was discovered but it was still the atom he labeled that in essence he created the atom in our minds.

While the evidence was "discovered", in labeling the atom, or developing a model, humans created the atom.

Like Lynne, C. J. had to accept some of the chemistry he encountered.

> ... but some stuff they say you know is true -- some stuff it's easy to see, but some things that are really too vague to actually. You have to accept it.

But, unlike Lynne, C. J. believed he could make sense of much of chemistry and relate it to other ideas and concepts. Furthermore, he was not hampered by those things he had to take on faith.

When relational learners learn instrumentally

C. J. liked to do problems that made sense. He enjoyed doing most of the interview problems because they were "logical" or related ideas. He did not like tasks that he considered to be sets of following rules, tasks where he did not perceive relational learning to be possible.

> When I like to think about stuff I like it to be -- orderly. I usually like to think about it -- like transitions between relationships, stuff like that. Its a pattern. But in this you've got a bunch of relationships sort of jumbled down in front of you and you have to sort them out. And I'm not too keen on that.

Problems should contain logical relationships or be meaningful. C. J. did not like anything that smacked of an instrumental approach to learning. This included chemistry tasks as well, such as balancing oxidation-reduction equations. These tasks, as presented in high school, were repetitive and time-intensive. C. J. did not believe that these were taught in a way that emphasized understanding. Thus, just to get by he cranked through the problems, without trying to understand. Now, when he encountered problems with oxidation and reduction in the college chemistry course and the interviews, C. J. experienced one of his few areas of difficulty. He disliked anything connected with the concepts of oxidation and reduction so much that when he attempted problems where oxidation and reduction occurred he could not draw upon previously-demonstrated conceptual knowledge and problem-solving strategies to approach the problem. Oxidation-reduction reactions were one area where he just tried to memorize, not understand.

A Multiplistic Approach to Chemistry

C. J. approached chemistry as a creative, relational endeavor. He tried to make sense out of what happened in lecture, lab and the demonstrations. He was frustrated when he did not know why something happened. His approach did not always lead to the "right" answers and it took a lot of time and effort. He was not always rewarded for his relational approach to learning. His teachers were not always comfortable with his approach and with his questions. However, C. J. was able to overcome these difficulties in most instances and emphasize conceptual relationships. He relied on both objective information and intuition. There was little compartmentalization of his knowledge because he expected ideas to relate. Chemical knowledge was a constructive enterprise to C. J.

Conceptions and constraints

A constructivist approach to learning focuses on the viability, not the truth, of knowledge (von Glaserfeld, 1981). Our conceptions are not exact replicas of true conceptions, they merely fit like a key fits a lock. We develop our conceptions in light of the constraints we encounter while learning. Lynne encountered very few constraints or limits to the concepts she could develop. In her belief system each concept was individual and unconnected, so there were no need for ideas to relate. Everything fit neatly into its own little box. It didn't matter if "atom" or atoms had distinctly different or even contradictory meanings in different contexts.

C. J. on the other hand, encountered many more constraints as he developed chemistry concepts. In his relational approach to learning, new concepts had to relate to previous constructions. New ideas had to make sense in light of what he already knew. As a result, there are very few misconceptions in C. J.'s transcripts.

How do Beliefs Arise?

Paarlburg (1968) offered the following explanation of myths, which is relevant to discussions of the origins of belief.

> People do not like to be without comprehension. They like to understand things, to have an explanation, to have some belief that permits an observed event to take on meaning. ... The motive leading to the propagation of a myth is not the scientific quest for fact; it is a subconscious desire for an individually acceptable answer to the question "Why?"

The constructivist perspective presupposes that people are goal-directed and continually struggling to make sense of their world. Beliefs develop during the constant struggle to make sense of the world, or at least one's part of the world, such as a school environment. As such, they are a reflection of this sense within the context of the individual. Beliefs arise from personal experience, which includes the socialization process (Erikson, 1986; Cobb 1986) as the individual constructs his or her knowledge of the world in order to meet goals or needs. Conceptions of mathematics and science arise through experiences with the world that the individual interprets as having relevance to the concepts discussed under the heading of school science and mathematics. These experiences can be classroom science and mathematics experiences, both implicit and explicit, and interactions with parents, teachers, and peers.

Much of the belief-making process is influenced by this implicit curriculum, and the socialization process in schools. Goodlad (1984) noted the following:

> Schools explicitly teach mathematics and have boys and girls learn to read, write, and spell, and so on. But they also teach a great deal implicitly through the ways they present the explicit curriculum -- for example, emphasizing acquiring facts or solving problems -- through the kinds of rules they impose, and even through the social and physical settings they provide for learning. Thus, they teach students to work alone competitively or to work cooperatively in groups, to be active or passive, to be content with facts or also seek insight, and on and on.

Science and mathematics teachers often express goals such as logical thinking, heuristic problem-solving and critical thinking. The implicit messages received through tasks, testing, teaching

behavior, etc., are often quite different and seem to emphasize recall, not higher intellectual skills and problem-solving (Goodlad, 1984; Stake and Easley, 1978; Thompson, 1984). Science and mathematics become bodies of facts and skills to be acquired as an end in themselves not tools for making sense of the world. They become subjects in schools which students "learn" in such a way as to meet their goals, whether these goals are course grades, looking smart, etc. If their methods of learning by rote are rewarded, by good grades on tests, "learning science" becomes synonymous with this sort of memorization.

Memorization and compartmentalization are necessary survival mechanisms in many chemistry courses. A typical general chemistry text is approximately 1100 pages long. The typical vocabulary in a year-long general chemistry course is as great or greater than that in most foreign language courses. In addition to language and conceptual knowledge, we attempt to teach mathematical skills, problem solving skills, and laboratory skills. Our evaluation system puts emphasis on "right answers" not understanding. Goodlad (1984) describes the situation this way:

> From the beginning, students experience school and classroom behaviors -- seeking "right" answers, conforming, and reproducing the known. These behaviors are reinforced daily by the physical restraints of the group and classroom, by the kinds of questions teachers ask, by the nature of seatwork exercises assigned, and by the format of tests and quizzes. They are further reinforced by the nature of the rewards -- particularly the subtleties of implicitly accepting "right" answers and behaviors while ignoring or otherwise rejecting "wrong" or deviant ones.

Osborne and Wittrock (1985) note that science classrooms do not encourage students to form links and construct and evaluate meaning. Bergerson, Herscovics and Nantais (1985) and Mason (1987) describe the evaluation system and its resultant focus on right answers as a driving force in creating the belief that mathematics consists of getting the answer and practicing computational techniques. This occurs because skills and facts are much easier to learn and evaluate.

Pines and West (1986) suggest the following.

> ... the science curricula of schools and colleges represent and present public knowledge--other people's knowledge--imposed with the power of authority on the students, who come with a wealth of beliefs about the world and the way it works - their own private understanding--which often conflict with what is taught and what is to be learned. It is hardly surprising, when we look at it this way, that some students do not bother to make sense of this formal, scientific knowledge.

This parallels Mason's (1987) statement that "Some students only experience other people's algebra ... without being encouraged to use algebra to express their own generality, to manifest their own inner perceptions in written form."

Classroom and textbook experiences such as these imply that science is something removed from reality; a world of arcane words and symbols which must be accumulated from some external source and memorized. Values, both implicit and explicit, are taught in science classrooms. Student goals are often quite different from those of the teacher. Views of the nature of the discipline, values, and goals have major implications for the way students then go about developing conceptions or "mis-conceptions".

Restructuring Beliefs

John, a student in the math anxiety study, can be used as an illustration of how beliefs about science and mathematics can change, leading to a necessity for conceptual change. John was failing his mathematics course when he enrolled in the math-anxiety program. At that time, mathematics for John, was a logical, authoritarian discipline. He viewed it as "strict formulas and rules and stuff." Mathematics was not creative; it was rigid, received knowledge. Buerke (1981) would describe John as an individual who had multiplistic views of knowledge with respect to most disciplines, but had a very rigid, dualistic view of mathematics. Like Lynne in chemistry, John was a received knower.

The mathematics anxiety course was designed to help students confront their mathematical beliefs and reflect upon these beliefs. About the third week of the course John suddenly came to regard mathematics as a creative enterprise, one where he could draw upon his intuition. His life, his classroom experiences with mathematics, and his grades changed dramatically (Yackel and Carter, 1987). More importantly, his view of mathematics and his ability to do mathematics changed. He could now think about math metacognitively, and this changed his strategies toward learning mathematics and his views of his ability. In rating his ability in mathematics he noted:

> I have to say "average", presently, because my grades reflect that. However, I feel I've developed an above-average

> sense of intuition, and it goes beyond the math book. ... I think presently I'm an average student because of my grades. My grades reflect that. More importantly, it's how it applies outside of class. I think I can probably do better than average. ... I feel I'm probably above average in that sense. In being intuitive or something.

He no longer ascribed to a rule-driven view of mathematics -- a view that had constrained his learning of mathematics until now.

John now believed mathematics classes should not stress strict rules.

> I think there's many different ideas. There's a lot of basic ideas to follow, in a problem or mathematics. I think when they say strict rule, I think it makes people limit themselves. They think "Oh I've gotta follow this strict rule." Then they don't really understand the concepts of it. So, when you think according to strict rule, you fail to understand the main idea of what's going on. You just apply the formula they give you, or the rule, and you fail to understand what it really means.

When he moved away from this perspective of mathematics as a series of rules it changed his perspective of himself as a doer of mathematics. It also changed his motivation, persistance and success in math classes. Restructuring his beliefs changed the questions in mathematics class from "how" to "why".

> I think that some people, even I, had the notion that mathematics is never gonna come to me, 'cause its something I have, a mental block inside my head and I'm not gonna be able to work around it. But I've been working harder lately at a lot of homework assignments. My grades went up. I think if you work hard at it and you think "Why am I doing this? What are you gonna accomplish out of it?" you can learn mathematics.

This change occurred because he started to view mathematics as a creative, relational discipline.

> When I started thinking of math in the creative sense I started doing better. When I was thinking about it as strict formulas and rules and stuff, I really had problems understanding it. But once you start thinking about it in creative ways it benefitted me more.

Creativity was central to his new approach to mathematics.

> I think that looking at problems differently means you understand the problem really well. ... I'm not trying to go against what the teacher says, but I really try to look at the problem and try and understand the way the teacher found the answer. Then I look at it my own way, approach it from my angle, and find it a lot easier.

John moved from a position where there was one way or a best way to solve a mathematics problem to one where there were many equally good ways to work problems. John now believed himself free to use his creativity and problem solving skills in his mathematics courses. The psychological context of math class no longer excluded this type of reasoning. Instead it was expanded to include conceptual relationships and understanding, not just rote, rules and remembered algorithms.

John had worked hard in his mathematics courses before, but he had only a poorly developed and loosely integrated conceptual knowledge of mathematics. It was only after changing his beliefs about mathematics that he could get rid of his "misconceptions" about mathematics and develop viable and powerful understanding of mathematical concepts.

A Perspective for Research

Looking piecemeal at misconceptions is like treating an illness symptom by symptom. The student's perception of what it means to learn science and mathematics is akin to a disease with symptoms manifested as misconceptions. Treating the symptoms is important, however it is not equivalent to treating the cause of the disease.

The misconceptions literature talks about the importance of having students confront their beliefs about specific concepts (Driver & Oldham). Researchers note that there is little or no classroom structure for this type of confrontation. Research on belief suggests that students not only need to confront their beliefs about specific concepts, but also about the nature of scientific and mathematical knowledge.

It is essential, however that the beliefs we must examine include our beliefs, values and goals as researchers, teacher beliefs about how we teach, what we teach, and how much we teach. To encourage relational learning we have to examine classroom practices, reward systems, the role of authority and power structures in schools and ways to encourage students to become autonomous learners. These are difficult tasks but potentially very powerful.

We also have to examine the nature of science and mathematics as it is taught in schools. School science is often taught as if science has no relationship to the real world, except the few cursory examples found in texts. Concepts are only related to other concepts in the relationships leading up to the equations. School science often gives the impression that ideas have some sort of divine origin. Science is seen as ideas discovered by very intelligent men, not as something created by ordinary human beings. Models are presented as facts to be learned. As Fee (1983) notes:

> The voice of the scientific authority is like the male voice-over in commercials, a disembodied knowledge that cannot be questioned, whose author is inaccessible.

With this picture of the nature of science it is little wonder that many of our students appeal to authorities for matters of science and mathematics and not to their own understanding.

Some of our students, such as C. J. and John, do view science and mathematics as relational. However, they believed they had to "go against" their teachers to be creative and use their conceptual knowledge. Only students with strong self-confidence can learn relationally in the face of such pressure to get "right answers." As long as we continue to expect and evaluate memory instead of understanding in school we only reinforce the compartmentalization of learning that leads to "misconceptions". Belenky, et.al. suggest a change in metaphor, from one of the teacher as banker doling out facts and algorithms to one of the teacher as a midwife helping students deliver their own thoughts. Misconceptions researchers already value students' "naive" understandings. Now we need to go a step further and focus on their understandings of understanding.

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OVERCOMING STUDENTS' MISCONCEPTIONS IN PHYSICS: THE ROLE OF ANCHORING INTUITIONS AND ANALOGICAL VALIDITY*

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Overview

This paper discusses an experiment conducted to test the effectiveness of lessons designed to overcome qualitative misconceptions in three areas of mechanics: static forces, frictional forces, and Newton's third law for moving objects. In developing the lessons, diagnostic tests were first used to identify qualitative problems which were answered incorrectly by a large percentage of students. These became target problems to be addressed in the lessons. Separate diagnostic tests were used to identify anchoring examples: i.e. problems which draw out beliefs held by naive students that are in rough agreement with accepted physical theory. The experimental lessons attempted to ground new ideas in the students' intuitions by using anchoring examples.

The lessons then attempted to convince the student that the target (misunderstood) example was analogous to the anchoring example. This can require a significant amount of effort if one is interested in the analogy making sense to the students. It is often helpful to use an intermediate third case-- a bridging analogy between the target example and the anchoring example. Lessons were taught Socratically in order to maximize classroom discussion of misconceptions and student generated examples. Visualizable models and empirical demonstrations were also used where appropriate. The experimental group achieved pre-post test gains that were two standard deviations larger than the control group's gains.

Thus the main instructional techniques used in this attempt to deal with qualitative misconceptions are: (1) the attempt to find and use anchoring examples in order to have the material make sense in terms of the student's physical intuitions; (2) the focus on developing analogies between examples in thought situations or thought experiments; (3) the use of bridging analogies as a particular method for doing this; (4) the idea that misconceptions can be used to advantage in the classroom when they generate a strong classroom discussion. With respect to the use of analogies, I will argue that analogous cases posed in the form of thought experiments or thought situations can be important key examples in instruction. With respect to the goal of tying ideas to student's physical intuitions, I will

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argue that researchers and curriculum developers should be paying as much attention to students' anchoring intuitions as they are currently to students' misconceptions.

The Problem

Because I cannot justify them here, in this section I will make a number of assumptions concerning the existence of certain problems facing teachers of physics. I will assume that there are areas where persistent misconceptions exist which constitute effective barriers to understanding. Evidence for this claim can be found in McDermott (1984), Helm and Novak (1983) and Halloun and Hestenes (1985), among others. In particular, I will assume that:

1. Many students, even those taking calculus-based college physics, harbor misconceptions at a basic qualitative level even though they may be proficient at the use of physics formulae. This indicates that courses need to place increased emphasis on conceptual understanding.
2. Many misconceptions are not "miscomprehensions" of presented material but are preconceptions that students bring to class with them.
3. It is clear that some preconceptions are more deep seated than others. As Chaiklin & Roth (1986) point out, incorrect answers to diagnostic problems may not always reflect deeply held preconceptions since students are willing to use beliefs they are uncertain about in problem solving. But there are several different types of evidence indicating that some preconceptions are deepseated, including post course tests where college students who have completed a physics course exhibit the same errors. Confidence measures also provide some evidence. Brown & Clement (this volume, 1987), found that students indicated they were fairly confident in their incorrect answers on a set of qualitative problems in this area after taking high school physics. Other indications of deepseatedness include resistance observed during tutoring, expressions of conviction in interviews, and historical precedents.

Additional references to studies supporting these assumptions can be found in Clement (1986).

One Teaching Strategy for Dealing with Misconceptions

In this section I describe one method for helping students overcome a persistent misconception. This is followed by a section describing a classroom teaching experiment using this method. Research on persistent misconceptions indicates that traditional instruction has not worked where deep seated preconceptions are concerned, and that significantly new teaching methods may need to be developed. The approach described here assumes that a good way to foster conceptual change in these areas is to have students build up their understanding at a qualitative, intuitive level before mastering quantitative principles. It also assumes that they need to become aware of their own preconceptions and to actively participate in criticizing and replacing or revising them.

In a diagnostic test 76% of a sample of 112 high school students indicated that a table does not push up on a book lying at rest on it. These were chemistry and biology students who would be eligible to take physics in the following year. Observations from classroom discussions and tutoring interviews indicate that many students believe that static objects are rigid barriers which cannot exert forces other than their own weight (sic). On the other hand, 96% of these students believe that a spring does push up on one's hand when the hand is pushing down on it. To the physicist, these beliefs are incompatible since he sees the two situations as equivalent. diSessa (1983) refers to the concept of springiness as a "phenomenological primitive" and describes aquiring skill in physics as depending on the evolution of such intuitions.

The hand on the spring situation is a useful starting point for instruction since it draws out a correct intuition from students. For this reason we call it an "anchoring example that draws out an "anchoring intuition". As used here, an anchoring intuition is a belief held by a naive student which is roughly compatible with accepted physical theory. Such a belief may be articulated or tacit.

Using analogical reasoning in instruction: The first strategy that suggests itself is that of presenting the the hand-on-the-spring and the book-on-the-table cases to students sequentially and asking them if they are not indeed analogous. Hopefully students will see that they are analogous and correct their view of the book on the table.

Unfortunately pilot tutoring interviews conducted by David Brown indicated that the simplest form of this strategy does not often work. Instead, students typically say that the table is not the same as a spring-- the table is rigid whereas the spring is flexible-- so the spring can exert a force while the table cannot. Thus there is a need for an additional instructional effort to help the student see how the analogy between the spring and the table can be valid. This effort fits with the more general plea of Posner, Strike, Hewson, & Gertzog (1982) to make science ideas plausible to students (e.g. having it make sense that tables push up) as well as comprehensible (knowing that tables push up).

Minstrell (1982) has reported some success in using key examples in Socratic teaching for the book on the table problem. In what follows we will build on his ideas by adding an explicit emphasis on anchoring intuitions, structural chains of analogies, and mechanistic models in such lessons.

The spontaneous use of analogies has been documented in thinking aloud interviews with scientists (Clement, 1981, 1986, to appear), and with students (Clement, 1987). Experts have also been observed to use special patterns of analogical reasoning in order to stretch the domain of a key analogue example and overcome a conceptual difficulty (Clement, 1982b, 1986). We suspect that these patterns are useful for overcoming conceptual difficulties in students as well. This can be done in a number of ways: (1) identifying salient features that are the same in the book and the spring; showing the existence of a conserving transformation between the book and the spring; and using a technique called bridging, described below.

The bridging strategy. In almost all cases students believe in the anchoring example that a spring can push back on one's hand, but many are still unconvinced that there is a valid analogy relation to the case of the book on the table. A useful strategy is to attempt to find an intermediate third case between the original case and the analogous case. This is termed a bridging analogy. Figure 1 shows a flexible board case used to help convince students that the analogy between the "hand on the spring" anchor and the targeted "book on the table" case is valid. Here, the idea of a book resting on a flexible board (case B) shares some features of the book on the table (case C) and some features of the hand on the spring (case A). The subject may then be convinced that A is analogous to B and that B is analogous to C with respect to the same important features, and thereby be convinced that A is analogous to C. Such bridges are not deductive arguments, but experts have been observed to use them as a powerful form of plausible reasoning. Presumably, this method works because it is easier to comprehend a "close" analogy than a "distant" one. The bridge divides the analogy into two smaller steps which are easier to comprehend than one large step.

Lessons can also use several intermediate bridging cases, as shown in the outline of the lesson on static forces shown in Figure 2. Visualizable models and empirical demonstrations are also used where appropriate. During the

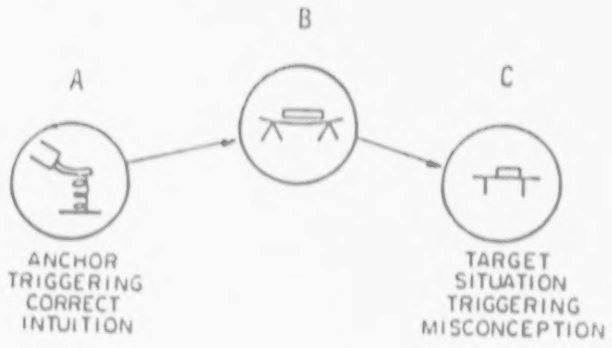

Figure 1

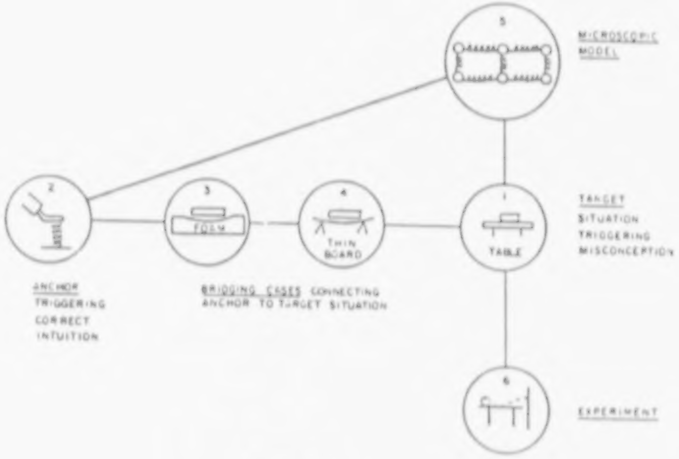

Figure 2

lesson, the class compares each of the thought experiment examples in a discussion led by the teacher. The teacher challenges students to say why they believe that cases are similar or different and avoids sharing his or her view until near the end of the class.

In this lesson the target problem -- the question of whether the table pushes up on a book -- is introduced first. Then the hand on the spring case is discussed and agreed upon as an anchor. The foam and flexible board cases are then introduced and students are asked to compare them with the spring and the table cases. The flexible board case usually promotes the greatest discussion, and a number of students switch to the correct view at this point, which may be about 35 minutes into the lesson. The teacher then introduces a microscopic model of rigid objects as being made up of atoms connected by spring-like bonds. Finally the students view a demonstration where a light beam reflecting off of a desk onto the wall is deflected downward when the teacher stands on the desk.

As a second example, the lesson on friction outlined in Figure 3 uses the target problem of a puck sliding on a floor. Students are asked what the direction of the forces are that act on the puck. Many students fail to identify a force of friction, or say that friction has no particular direction, or that it acts in a downward direction, whereas the physicist identifies a force of friction exerted by the floor on the puck acting horizontally to the left. The next example introduced in this lesson is a brush being pulled to the right while resting on top of another brush as shown in Figure 3. The anchor situation in the

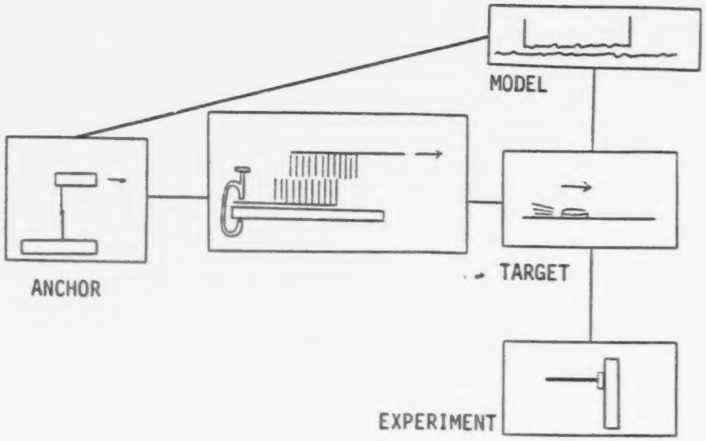

Figure 3

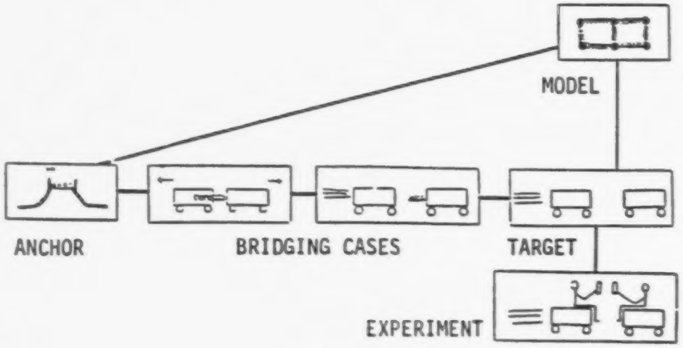

Figure 4

left-most drawing is the same experiment with only two single bristles on each brush in contact. Students see that the lower brush exerts a horizontal force to the left on the upper brush. The teacher also introduces a model of surfaces as being very rough in a microscopic view. After discussion, a demonstration is conducted where students are asked the direction of the force which holds up a book pressed against a wall with a ruler. Other more complex models of friction such as "temporary welds" and Van der Waals forces are included at the end of the lesson to extend this first order model, but the sequence in Fig. 3 is the one used to motivate an intuition for the direction of the force.

As a third example, a lesson on Newton's third law in collisions uses a target problem of a moving cart colliding with a stationary cart of the same mass, as shown in Figure 4. The anchoring example here involves the idea that both of one's hands feel the same force when compressing a spring between them. The first bridging example is a pair of exploding carts, where one cart contains a cocked spring. Here again the force on cart A is the same size as the force on cart B. In the second bridging case students argue about whether the forces are equal in a collision where B has a spring attached to it. They are then asked if this is equivalent to the target problem if we consider the surface of cart B to made up of spring-like bonds between its atoms. At the end of this lesson an experiment is done where a student riding on a cart collides with another student on a stationary cart. Bathroom scales held by each student out in front of the carts receive the impact of the collision.

Students predict whether one reading will be consistently larger. These give very rough measurements, but they are accurate enough over several trials to support the idea of equal forces. A second lesson extends this idea to unequal masses.

Thus the lessons use a sequence of analogous cases to connect an anchoring example to the target problem, and also to develop a visualizable model of the mechanism(s) providing forces in the target problem. Demonstrations are used primarily to disequilibrate students' preconceptions or to support a key aspect of the analogue model such as the presence of deformation in rigid objects.

Classroom Teaching Experiment

Method. The strategy described above was evaluated by designing a set of experimental lessons and giving identical pre and post tests to experimental and control classes. Altogether, five experimental lessons were designed in the three areas as described above: static forces, frictional forces, and Newton's third law for moving objects. Members of the design team were David Brown, Charles Camp, John Clement, John Kudukey, James Minstrell, Klaus Schultz, Melvin Steinberg and Valerie Veneman. Three experimental classes were taught by one teacher and two taught by another, both in the same high school. The lessons were introduced at the different points where they fit into the students' study of mechanics during the first five months of the year. The four control classes were also first year courses in physics taught by two separate teachers in two other schools. Control classes studied their normal curriculum in physics.

The test, described in Brown and Clement (this volume, 1987b), consisted of 16 questions on common misconceptions and contained both near and far transfer questions. The identical pre and post tests were given about 6 months apart: in the second month of the course just before the first experimental lesson and again two months after the final experimental lesson.

Quantitative Results. Results are highlighted in the right hand column of Table 1. The experimental group achieved significantly larger pre-post test gains than the control group, both overall, as shown in Table 1, and in each of the three areas, as shown in Table 3. The difference between the overall gains was larger than two standard deviations.

We interpret these results with some caution, since the tests were multiple choice tests, and more accurate assessment of students' understanding requires clinical methods. Also, matching of experimental and control groups was limited by characteristics of the available classes. The school where the experimental classes were conducted and one of the control schools had upper honor level classes and lower standard level classes, while the second control school grouped students homogeneously.

However, as shown in Table 2, even when the lower level experimental classes (with a teacher who was relatively new to physics teaching) are compared with the homogeneous control classes (with an experienced person considered to be a master physics teacher), there are large significant differences in favor of the experimental group.

		Average Pretest Score	Average Posttest Score	Average Gain
Control n = 50		5.62 (29.6%)	8.00 (42.1%)	2.38 (+12.5%)
	s.d =	(2.44)	(3.13)	(2.83)
Experimental n = 82		5.63 (29.6%)	14.56 (76.6%)	8.93 (+47.0%)
		(2.38)	(2.90)	(2.84)

Experimental group had larger gain (t = 12.72, two-tailed, $p < .00005$).

Table 1

		Average Pretest Score	Average Posttest Score	Average Gain
Homogeneous Control Classes n = 23		5.17 (27.2%)	8.26 (43.5%)	3.09 (+16.3%)
	s.d.=	(2.25)	(3.76)	(3.29)
Lower Experimental Classes n = 46		4.83 (25.4%)	13.63 (71.7%)	8.80 (+46.3%)
		(2.07)	(3.03)	(3.30)

Experimental group had larger gain (t = 6.80, two-tailed, $p < .00005$).

Table 2

FORCES FROM STATIC-OBJECT EXPERIMENTAL VS. CONTROL

Points Possible = 6

	Pretest	Posttest	Average Gain
Control n = 50	1.14 (19.0%) (1.34)	2.58 (43.0%) (1.54)	1.44 (+24.0%) (1.39)
Experimental n = 82	1.13 (18.8%) (1.16)	4.83 (80.5%) (1.02)	3.70 (+61.7%) (1.35)

Experimental group showed larger gain (t = 9.15, two-tailed, p<.00005)

FRICTION FORCES - EXPERIMENTAL VS. CONTROL

Possible points = 6

	Pretest	Posttest	Gain
Control n = 50	1.08 (27.0%) (0.778)	1.58 (39.5%) (0.835)	0.50 (+12.5%) (1.05)
Experimental n = 82	1.10 (27.5%) (0.795)	2.72 (68.0%) (1.10)	1.62 (+40.5%) (1.25)

Experimental group showed larger gain (t = 5.5, two-tailed, p<.00005)

DYNAMIC THIRD LAW: EXPERIMENTAL VS. CONTROL

Points Possible: 6

	Pretest	Posttest	Gain
Control n = 50	1.26 (21.0%) (1.14)	1.42 (23.7%) (1.64)	0.16 (+2.7%) (1.68)
Experimental n = 82	1.12 (18.7%) (1.06)	4.50 (75.0%) (1.48)	3.38 (+56.3%) (1.53)

Experimental vs. control (t = 11.03, two-tailed, p<.00005)

Table 3

Qualitative results. Qualitative observations from video tapes of these classes indicate that: 1) students appear to readily understand the anchoring cases; 2) however, many students indeed do not initially believe that the anchor and the target cases are analogous; 3) the bridging cases sparked an unusual amount of argument and constructive thinking in class discussions; 4) the lessons led many students to change their minds about or degree of belief in statements such as: "A table cannot exert an upward force on a book at rest on it"; and 5) students were observed generating several types of interesting arguments during discussion, such as: generation of analogies and extreme cases of their own; explanations via a microscopic model; giving a concrete example of a principle; arguments by contradiction from lack of a causal effect; generation of new scientific questions related to the lesson; and even spontaneous generation of bridging analogies. This last observation gives us some reason to believe that even though lessons were designed primarily with content goals in mind, process goals are also being achieved as an important additional benefit.

Experimental classes did not achieve high post test scores on all transfer problems. And discussions in different classes varied somewhat unpredictably from very exciting to dull. Thus there is still considerable room for improvement. Nevertheless, we are encouraged enough by the results to suspect that the techniques being tried are having an effect in helping students overcome misconceptions.

Discussion

In this section I propose some interpretations of the above results. The experimental teaching method described attempted to ground the student's understanding on physical intuition. Here we are faced with a paradox: in order for difficult conceptual material to make sense to the student, it is necessary to connect somehow with the student's existing knowledge; but the student's existing intuition in the area is incorrect. A way around this paradox was found by using anchors. This method relies on the fact that students are globally inconsistent from a physicist's point of view; the student can simultaneously harbor in memory an anchoring intuition and a misconception that are diametrically opposed. This is presumably because the student's knowledge schemas are packaged in much smaller pieces than the physicists' knowledge (diSessa, 1985), and because each schema is activated only in certain contexts. The intuitions used for an anchoring example are inconsistent (from a physicist's point of view) with their view of the examples where a misconception is dominant. The teaching strategy takes advantage of this fact by using discussion and bridging to promote dissonance between the anchor and the misconception, thereby encouraging conceptual change.

The notion of searching for anchoring intuitions opens up a large field for needed research that should complement the ongoing research effort on misconceptions. Potential anchoring examples can be listed by skilled teachers, but they require empirical confirmation. For example, we hypothesized that hitting a wall with one's fist might be an excellent anchoring example for the idea that a static object can exert a force. Surprisingly however, only 41% of pre-physics students tested agreed that the wall would exert a force on one's hand. Empirical studies can determine which situations will appeal the most to students' intuitions. Thus one must find the "right" analogous case to use as an anchor -- not just any concrete example that makes sense to the teacher will work.

An anchor can also serve as the metaphorical basis for a visualizable model such as the idea of atoms with spring like bonds between them. Such models are not simply a set of common features abstracted from observed phenomena (we cannot observe atoms or springs inside of tables). Like other models in science they are imaginative constructions which have metaphorical content.

In areas where students have insufficiently developed anchoring intuitions they may need to be developed by real or simulated experiences such as Arons' activity of having students push large objects in a low friction environment, McDermott's (1984) use of air hoses to accelerate dry ice pucks, or diSessa, Horwitz, and White's use of dynaturtle (White, 1984).

Analogies and Bridging in the History of Science

According to legend Galileo performed an an empirical test by dropping light and heavy objects from the tower of Pisa, but this legend has come under serious doubt. However it is known that Galileo and his predecessor, Benedetti, did use thought experiments like the following one to argue their side in this issue. Figure 5a shows two equal objects of one unit each being dropped while Figure 5b shows a heavier object being dropped that is equal to the two smaller objects combined. According to Aristotle the one-unit objects will fall much more slowly than the larger object. Galileo claimed that they will reach the ground at nearly the same time. In saying this he was effectively proposing an analogy between cases A and B in Figure 5 to the effect that each body falls according to the same rule irrespective of its weight.

A marvelous bridging case used to support this analogy is the case shown in Figure 5c. The argument was first published by Benedetti (1969) and a similar argument was given by Galileo (1954). Imagine the two unit objects in A to be connected by a thin line or thread. Does the mere addition of this tiny thread, which makes the two objects become one, cause their rate of fall to increase by a large amount? Because this is implausible, the bridge argues that A and B are indeed equivalent with respect to rates of fall. In an insightful Gedanken experiment, the lightest thread can apparently make all the difference. Apparently Benedetti and Galileo felt that thought experiments which establish the validity of an analogy are a powerful method of argument and instruction.

Educational Implications

When students harboring misconceptions produce incorrect answers in the classroom, the instructor may in some cases assume that the cause is "low intelligence" or poorly developed reasoning skills, when in fact the cause is the student's alternative knowledge structures. It is important for teachers to become sensitive to such distinctions because the indicated teaching strategies are quite different in each case. Avoiding this confusion might have an impact on the way teachers view students and in turn, on the way students view themselves.

The major ideas involved in the teaching approach investigated here were:

(1) *Anchoring intuitions* can be used as starting points for lessons which attempt to overcome misconceptions in physics. An important implication for curriculum development related research is the need to search for such anchoring intuitions.

(2) *Forming analogies* between more difficult examples and an anchoring situation is an important instructional technique. However, *more energy than is commonly realized must be invested in helping students to believe in the validity of such analogies*. Where a misconception is present in the target case, the problem is that students will often not be able to understand how the target case can possibly be analogous to the familiar anchoring case. Presenting the right analogy is not enough -- the student must also come to believe in the validity of the analogy.

(3) The technique of *bridging* by using structured chains of analogies combined with discussion to encourage active thinking appears to be helpful for this purpose. The bridging cases used here seem to work at the level of a person's physical intuitions, not at a level that uses formal notations. Bridging appears to be an important tool for stretching the domain of applicability of an anchoring intuition to a new situation, i.e. for making the intuition more general and powerful. Analogies and bridging may

therefore be important tools for developing and refining physical intuitions.

(4) Many anchors and bridges can be introduced as thought situations or thought experiments. Thus, thought experiments are potentially powerful tools in instruction, as has been noted by Helm and Gilbert (no date). Empirical information is therefore not the only means recommended for dealing with misconceptions. (See Brown, (in progress), and Brown & Clement (1987) for evidence supporting this position.)

(5) Misconceptions can be used to advantage in instruction. Topics where students feel that the accepted physical theory is counterintuitive are sometimes frustrating to them, but such topics are also potentially more interesting because of their surprising nature. In a sense, they have more "news value" as something unusual to be learned. Also, when the conviction of a misconception can be brought into conflict with another conviction within the student's head, this produces more dissonance and more potentially useful energy to be harnessed for learning than ordinary topics which do not threaten beliefs held with conviction. In this case the students can become internally motivated to understand the issue and resolve the conflict. The above five strategies have also been implemented in an experimental instructional computer program described in Schultz, Murray, Clement, and Brown (this volume, 1987).

(6) Socratic discussions can help students achieve conceptual change. Two types of dissonance can be used: the tension between a misconception and a correct conception in the same student; and the tension between students who hold the correct point of view early on and students who do not. In the first type, one attempts to draw out both correct and incorrect conceptions which are activated in slightly different contexts in the same student and play them off against each other. In the second, one encourages controversy centering on opposite views held by different students. These tensions have the potential to create some unusually exciting and motivating discussions in the classroom that should act to increase student involvement and retention. Skillfully led classroom discussions appear to be one effective vehicle for fostering dissonance, internal motivation, and conceptual restructuring.

The approach described above attempts to depart significantly from a model of teaching where knowledge is "piped" directly from teacher to the empty vessel of the student. It does so by drawing out and developing prior knowledge in the subject. The teaching approach attempts to interact with this knowledge rather than to transfer knowledge. It interacts with prior knowledge of two types: anchoring intuitions that are in agreement with accepted theory and misconceptions that are not.

However, it takes a research effort to develop and optimize this type of approach. Maps of students' misconceptions and anchoring intuitions are needed. Knowledge of students' intuitive reasoning skills is also needed. The present study provides some encouragement regarding the possible payoffs of such an effort.

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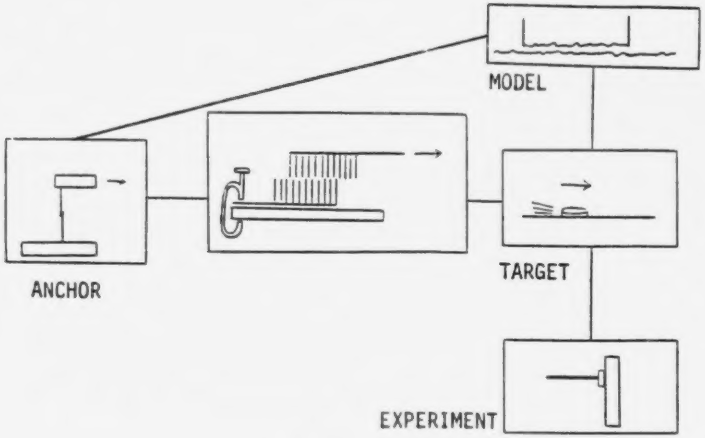

Figure 3

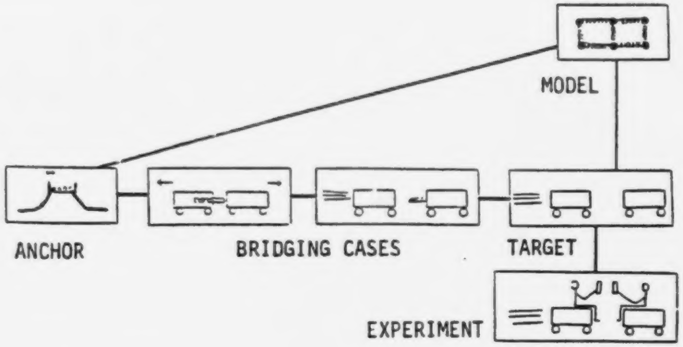

Figure 4

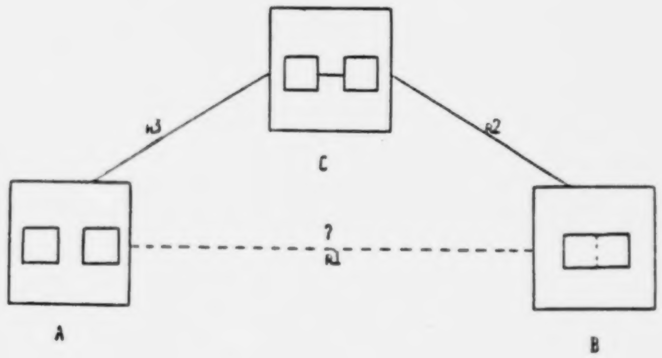

Figure 5

Fractional understanding of fractions: Variations in children's understanding of fractional concepts, across embodiments (Grades 2 through 5)

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1. INTRODUCTION

Over the last decade it has been argued, consistently, that partitioning is a constructive mechanism which can facilitate the development of mature fractional concepts (Behr, Lesh, Post, & Silver, 1983; Hunting, 1983, 1986; Kieren, 1976, 1980; Vergnaud, 1983). In particular, attention has been drawn to the difficulty many children experience in partitioning discrete sets of objects, and Hunting (1986) has suggested that since children have been found to have knowledge of subunits and relationships among subunits in different ways it would be sensible for such knowledge to be assessed before providing children with instruction on fractions. Thus, Hunting says, children with incomplete partitioning knowledge should be taught partitioning prior to commencement of fractions instruction. According to Hunting (1986, p. 214), actions used to subdivide continuous quantities are different from those used to subdivide discrete quantities and, therefore, children's meanings and conceptions of fractions may be quite different if restricted to either contextual setting.

Mathematics educators have also begun to investigate the ways children process mathematical information which is conveyed to them and the ways they communicate their meanings of mathematics to others (Del Campo & Clements, 1987). While there were some notable exceptions (eg. Erlwanger, 1975), until recent times much of the research into mathematics learning made extensive use of pen - and - paper tests. The use of such tests tended to confirm the idea that mathematics was an external body of knowledge which children had to struggle to receive. In recent times, however, there has been an emphasis, in keeping with the growth of constructivist ideas, on observing children while they construct and communicate mathematics (Kamii, 1984; Labinowicz, 1985).

Table 1 (reproduced from Del Campo and Clements, 1987, p.12) sets forth the various receptive and expressive modes to which attention is being drawn (notice that one can communicate with oneself through the imagination).

MODES OF COMMUNICATION

MODE	RECEPTIVE LANGUAGE (processing someone else's communication)	EXPRESSIVE LANGUAGE (your own language)
SPOKEN	listening	speaking
WRITTEN	reading	writing
PICTORIAL	Interpreting diagrams, pictures	drawing
ACTIVE	Interpreting others' actions	performing, demonstrating
IMAGINED	-	imagining

Table 1: Ways of receiving and communicating mathematics

We were interested in studying the language which children use to express fraction ideas, and the relationships which this had with their ability to comprehend the formal language of fractions as it appears in textbooks and pen-and-paper tests. Also, we wanted to investigate the extent to which performance on standard pen-and-paper items on fractions related to children's ability to partition, in situations when the children were confronted with familiar and unfamiliar contexts in which partitioning was possible. To use Pines and West's (1983, p.47) vine metaphor, we were intent on exploring both the upward vine of fraction knowledge, originating from the child's intuitive knowledge of the world, and the downward vine, originating from formal instruction. Since we wished to be informed about these **before** the intertwining of the two vines we decided to work with children in Grades 2, 3, 4, and 5 (aged from about 7 to 11 years). Specifically, we set out to investigate how, and to what extent, children's meanings and conceptions of the fractions one-half, one-quarter, and one-third differ across different kinds of continuous quantities and different displays of discrete sets of objects.

2. METHOD

Sample

Altogether 1024 children (284 in Grade 2, 390 in Grade 3, 283 in Grade 4, 67 in Grade 5), in 45 different classes in 20 elementary schools in Melbourne, Australia, were involved in the study. All of the children in the 45 classes attempted all of the questions on a 36-item pen-and-paper test on fraction knowledge, and then 240 children (12 from each of the 20 schools) were interviewed by mathematics education graduate students trained by the writers. The average length of each interview was about 45 minutes, and during an interview the child was presented with a wide range of materials, which were possible fraction embodiments, and challenged to use these materials to construct representations of "one-half", "one-quarter", and "one-third".

The pen-and-paper instrument

Of the 36 items, 24 involved the recognition of 1/2, 1/4, or 1/3 when embodiments were presented pictorially, in either discrete or continuous forms (see Figure 1(a) for three of these items - the child had to put a tick in the appropriate box); 6 of the items involved equal sharing of a number of objects (lollies) among a specified number of people (see Figure 1(b) for two of these items - the child had to put a ring around the number of lollies each person would get); the remaining 6 items required the child to indicate which of a set of four pictures illustrated a specified fraction. (Figure 1(c) shows one of these items.) Before the children attempted the items they were given practice examples which familiarized them with the modes of response. Persons who administered the 36 items to the 45 classes were convinced that no child misunderstood how answers were to be given and, since unlimited time was permitted, all 1024 children responded to each of the 36 items. The 36-item instrument was devised by the writers.

The interview schedule and sample

Twelve children, three high-performing girls, three low - performing girls, three high - performing boys and three low - performing boys (the criterion for performance being number correct on the 36-item pen-and-paper instrument), were selected from each of the 20 co-operating schools and were individually interviewed according to a set schedule. The twelve children in each school were usually from Grades 2, 3 and 4 (four from each grade) but in two schools Grade 5 children were interviewed. The 240 children consisted of 72 children in Grade 2, 72 in Grade 3, 72 in Grade 4, and 24 in Grade 5. Equal numbers of girls and boys were interviewed, but since sex-related differences was not a focus of the study no data pertaining to the sex factor will be presented.

After a child had been made to feel as comfortable as possible, at the beginning of an interview, the interviewer then said: "Tell me the first thing that comes into your head when I say ....". The sentence was completed, successively, by the words "one-half", "one-quarter", and "one-third", and the interviewer summarised the responses, by writing and/or drawing, in the appropriate sections on an "Interviewing Schedule" sheet (which had been prepared by the writers). After this, the interviewer asked the child to show 1/2 (or 1/4 or 1/3) of each of a sequence of ten possible fraction embodiments. For example, in one possible embodiment the child would be given a length of plasticine (10cm long) and a balance and asked to use the balance to get one-half (or one-quarter or one-third) of the plasticine; in another possible embodiment, the child would be presented with a circular paper disc and some scissors and asked to use the scissors to get one-half of the circle. (When this was completed the child would be given another paper circle and asked to use the scissors to get one-quarter of the circle; and then the same process would take place for one-third.) As the child responded the interviewer would summarise what was being done, and said, in the appropriate section of the "Interviewing Schedule". The interviews were audio-, but not video-, taped.

Figure 2 shows a summary of the structure of the interviews. Of the ten. possible fraction embodiments seven were of the continuous variety (involving circular discs, pieces of string, paper rectangles, paper (equilateral) triangles, spherical balls of plasticine, and jars wih water), two were of the discrete variety, and one, which involved the perimeter of a triangle, had both continuous and discrete features.

Figure 3 shows how one of the interviewers completed the interview Schedule for a Grade 4 child. Item 1 was for the initial request ("Tell me the first thing ..."), and items 2 to 11 were for the ten possible fraction embodiments. Notice that for each item the child was asked about 1/2 first, then 1/4, then 1/3. Interviewers were instructed not to subject interviewees to time pressure.

Foci of the investigation

The following four questions provided the foci of the investigation:

1. Can the meanings which young schoolchildren give to the terms "one-half", "one-quarter", and "one-third" be identified, and how are these meanings affected as the children progress through the middle grades of elementary schools?

2. Do the meanings which a child gives to the terms "one-half", "one-quarter", and "one-third" vary, depending on the context in which the terms are confronted? In particular, do the meanings vary for different continuous embodiments, and is there a difference between meanings given to continuous and discrete embodiments?

3. Do children generally acquire an understanding of 1/2 (in continuous and discrete forms) before 1/4, and is 1/3 generally acquired after both 1/2 and 1/4?

4. Do children possess partitioning concepts independent of their acquisition of receptive and expressive understandings of the fractions 1/2, 1/4, and 1/3?

Obviously, these questions are not only interdependent, but are also very large in their scope. We shall now consider the light which data from the present investigation throw on each question. The discussion of the first question will be more detailed than that for the others, mainly because many of the points arising out of the first question are pertinent to the other questions.

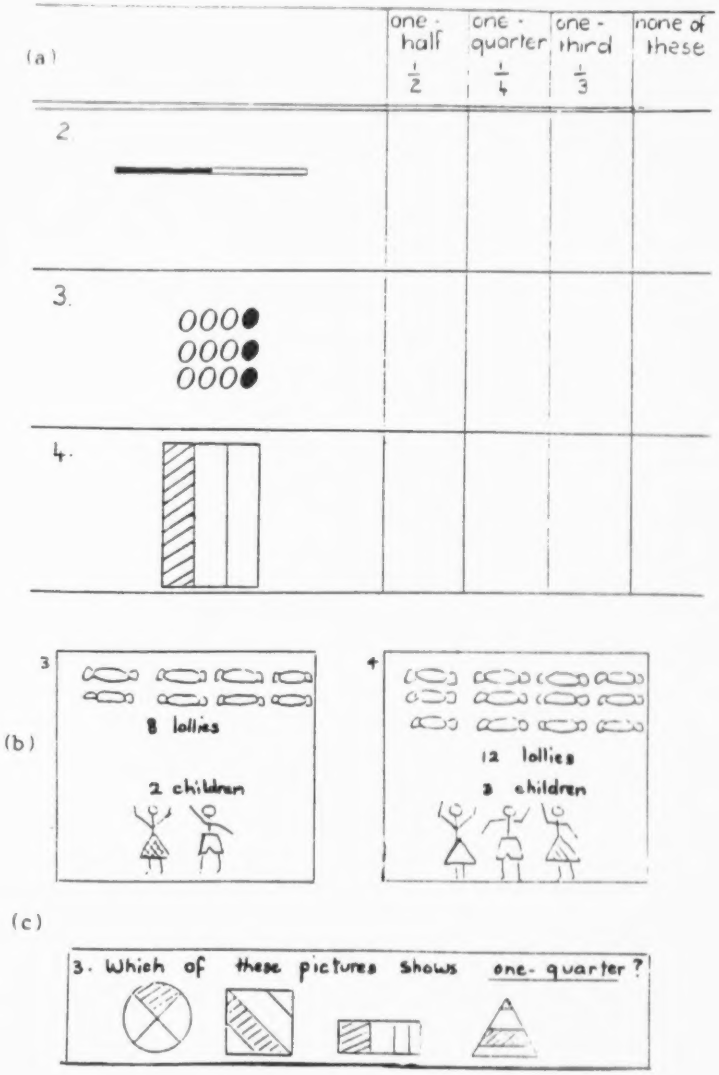
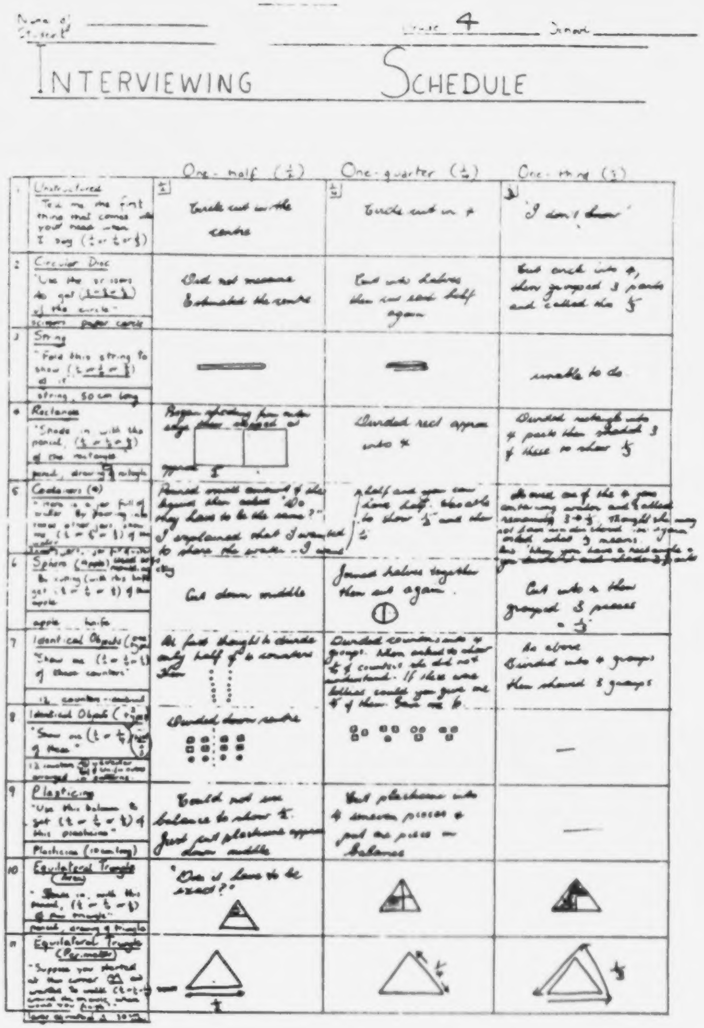


Figure 1: The three types of pen-and-paper items

Type of Embodiment	Continuous or Discrete	Linear, Area, Volume, Mass Other	Action	Equipment needed
Circular disc	continuous	area	cutting	scissors, paper disc
String	continuous	linear	folding	string
Rectangle	continuous	area	drawing	drawings of rectangles, pencils
Containers	continuous	capacity/ volume	pouring	4 clear identical cylinders, coloured fluid
Sphere	continuous	volume	cutting	apples
Identical objects	discrete	(other)	sharing	12 Unifix cubes (same colour)
Two kinds of objects	discrete	(other)	sharing	8 identical, little squares, and 4 identical little circles
Plasticine	continuous	mass	dividing, weighing	plasticine, a balance
Equilateral triangle	continuous	area	drawing	drawings of equilateral triangles, pencils
Equilateral triangle	continuous discrete	perimeter	drawing	3 drawings of equilateral triangles, pencil

Figure 2: The structure of the interviews.



Name of Student ____________ Grade 4 ________ School ____________

INTERVIEWING SCHEDULE

		One-half (½)	One-quarter (¼)	One-third (⅓)
1	Unstructured "Tell me the first thing that comes into your head when I say (½ or ¼ or ⅓)"	½ Circle cut in the centre	¼ Circle cut in 4	⅓ 'I don't know'
2	Circular Disc "Use the scissors to get (½ or ¼ or ⅓) of this circle" scissors paper circle	Did not measure. Estimated the centre	Cut into halves then cut each half again	Cut circle into 4, then grouped 3 pieces and called this ⅓
3	String "Fold this string to show (½ or ¼ or ⅓) of it" string, 50 cm long			unable to do
4	Rectangle "Shade in with the pencil, (½ or ¼ or ⅓) of the rectangle" pencil, drawing of rectangle	Began shading from outer edge then stopped at approx ½	Divided rect approx into 4	Divided rectangle into 4 parts then shaded 3 of these to show ⅓
5	Containers (4) "Here is a jar full of water. By pouring into these other jars, show me (½ or ¼ or ⅓) of the water" [illegible]	Poured small amount of [illegible] then asked "Do they have to be the same?" I explained that I wanted to share the water - I [illegible]	half and you can [illegible] half. Was able to show ¼ and [illegible] ½	[illegible]
6	Sphere (apple) "By cutting (with this knife) get (½ or ¼ or ⅓) of this apple" apple knife	Cut down middle	Joined halves together then cut again.	Cut into 4 then grouped 3 pieces = ⅓
7	Identical Objects (one type) "Show me (½ or ¼ or ⅓) of these counters" 12 counters - coloured	At first thought to divide only half of the counters then	Divided counters into 4 groups. When asked to show ¼ of counters she did not understand. If these were lollies could you give me ¼ of them. Gave me 6.	As above. Divided into 4 groups then showed 3 groups
8	Identical Objects (2 types) "Show me (½ or ¼) of these" 12 counters [illegible] arranged in patterns	Divided down centre		—
9	Plasticine "Use this balance to get (½ or ¼ or ⅓) of this plasticine" Plasticine (10 cm long)	Could not use balance to show ½. Just cut plasticine approx down middle	Cut plasticine into 4 uneven pieces & put the pieces on balance	—
10	Equilateral Triangle (Area) "Shade in, with the pencil, (½ or ¼ or ⅓) of this triangle" pencil, drawing of triangle	"Does it have to be exact?"		
11	Equilateral Triangle (Perimeter) "Suppose you started at the corner and wanted to walk (½ or ¼ or ⅓) around the triangle, where would you finish?" [illegible] 10 cm	½	¼	⅓

Figure 3: How the Interview Schedule was completed for a Grade 4 girl.

3. THE DATA

1. The meanings which young school children give to the terms "one-half", "one-quarter", and "one-third".

One-half

Our data from the pen-and-paper items make it clear that children in Grades 2 - 4 give similar meanings to the term "one-half" when this arises in the context of a drawing, on paper, of familiar or reasonably familiar elementary closed shapes which have at least one line of symmetry (for example, squares, rectangles, isosceles or equilateral triangles, circles, regular hexagons). Nearly all the children in Grades 2 through 5 were able to identify correctly such instances of one-half among pen-and-paper items, although it is interesting to observe that 17%, 14%, 12% and 12% of children in Grades 2, 3, 4, and 5 respectively gave an incorrect response to the pen-and-paper item shown in Figure 4. The percentage of children in each grade who did not associate the shaded triangle in Figure 5 with the fraction one-half was much lower.

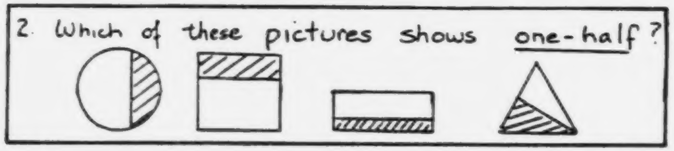


Figure 4: The line of symmetry for the triangle is oblique.

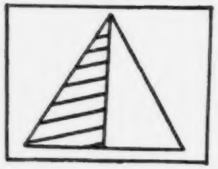


Figure 5: The line of symmetry for the triangle is vertical.

This result suggests that when the line of symmetry is drawn obliquely rather than horizontally then some children no longer are prepared to identify the symbolic representation with one-half. The veracity of this observation was emphasized in the interviews when children actively made representations of one-half by cutting circular discs, shading rectangles, and shading equilateral triangles. Invariably, when axes of symmetry were defined by cutting or drawing, these axes were "vertical" or "horizontal" with respect to the child. This suggests that children hold in their long-term memories certain kinus of visual images of halves of continuous shapes, and that some children experience difficulty if the orientations of pictorial representations do not correspond to their visual images.

An important finding from our interview data was that very few children at any of the grade levels felt the need to check their representations of 1/2, 1/4 and 1/3. Typically, children would estimate the fractional quantity, proceed to carry out an appropriate operation (e.g. cutting with scissors), and then intimate that they had done all that was needed to be done. During the interviews this lack of awareness of the need to check or, indeed of the possibility of checking, was especially apparent. Of course, such a lack was not obvious from the pen-and-paper data.

The data with respect to discrete embodiments of 1/2 were surprising in that they revealed that many children, even in Grade 5, do not, unless prompted, associate the expression "find one-half of" with the physical act of partitioning the set into two equivalent subsets. The pen-and-paper items which asked for 8 and 10 lollies to be shared equally between two children produced data which showed that children in Grades 3, 4, and 5 had little difficulty with the idea. Less than 5% of children at each of these grade levels gave incorrect respnses. In Grade 2, however, only 53% of the children gave correct responses. When, in the interviews, the Grade 2 children were asked to show the interviewer one-half of a set of 12 identical objects an even smaller percentage (24%) gave the correct answer. During the interviews the spatial configuration of the 12 objects was not symmetrical and, for most children, the interviewers' request to be given one-half of the objects neither stimulated thinking about symmetry nor about sharing.

It is interesting to reflect on the percentages of children who nominated one-half as the fraction to be associated with either or both of the two pictures shown in Figure 6. Table 2, below, shows the percentages correct on these pen-and-paper items.

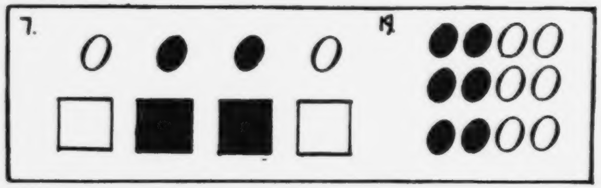


Figure 6: Two discrete representations of one-half

Table 2: Percentages correct on two discrete 1/2 questions (Grades 2 through 5)

		Grade			
		2 (n=284)	3 (n=390)	4 (n=283)	5 (n=67)
Item	7	23%	28%	36%	40%
	19	29%	37%	52%	65%

During the interviews it became clear that many children simply could not think of a discrete set of objects as a unit which can be partitioned. As one Grade 4 child said, "There are 12 blocks here so how can you get a half? What do you mean? Half a block?"

In the interview situation some of those who did appreciate the need to partition the given set of objects into the two equivalent subsets still gave incorrect responses. This was especially true in the interview task which involved 12 objects, 8 of one kind and 4 of another, the children being asked to give the interviewer one-half of the objects. Many children gave the interviewer six objects, but not four of one kind and two of another. Partitioning in the interviews was usually done by a "one for you, one for me" procedure (which sometimes yielded an incorrect response especially when all the objects were not the same). In Grades 4 and 5 , however, it was common for children to divide by two rather than employ a one-for-one procedure. Sometimes a division by two did not yield a correct answer, but the lack of symmetry of the two subsets was rarely a matter for concern. By contrast, some children rearranged the objects so that the "new" arrangement had line symmetry. In such cases partitioning was usually a visual matter, with no attention being paid to the numerosity of the subsets.

The interview task in which children were asked to imagine where they would be if they walked half-way around an equilateral triangle (starting at the bottom left vertex and walking clockwise) is interesting in that, arguably, it is simultaneously a continuous **and** a discrete possible fraction embodiment. It is continuous because it involves length of a continuous path; it is discrete because the triangle has three sides, and the number of sides might be expected to be poignant so far as children's thinking about the task is concerned.

This perimeter task was extremely difficult for children in Grades 2 through 5, with 4%, 8%, 18% and 37%, respectively, giving correct solutions. Clearly, the difficulty arose because of the children's lack of familiarity with the task. The children simply did not comprehend the instruction, "Suppose you started at this corner and wanted to walk one-half of the way around the triangle, in this direction ..." The most common error was to indicate a point half-way along the "first" side of the triangle; another common error was to indicate that the "first vertex around" was the answer. For almost all children in Grades 2 and 3 the task was essentially a spatial one, in that it did not occur to them that the number of sides which the triangle had could possibly be relevant. At Grades 4 and 5, however, 12 children (17%) and 11 children (46%) of the respective interview samples took the number of sides into account - but not all of these could cope the fact that "one-half of 3 is one-and-a- half".

In summary, most children in Grades 2 through 5 have a good receptive understanding of 1/2 if by this we mean the ability to respond correctly to pen-and-paper items involving pictures of continuous embodiments (like circles, squares and triangles). A few Grade 2 children, however did not recognize pictorial representations of a "continuous" one-half when the line of symmetry was neither vertical nor horizontal. In the interview situation, Grade 2 children rarely felt the need to check that their physical representations of one-half was identical to the half "left over", but in pen-and-paper items the need for halves to be identical was realized by a great majority of children.

One-quarter

Data from pen-and-paper items indicate that while most children in Grade 2 can share 8 lollies equally among 4 recipients they do not associate pictorial representations of "continuous" 1/4 (e.g. those shown in Figure 7) with the expression "one-quarter". Even at Grade 5 level many children do not have this association. In our study, many children associated pictures like those in Figure 7 with the fraction 1/2. The interviews made it clear why this was done. Even children in Grade 2, and certainly those in Grades 3 through 5, have acquired the notion that "one-quarter is a half of a half"; they say the words, and can often physically demonstrate their meaning by cutting or folding, etc. However, this iterative procedure is only partially understood, in the sense that while children know that "halving" and "getting quarters" are related, they are not sure what the relationship is. Much of the difficulty derives from their uncertainty in respect of the definition of the "whole". For example, with the rectangle in Figure 7, many children think that the shaded sub-rectangle is half of the two sub-rectangles on the left side of the rectangle; thus, they conclude that the shaded sub-rectangle stands for 1/2, not 1/4.

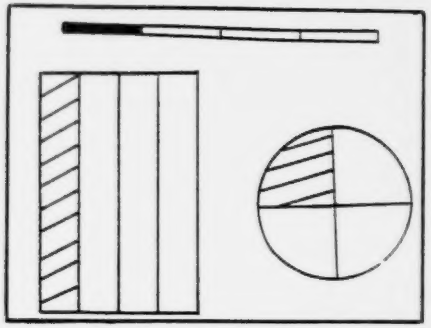


Figure 7: Three pictorial representations of 1/4

This uncertainty with respect to the relationship between getting "half of a half" and "one-quarter" was often evident in the interview situation. When asked to fold a piece of string to show one-quarter, for example, most children folded the string in half, and then folded the "double half" in half again. But when asked "where the quarter was", they often could not answer. All they knew was that "you get one-quarter by getting half of a half".

Another common error on the "continuous" 1/4 pen-and-paper items (such as those shown in figure 7) was to say each picture represented "one-third" because "one section was shaded and three were not". This kind of response relied on the word association of the numbers "one" and "three" with the expression "one-third". This word association remained strong throughout elementary schooling.

Despite the fact that in this study the expression "one-quarter" was used and the expression "one-fourth" was not, most children knew that, somehow, the number "four" and the expression "one-quarter" were linked. When children were asked, at the beginning of the interviews, to say the first thing that came into their heads when they heard the expression "one-quarter", almost one-third of the interview sample (in fact, 31%) replied "four".

From the **receptive** point of view, most children realised that for "one-quarter" you need to have four identical subunits. Thus, despite the obliqueness of the diameters for the circle in Figure 8, most children responded correctly to the item.

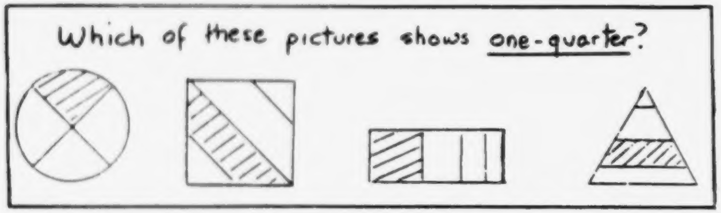


Figure 8: A pen-and-paper "one-quarter" item

On tasks requiring expressive demonstration of the need for four identical subunits, the children had more difficulty. For example, in one of the interview tasks children were given a 10cm length of plasticine and a balance and were asked to find one-quarter of the plasticine. Typically, children broke the plasticine in half, then after breaking one, or both, of the halves in half again, pronounced that they had obtained "one-quarter". However, when asked where the "one-quarter" was they often floundered. Furthermore, when challenged to use the balance they rarely had any idea what use it could be in the exercise. Some picked up one of the bits of plasticine they had obtained and, putting it on one side of the balance, asked, "Is that what you mean? That's one-quarter?" Others put one bit of plasticine on one side of the balance and three bits on the other, or two bits on one side and two bits on the other, or even four bits on one side.

This lack of awareness, in expressive demonstrations of one-quarter, of the need to obtain four **identical** subunits, was particularly obvious in many responses to the interview task of cutting a paper circle to show "one-quarter". While the children recognized pictures of "one-quarter" (see the comments on Figure 8), they often could not cut a circle into four equal sectors. It was more common for them to cut the circle in half (approximately) and then to cut the halves into sections by cuts which were parallel to the diameter defined by the first cut.

At the Grade 5 level, while 10 of the 24 children interviewed immediately cut out a sector from the paper circle which more or less corresponded to one-quarter of the circle, none of the 10 who did this felt the need to check their response. Of the other 14 Grade 5 interviewees, 7 obtained 4 pieces by making 3 parallel cuts. At the Grade 2 level no child cut out, directly, a sector which was close to 1/4. It seems to be the case that by the Grade 5 level many children have developed, or are developing, a visual image of a sector corresponding to the expression "one-quarter" - but this kind of image is not present in Grade 2 children.

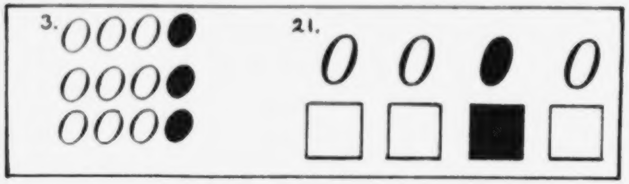


Figure 9: Two discrete representations of 1/4

We turn briefly to our data on the meanings children give to "one-quarter" in discrete contexts. In our pen-and-paper sample it was only at the Grade 5 level that more than 50% of children associated the fraction "one-quarter" with the pictures shown in Figure 9 (the percentages of children who did this were 18% in Grade 2, 22% in Grade 3, 30% in grade 4, 41% in Grade 4, and 52% in Grade 5). The most common errors on the two representations in Figure 9 were 1/3 (for Item 3 , and 1/2 (for Item 21), suggesting that the respondents simply associated the three black "ovals" in Item 3 with "one-third" (because of the word association), and the two black shapes in Item 21 with "one-half". Clearly the difficulties children experienced with discrete "one-half" tasks carried over to discrete "one-quarter" tasks. The "half of a half" strategy was attempted, usually unsuccessfully, by some Grade 4 and 5 children in the interview sample. Only a very small proportion of the interview sample had any idea about how to attempt the equilateral triangle perimeter task for "one-quarter." It seemed to be the case that this task evoked no links with the children's existing cognitive structures. The children in the interview sample had no idea on what the unit was for this task, and since they could not define the unit they could not obtain four identical subunits.

In summary, we were surprised by the extent of the difficulty experienced by children in Grades 2 through 5 on 1/4 questions of both the pen-and-paper and interview varieties. As a result of schooling, children had acquired a "half of a half notion", but often they did not understand how this procedure was associated with the expression "one-quarter". Also, if in a "continuous" pen-and-paper 1/4 question one section of a diagram was shaded and three were not, then, throughout the grades, many children thought the diagram represented "one-third" because of the word association of "third" with "three".

In "discrete" 1/4 questions, of both the pen-and-paper and interview varieties, children throughout the grades tended not to be able to identify the "whole" and, therefore, were not able to partition, even though they knew four subunits were required. Being asked to walk one-quarter of the way around an equilateral triangle confused most of the interview sample. Children simply did not know what the expression "one-quarter" meant in this context. By Grade 5, most children could respond quickly to "continuous" 1/4 pen-and-paper tasks and could apply the "half of a half" procedure in interview tasks involving "continuous" 1/4. However, despite the fact that almost all fifth-grade children were successful in sharing eight lollies among four children (on a pen-and-paper item), about one-half of the fifth-grade interview sample could not cope with "discrete" 1/4 questions.

It is only by Grade 5 that some children seem to have developed a mental image of the shape of "one-quarter" in certain common "continuous" embodiments (for example, a circle).

One-third

Three pen-and-papers items asked children to share 6, 12, and 18 lollies respectively among three children. These questions were done extremely well by almost all children in Grades 2 through 5. During the interviews it became clear that children in Grade 2 tended to use a "sharing" procedure to partition the given sets of lollies. This procedure continued to be used by the children in Grades 3, 4, and 5 but by Grades 4 and 5 the appropriateness of a division procedure ("eighteen divided by three equals six") was recognised by more than 50% of the interview sample.

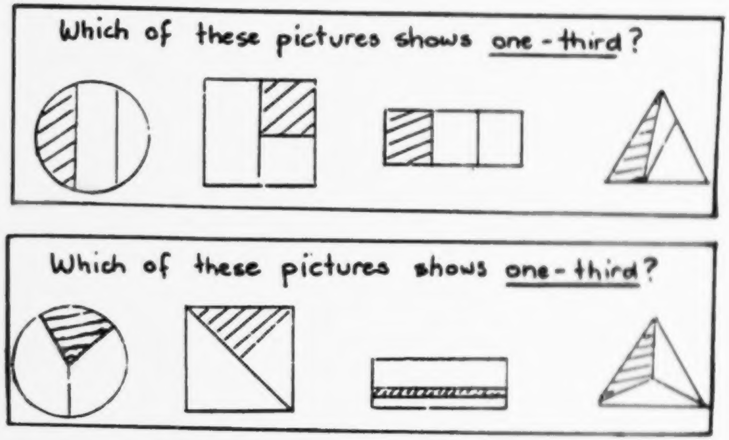

Which of these pictures shows one-third?
Which of these pictures shows one-third?

Figure 10: Two pen-and-paper 1/3 questions

Most Grade 2 children found the questions shown in Figure 10 difficult, with only about 30% of responses being correct. By Grade 5 more children recognised the need for identical subunits, but still 25% gave incorrect responses. The interviews made it clear that children are tempted to cut circles vertically in attempts to obtain "one-third" subunits. They have no visual image of 120° sectors.

On pen-and-paper 1/3 items requiring children to associate a fraction with a given picture, children in Grades 2 through 5 performed little better than what might have been expected from random responses. Except for the "sharing" idea, which most had acquired and could apply well, the children's understanding of "one-third" left much to be desired.

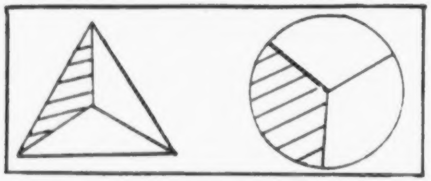


Figure 11 Two "continuous" 1/3 pictures

Children looking at the pictures in Figure 11 seemed to know, intuitively, that the shaded portions were not "one-half", despite the fact that there was one shaded section and two unshaded sections. If it was not "one-half", then they had no procedure for working out what the fraction might be. They opted for "one-quarter" because this was the only other fraction they had heard much about.

This identification of "one-third" with "one- quarter" was very common in the interviews. Often the child would use a "half of a half" procedure and then say that the "one-quarter" which was obtained was, in fact, "one-third". Also, when this was done it was not uncommon for the "three-quarter" sections which were obtained to be associated in the children's minds with "one-third". This linking of 3/4 and 1/3 seemed to derive from the word association of "three" (in "**three**-quarters) and "third" (in "one-**third**"). Certainly many children did not know that "one-third" is less than "one-half" and, in the interviews, children often said that a container almost full with water was "one-third". Figure 12 shows two responses from Grade 2 children who had been asked to shade "one-third" of a circle and "one-third" of a triangle. In both cases the children emphasized that it was the **shaded** part that was "one-third".

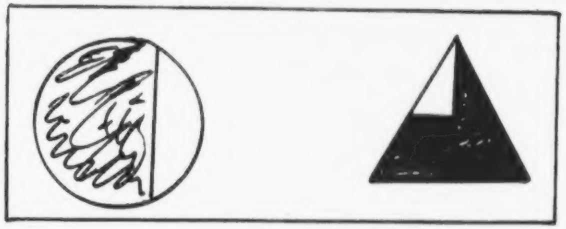


Figure 12 Children's representations of "one-third"

With the 10cm length of plasticine, many children broke the plasticine into two unequal sections and said the longer part was "one-third". Those who did break the plasticine into three parts rarely obtained three **equal** parts, and were nonplussed when asked to use the balance to see if they had obtained "one-third". Indeed, even those who succeeded in obtaining three roughly equal parts rarely knew what to do with the balance to check that they had obtained "one-third".

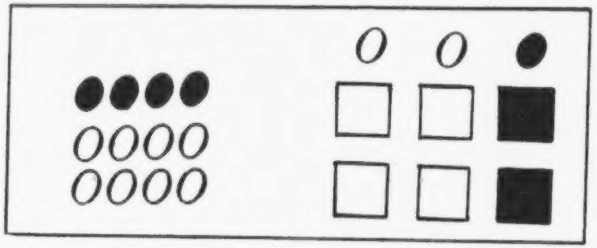


Figure 13: Two "discrete" arrays showing 1/3

Children had even less idea of "discrete" 1/3 than they did of "continuous" 1/3. In the arrays shown in Figure 13 the responses "1/2", "1/4", and "none of these" (i.e. none of 1/2, 1/4 and 1/3) were as numerous as "1/3" for children in Grades 2 through 5. In the interview situation when children were given 12 identical counters and asked to show "one-third" of them, they rarely knew how to begin. Yet, if they had been given 12 lollies and asked to share them equally among three people most would have been able to do it.

The almost total lack of understanding of "one-third" was especially apparent on the interview task involving going "one-third" of the way around the equilateral triangle. We had thought that perhaps this was a "one-third" embodiment which the children would grasp because the triangle had three sides of equal length. However, we were mistaken. Indeed, even at the Grades 4 and 5 levels most children did not relate "one-third" to the sides of the equilateral triangle, and less than 10% of all the children interviewed obtained the correct answer. The vertex for "two-thirds" was nominated more often than the vertex for "one-third".

In summary, while children know how to share a reasonably small number of objects among three people (provided the sharing can be done equally), they do not identify appropriate "wholes" for "continuous" and "discrete" 1/3 tasks in both the pen-and-paper and interview varieties. Since they do not identify the "whole" they are not able to carry out partitions corresponding to "one-third". For "continuous" tasks, they have no visual image of what "one-third" of a shape looks like and therefore nothing to guide them in their choice of strategy. The only hint they detect of the meaning of "one-third" is the word association of "third" with "three". When, in the interview situation, the children were asked "What is the first thing that comes into your head when I say one-third?" the most common response was "three". But the meaning they gave to the linking of "three" with "one-third" was highly idiosyncratic. If, in "continuous" 1/4 questions they saw a picture with one part shaded and three parts not shaded, then this might be "one-third". If they think they can see "one-quarter" of a "continuous" shape then the other larger section might be "one-third" because it is "three-quarters" and there is a word association of "third" with "three". So, in the interviews, many children produced representations which were more than "one-half" of a defined "whole" when they were asked to show "one-third". As several interviewees commented, "That's about three-quarters, and three-quarters is a third, isn't it?" It is not surprising, then, that virtually no child in the interview sample, at any grade level, could confidently assert that "one-third" is more than "one-quarter" but less than "one-half".

2. Do the meanings children give to "one-half", "one-quarter"' and "one-third" depend on context?

When mathematics educators have written about children's conceptions of fractions they have tended to concentrate on children's abilities to identify appropriate "wholes", to partition "wholes" in ways corresponding to given fractional quantites so that identical subunits are obtained, and to demonstrate fractional quantities by shading or cutting or folding etc. on a very limited range of tasks (e.g. circles, rectangles, and plasticine). Our data indicate that the notions of "wholes", and "partitioning", and "equal parts", are not part of the cognitive structures of elementary schoolchildren.

In our study, children in Grades 2 through 5 did best on pen-and-paper "continuous" 1/2 and 1/4 questions in which circular, rectangular, and linear pictorial representations were given. They also did well on the pen-and-paper "continuous" 1/2 question in which an equilateral triangle with a vertical axis of symmetry was given. The success on these questions seemed to be more related to standard visual images being stored in the children's long-term memories than to children's analyses of the pictorial representations. Clearly, the above-mentioned representations (circles, rectangles, lines, and triangles) are often used by teachers when teaching the "fractions" 1/2 and 1/4.

Children don't think about "wholes" or partitioning; rather, hearing a verbal stimulus (e.g. "one-half" or "one-quarter") causes them to consult their long-term memories and to evoke appropriate visual images. In most cases, this image has a vertical or horizontal line of symmetry. So, on pen-and-paper questions where the line of symmetry is neither vertical nor horizontal, children experience greater difficulty. Of course, it is very unlikely they can verbalise the fact that they are using visual imagery, and our interview tasks had the children attempting to explain what they were doing using words and concepts which they had heard their teachers use. Of course, the children did not often really understand what the teachers had told or shown them. Thus, for "one-quarter" tasks, "half of a half" iteration ideas and word-association procedures were often erroneously used.

Our data make it abundantly clear that "discrete" 1/2, 1/4, and 1/3 questions, whether in pen-and-paper or interview form, are much more difficult than "continuous" questions. Children simply have not stored visual images in these cases and are therefore forced to use their misunderstandings of their teachers' ideas of "wholes" and partitioning, and "equal shares". The same comment applies to the interview task involving the perimeter of an equilateral triangle. We believe, then, that the meanings children give to 1/2, 1/4, and 1/3 depend very much on whether the context is "continuous" or "discrete"; also, for "continuous" representations, certain shapes are easier than others because of the visual imagery factor. Interestingly, in the past, neither children, nor teachers, nor educational researchers have paid much attention to imagery in connection with children's learning of fractions.

3. Is 1/2 acquired before 1/4, and is 1/4 acquired before 1/3?

Table 3 summarises our response to this question.

Table 3 The relative difficulties of 1/2, 1/4 and 1/3

	1/2		1/4		1/3	
	rec*	exp**	rec	exp	rec	exp
continuous	1	3	2	4	5	11
discrete	6	8	7	9	10	12

* rec. stands for "receptive mode" (e.g. pen-and-paper item)
** exp. stands for "expressive mode" (as in interview tasks)

The entries in Table 3 indicate the relative difficulty of typical questions in that category. Our data indicate that "continuous" 1/2 receptive-mode questions are the easiest, "continuous" 1/4 receptive-mode questions are the next easiest, and so on, with "discrete" 1/3 expressive-mode questions being the hardest.

It is fallacious to say that 1/2 questions are easier than 1/4 questions which, in turn, are easier than 1/3 questions. Table 2 makes it clear that whether a question is "continuous" or "discrete", or involves receptive or expressive modes, are crucial considerations for predicting the likely difficulty of fraction tasks for elementary schoolchildren. Textbook writers and teachers might do well to ensure that tasks associated with each of the 12 cells in Table 2 are given due attention in school mathematics programs.

4. Do children possess partitioning concepts independent of their understanding of the fractions 1/2, 1/4, and 1/3?

The short answer to this question is "yes". Our pen-and-paper items involving equal sharing of lollies with given numbers of people made it clear to us that children who can share equally are not necessarily able to give meaning to the expressions "one-half", "one-quarter", and "one-third". The ability to partition seems to be largely independent of children's knowledge of fractions. Putting it another way, in most contexts the notions of "equal sharing" and "fractional quantities" are not related in children's cognitive structures. This applies to children in Grades 2 through 5. The only exception would appear to be with "continuous" 1/2, 1/4, and 1/3 tasks for circular or rectangular embodiments. Sometimes children associate these with an event like sharing sections of a birthday cake. Often they associate circles and rectangles with classroom episodes on fractions.

4. SOME FINAL COMMENTS

Many elementary schoolchildren all around the world find the learning of fractional concepts difficult. It is our view that this is because teachers, textbook writers and educational researchers, when addressing the issues associated with the teaching and learning of fractions, use language and physical demonstrations which confuse children. Children give different meanings to fraction words, in different contexts, and the meanings they give are not even known to themselves, let alone to adult observers. In future, we need to pay more attention to visual imagery, to the distinction between receptive and expressive modes, to the distinction between "continuous" and "discrete" embodiments, and to the peculiar features associated with the actual embodiment. Thus, investigating fractional understandings in situations involving 2-D drawings of shapes, or manipuilating actual 3-D shapes, or capacity and volume, or perimeter, or mass, or time, might yield quite different data so far as children's thinking is concerned.

Our challenge, then, is to acquire an understanding of the meanings a child attaches to a fractional expression (like "one-third") in different contexts. Careful consideration of Table 3, in this paper, might be especially helpful in this regard. Certainly the notions of a "whole", and "partitioning" and "equal shares" are important and must always be kept in mind by teachers. But let us be wary of imposing adult conceptions on children's minds. If we do this, then children will continue to have fractional understanding of fractions.

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Student Representations of Simple Electric Circuits

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Alternative frameworks in electricity are expected to be less stable than in mechanics: since electric phenomena are not as pervasively present to the conscience of the children as mechanical ones, their common science representations or models appear later in life, they are not as compelling and are easier to change. But this does not mean that the correct scientific view is more easily established.

In a collaboration with J.L.Closset we have been interested in following the evolution (1) of some of the already known frameworks which students utilize when studying electric circuits.

Closset (2) described the "sequential" method as one of the most diffuse ones : the circuit is examined step by step starting from the battery, the current "decides" locally what to do, e.g. it divides evenly at branch points since it "does not know what comes next", and if the circuit is modified only the elements downstream are affected by the change. This model needs obviously to know the sign of the travelling charges since it has to establish the direction of the current flow.

Another common-sense procedure which we call the "constant value" is to endow the electric properties directly to the circuits elements: when they are on, a battery produces always the same amount of current, a lamp lights up always the same.

Notice that the constant-current battery representation is the only one which entails a certain feeling of the circuit as a system, since the current made available by the battery has to be divided up among all the circuit elements.

We used together two of Closset's tests, the "hot iron" and the "three lamps", with the idea of probing the coherence in the use of frameworks. We proposed them to high-school pupils either before (st (b)) or after (st (a)) formal instruction on electric circuits, and to university students in biology (Bio) and in physics (Phy) before they studied circuits at university; in this case the difference is made by spontaneous motivation selection.

The answers have been grouped into blocks: besides the "OK" one and the "No" answers, the "Sequential", the "Constant Value" and "Others".

The hot iron test (fig.1) seems to be easier but that is due only to an ambiguity, since the problem is not enough defined to yeld one answer: as the heat developed by the iron is maximum when its resistance is adapted to that of the circuit, and since the test does not specify the ratio of the resistances of the two lamps to that of the iron, the required increase of the heating could be achieved either by an increase or by a reduction of the iron resistance. But student has ever discovered this difficulty: they always reason in terms either of an increase of the resistances or of the current, but never think of their interaction.

Accepting as "correct" both an increase or a decrease of luminosity of the lamps, the hot iron test scores well (between 20 and 55%) (fig.2), although never for the correct reason.

The sequential reasoning reveals itself in two ways: by a lower luminosity of the downstream lamp to begin with, and by a further dimming of the same lamp after the iron increased its absorption, coherently with the common-sense expression of "current consumption". But for others the iron behaves more like a dam which holds up "the current", thus increasing at the same time the light in the upstream lamp. Notice that this is a very interesting interpretation of conservation of current.

The sequential model features two different sets of answers, depending on the flow direction of the current,i.e. the sign of the carriers. The ratio of positive to negative current shows an interesting pattern (fig.3), which suggests that formal instruction reduces the worry about the "real" current being made of electrons flowing the other way round.

The "three lamps" test (fig.4) has the advantage that the correct answer can only be obtained the correct way; its lower scoring (from 0 to 20%) (fig.5) is probably only due to this. The sequential reasoning is revealed by the fact that the luminosity of lamp 1 is unaffected when the circuit is modified; this model features a consistently high score.

As for the constant value procedure, if it applies to the battery, the two lamps give more light afterwards, since they share the same current which before had to be divided by three; if the scheme applies to the lamps, both their luminosities remain unchanged.

Analysing both tests together gives rise to a matrix (fig.6 e fig.7), on which those students which are completely coherent reside on the main diagonal. Before instruction there is a certain amount of them which are coherently "constant energy", and only few are completely "sequential". This changes afterwards, the impression one gets is that the school training increases the sequential reasoning (which is already known to appear later (1), a probable effect of the school). The second effect of instruction is to spread the answers over a wider spectrum: starting from a fairly coherent situation, either sequential or constant value (but with only one correct answer!), the students scatter much more over the matrix, to reach again a less dispersed situation (but with a high percentage of sequentials!) only with the physics students, a population sample selected by motivation.

The scattered or missing answers, being 25% before instruction, rise after it to over 50%, and remain there to descend again to 25% only for the physics students sample.

We gave a more complicated circuit at a real university examination to 230 biology students (the "too simple" research-circuits are thought to be too easy in the normal school routine), asking to compare the luminosity between selected couples of a 10 lamps circuit (fig.8). The sequential reasoning appears most dramatically when comparing lamps 5 and 6, which lights up differently in 28% of the sample, in a way which depends on the current flow direction. The most common error (80% of the sample) states the equal brightness of lamps 1 and 7: usually this answer is due to a sequential model, but often it explicity demonstrates a new type of constant value reasoning: the resistances are from the beginning defined as being either "series" or "parallel", these featuring 1/2 or 1/4 etc. of the original current, depending on their level in the hierarchy. In this framework 1 and 7 are parallel of the first level.

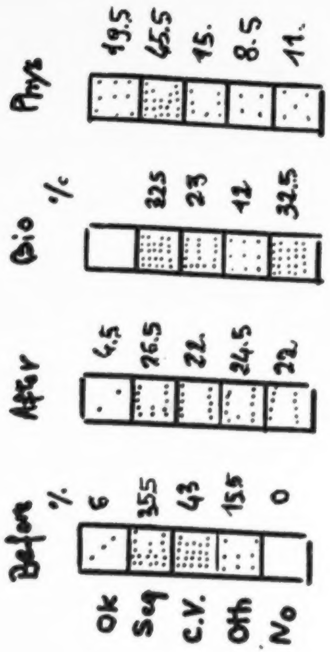


Fig 5

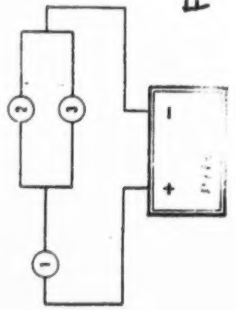


Fig.4

a) The three lamps are identical. Is there a lamp which is brighter than the others? If yes, which one?

We cap the wire both sides of lamp 3 and take the lamp away:

b) L2 is brighter than before
L2 is less bright than before
L2 is as bright as before

c) L1 lights up more than before
L1 lights up less than before
L1 lights up the same than before

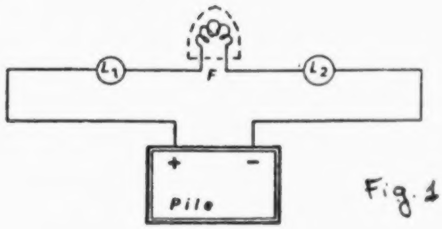


Fig. 1

L1 and L2 are two identical lamps, F a toy hot iron, heated as a real one by the battery:

a) L1 is brighter than L2 ☐
L1 is less bright than L2 ☐
L1 is as bright as L2 ☐

F is substituted by a bigger iron F1, which heats more than F:

b) L1 is brighter more than before ☐
L1 is less bright than before ☐
L1 is as bright as before ☐

c) L2 is brighter more than before ☐
L2 is less bright than before ☐
L2 is as bright as before ☐

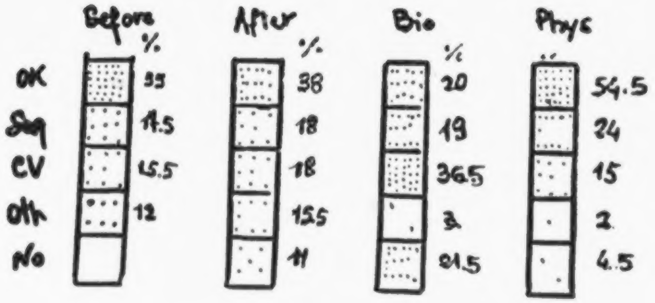
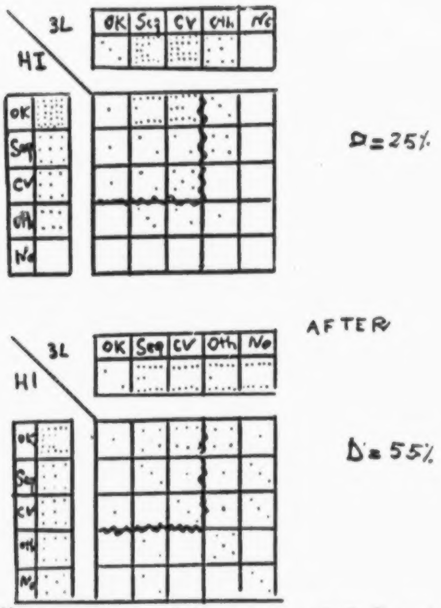


Fig 2

HI	Bef.	Aft.	Bio	Phys
$\frac{Seq(+)}{Seq(-)}=$	0.5	3.0	1.0	2.7

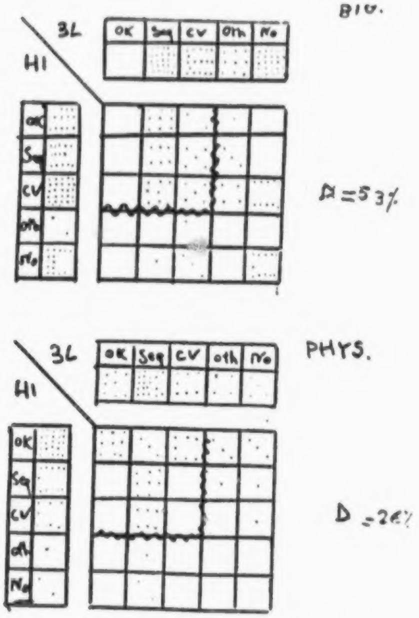
Fig 3



237

D is the percentage of the answers out off the red square

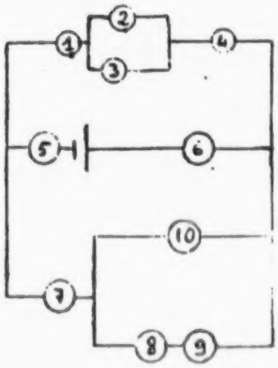
Fig 6



D is the percentage of the answers out off the red square

Fig 7

The ten lamps of this circuit are all identical. Mark for each of the indicated couples which is the brighter one or if the two lamps are equally bright, and justify your choices.



				% No correct
	1	>	2	13.8
	1	=	4	12.3
→	1	<	5	42.7
→	1	<	7	19.7
	1	>	8	24
	2	=	3	12
	3	<	4	12.3
→	5	=	6	29
	7	>	8	12
→	9	<	10	55.9

fig 8

HIERARCHICALLY STRUCTURED PROBLEM SOLVING IN ELEMENTARY MECHANICS: GUIDING NOVICES' PROBLEM ANALYSIS [*]

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A major problem in education is to convey a sense of how problems in a domain of inquiry are approached by an expert. This means not only providing the student, or novice, with some knowledge of the body of facts relevant to the domain, but also imparting a sense of how the facts are interrelated and of what heuristics are useful for solving problems. The research reported in this article investigates the effects of structuring the problem solving activities of novices in the domain of classical mechanics in a way consistent with the problem solving approaches used by experts. Such structuring was accomplished through the use of a hierarchical, computer-based problem analysis tool, which interactively developed equations appropriate for solving mechanics problems.

That novices and experts store and use domain-specific knowledge in distinctly different ways is the consensus of a number of studies in such diverse fields as chess (Chase & Simon, 1973), computer programming (Ehrlich & Soloway, 1982), electrical circuits (Egan & Schwartz, 1979), and classical mechanics (Larkin, 1979). Experts tend to store information in hierarchically structured clusters related by underlying principles or concepts. When attempting to solve a problem, experts initially focus on the principles and heuristics that could be applied to solve that problem. In contrast, the knowledge base of novices is more homogeneous and has fewer interconnections. When solving problems, novices do not focus on principles or heuristics that could be used to construct a solution strategy, but rather focus on the actual equations that could be manipulated to yield an answer (Chi, Feltovich & Glaser, 1981; Larkin, McDermott, Simon & Simon, 1980; Mestre & Gerace, 1986).

Recent studies (Eylon and Reif, 1984; Heller and Reif, 1984) suggest it may be possible to improve the problem categorization skills and the problem solving performance of novices by using an instructional approach that imposes a hierarchical, expert-like organization both on information,

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and on problem solving heuristics. Eylon and Reif (1984) found that subjects presented with a physics argument organized in hierarchical form performed significantly better on recall and problem solving tasks than did subjects who were presented the same argument non-hierarchically. In addition they found that the hierarchical presentation of a set of rules needed to solve a particular class of physics problems resulted in better performance on similar problems than a non-hierarchical presentation of the same set of rules. These results suggest that the organization of the information imparted in teaching may be as important as the information itself, since the organization has an effect on intellectual performance.

In another study, Heller and Reif (1984) showed that novices taught to perform qualitative analyses of force problems, considering principles, concepts, and heuristics, improved in ability to construct problem solutions. Interestingly, qualitative analyses are commonly performed by experts, yet novices are not even implicitly taught how to perform such analyses. Even novices who had received high grades in their mechanics course were unable to perform meaningful qualitative analyses before training. Together, the two Reif studies indicate that the performance of novices on several types of tasks can be improved when they are taught using pedagogical approaches designed to reflect expert-like knowledge organization and behavior.

In the present study novices actively participated in problem solving activities which were structured to reflect our best understanding of how physics experts analyze problems. The treatment involved five one-hour sessions during which subjects solved a total of 25 classical mechanics problems using a menu-driven, problem-analysis program, termed the Hierarchical Analysis Tool. Problem analysis activities were structured by presenting the novice with a series of questions and options dynamically generated by computer software.

The user begins the analysis from the initial menu where s/he is asked to select the appropriate principle that could be applied to solve the problem under consideration. Subsequent menu selections are dependent on prior choices, and involve both concepts and heuristics. The analysis is termed hierarchical because the selections become increasingly specific to the problem situation as one progresses. At the end of the analysis, the user is provided with a set of equations which are consistent with the menu selections made during the analysis, and which could be used to solve the problem. The subject must then manipulate these equations to construct a final solution.

The effects of this treatment were compared with that of two other treatments, each intended to reflect some aspect of the problem solving behavior observed in novices. In one treatment, novices were provided with the textbook that they had used in their physics course and asked to solve the set of 25 mechanics problems as they would in doing homework. In the other treatment, novices used an equation data-base program, termed the Equation Sorting Tool, to obtain the equations needed to solve the same 25 problems. This sorting tool contained all the useful equations found in the commonly used freshman mechanics text written by Resnick and Halliday (1977). The body of equations could be reduced to a small, manageable number by performing sequential sorts according to principles or to surface-feature attributes similar to those employed by novices in categorization and problem solving tasks.

The effect of these treatments on problem categorization and problem solving skills was assessed using two measures. One measure was a test designed to be similar to a traditional final exam in a freshman level classical mechanics course and was intended to measure changes in problem solving skills. The test contained a mixture of problems requiring the application of either one or two principles for solution.

Results from the Reif studies suggested that subjects who used the Hierarchical Analysis Tool might improve more in problem solving performance than either of the other two comparison groups. This suggestion should be qualified by noting that the current approach is significantly more ambitious than that employed in the Reif studies, which focused on specific classes of problems, rather than the entire field of classical mechanics. Unlike the treatments employed in the Reif studies those used here did not present organized, domain specific information, nor did they provide explicit instruction in problem solving strategies.

The other measure, constructed specifically for this study (Hardiman, et al., 1987), was a judgment task designed to detect shifts toward reliance on a problem's deep structure (i.e., principle used for solution), rather than its surface features (i.e., objects or terminology used in the problem), in problem categorization. The task consisted of 32 items, each composed of a model problem and two comparison problems. Subjects were asked to judge which of the two comparison problems would be solved most like the given model problem.

Each comparison problem was designed to match its associated model problem in either surface features (S), deep structure (D), both surface features and deep structure (SD), or neither surface features nor deep structure (N). Comparison problems were paired together, such that only one of the comparison problems matched the model problem in deep structure. This meant that comparison problems could be paired in four ways: 1) S-D, 2) S-SD, 3) N-D, and 4) N-SD.

The variation of surface features and deep structure allowed us to investigate shifts from novice-like, surface feature-based categorization to expert-like, deep structure categorization (Chi, et al., 1981). Assuming a categorization strategy based strictly on surface features, the following predictions should hold for performance in the four categories: 1) S-D: always chose S, so 0% deep structure choices, 2) S-SD: S and SD both match in surface features, so both are equally good choices, implying 50% deep structure choices, 3) N-D: neither alternative is a good choice, predicting 50% deep structure choices, and 4) N-SD: the surface feature match is consistent with the deep structure match, so 100% deep structure choices. The predicted pattern of performance for experts, assuming a categorization strategy based strictly on deep structure, would be 100% for all four item types. Thus, a shift to more expert-like categorization strategies should be evidenced by an increase in percentage of deep structure choices in the first three categories, particularly on S-D items.

DESCRIPTION OF COMPUTER-BASED TOOLS

Since understanding the findings of this study hinge on understanding the treatments that subjects received, this section is devoted to describing the architecture and functioning of the two computer-based tools. Both tools were designed as resources to help novices construct problem solutions. As such, neither tool provided the user with an actual answer to the problem under consideration. The final result in both cases was one or more equations that could be applied to solve the problem. After using either tool the subject still had to synthesize and manipulate the equations generated during the analysis to write out a formal solution. More over, if the computer-based tools were used inappropriately, then the equations generated would be incomplete or inappropriate for solving the problem.

Hierarchical Analysis Tool

The Hierarchical Analysis Tool presents the user with a series of menus and options that require responses to well-defined questions that an expert might pose when analyzing a classical mechanics problem. This Analysis Tool does not explicitly tutor or provide feedback to the user --

it only provides a mechanism that constrains the user to an expert-like approach to problem analysis. The best means of understanding its structure and function is through an example. Let us consider the following problem:

PROBLEM 1

A small block of mass M slides along a track having both curved and horizontal sections as shown. The track is frictionless. If the particle is released from rest at height h, what is its speed when it is on the horizontal section of the track?

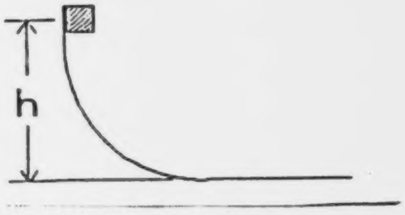


Figure 1 contains the series of menus and menu selections that appropriately analyze Problem 1 (we have placed an asterisk next to the appropriate choices to facilitate discussion). Several features of Figure 1 should be noted. The first menu asks the user to select among four fundamental principles that could be applied to solve the problem. Since this problem can be solved most easily using work and energy principles, menu selection #4 is the appropriate choice. The second menu is more specific and asks the user to describe the mechanical energy of the system. Explanatory information is provided (enclosed in parentheses) to help the user decipher the choices presented.

Heuristics dictate the choices presented in menu level 3: the user is asked to classify the changes in mechanical energy by considering one body at a time at some initial and some final state. In problem 1, the block starts out with only potential energy and ends up with only kinetic energy (assuming one takes the potential energy to be zero, level with the horizontal portion of the track). The fourth menu level asks the user to characterize the changes in kinetic energy, which in this case are comprised purely of changes in translational kinetic energy. The user must then specify the boundary conditions (i.e., conditions at the beginning and end points). Then, the cycle of questions is repeated to describe the changes in potential energy in menu levels 6 and 7.

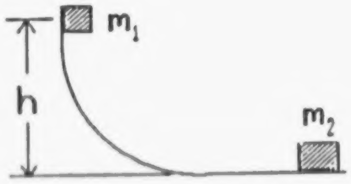
At menu level 8, the user is asked whether there is more than one body in the system. Since there is not, the next screen gives a summary of the solution plan generated thus far, which includes the principle that was selected at the first menu level, now stated in a general equation form, as well as the specific equations dictated by the selections made during the analysis. If appropriate selections were made, then the general and specific equations can be combined to generate a correct answer to the problem. For Problem #1, the user would have to manipulate the equations given in menu level #9 of Figure 1 to obtain the correct answer, namely, $v = \sqrt{2gh}$.

If the user makes an inappropriate selection at any menu level during the analysis, the end result is a set of equations that is consistent with the classification scheme selected, but that may be inappropriate for use in solving the problem. The user becomes aware of errors he or she committed during the analysis only by recognizing that a particular set of menu options, or that the final set of equations, do not fit the problem being analyzed. In this case, the user may back up to some previous menu and change a selection or, if s/he desires, return to Menu 1 to restart the analysis. To keep track of his/her menu selections the user does have the option of listing all the menu selections made previous to the current menu. If any of the terms that appear in the menu choices are unfamiliar, the user can press a key to look up that term in a glossary. The Hierarchical Analysis Tool then returns to the analysis without loss of continuity.

At the final menu presented in Figure 1, the user must decide whether or not the analysis for the problem is complete. If the problem can be solved with the set of equations that was generated, then the user can say that the analysis is finished. If necessary, the user can review the set of equations generated once more. However, if the problem has more than one part and requires the application of more than one principle, then the user can opt to continue the analysis of the problem. For example, consider the following twist to Problem #1:

PROBLEM #2:

A block of mass m_1 is released from rest at height h on a frictionless track having both curved and horizontal sections as shown. When the block reaches the horizontal section, it collides and sticks to another block of mass m_2. Find the final speed of the two block system.



Problem 2 must be solved using a sequential application of work-energy and linear momentum principles. To obtain the speed of block m_1 when it reaches the horizontal part of the track the user would use conservation of energy, just as in Problem 1. Then the user would return to the first menu to continue the solution, selecting "Linear Momentum" in order to determine the final speed of the two-block system after the collision. The end result is two "equation screens" that may be used to solve the problem: one allowing the computation of the speed of m_1 when it reaches the bottom of the ramp, and the other allowing the compution of the final speed of the two-block system.

The Hierarchical Analysis Tool can also be used to analyze problems that could be solved using distinctly different methods. For example, the following problem could be solved by selecting either "Work and Energy" or "Newton's Second Law or Kinematics" at the first menu level:

> A ball is released from rest at height h in the earth's gravitational field, and allowed to fall freely. What is its speed just prior to hitting the ground? Neglect air resistance.

The selection of "Work and Energy" would lead to an analysis similar to that presented in Figure 1. Choosing "Newton's Second Law or Kinematics" would put the user onto a path leading to the kinematic equations governing motion under the influence of a constant acceleration; in this case, the acceleration is caused by the earth's gravitational field. Both paths would result in final menus containing equations that would be appropriate for solving the problem.

Equation Sorting Tool

The Equation Sorting Tool is intended to be consonant with the problem solving habits that have been observed in novice physics students. It is a data base of equations used in classical mechanics that can be sorted in three different ways: 1) by Problem Types, such as "inclined plane" and "falling bodies," 2) by Variable Names, such as "mass" and "velocity," and 3) by Physics Terms, such as "potential energy" and "momentum." The Sorting Tool was designed to reflect the fact that novices tend to focus their efforts on finding the appropriate set of equations that can be manipulated to yield an answer. The "formula sheet," a solid mosaic of equations, that many students are allowed to bring to exams offers blatant evidence of this approach. Further, novices appear to cue on a problem's surface features in deciding what equation to use. These surface features can be classified via the three categories listed above.

The data-base contains 178 equations from the first fourteen chapters of the commonly used classical mechanics text, Physics, by Resnick and Halliday (1977). This data-base can be reduced to a small number of related equations by performing sequential sorts. For example, to analyze Problem 1 the user may first choose to perform a sort according to the Variable Name "height." This produces a list of those equations containing the variable "h." The user then has the option to browse through the reduced equation list, or to perform another sort. If the user chooses to perform another sort, a logical choice might be the Problem Type "sliding bodies." The data-base is then reduced to those equations that both contain the variable "h" and are pertinent to sliding bodies. After a few such sorts, the number of equations is reduced to a small, manageable number, from which the user can select the one or two equations needed to solve the problem.

PARTICIPANTS IN EXPERIMENTS 1 AND 2

Subjects

Forty-nine undergraduate students at the University of Massachusetts who had completed the first semester physics course for majors or for engineers, and received a grade of B or better, participated in this study. Seven of these subjects were eliminated, five due to attrition, one because of an extremely low level of performance, and one because of ceiling-level performance, leaving a total of 42 subjects. The subjects participated in ten hour-long experimental sessions, for which they were paid fifty dollars.

Sessions

In the first session, a problem solving pretest was given (Experiment 2 reports pre- and post-test results). On the basis of pretest scores, the 42 subjects were divided into three treatment groups of 14 subjects each. The pre-treatment similarity judgment task was given in session 2 (pre- and post-treatment similarity judgment task results are reported in Experiment 1), followed by the pre-treatment explanation task in session 3 (reported in Touger et al., 1987, these Proceedings).

The treatment phase of the experiment occurred in sessions 4 through 8. In each of the five sessions of the treatment phase, the subjects solved a set of five problems. These problems were of different types based on the solution principle(s) that could be applied to solve them: 1) Force, 2) Energy, 3) Linear Momentum, 4) Angular Momentum, and 5) Combinations of 1-4. The five problems within a session were randomly ordered, and the five sets were randomly distributed across sessions.

The three treatment groups varied in the type of problem solving activity that they participated in, in sessions 4 through 8. Members of the Hierarchical Analysis Tool group (HAT group) used the Hierarchical Analysis Tool to generate equations tailored for the problem situation prior to solving each problem. The Equation Sorting Tool group (EST group) used the Equation Sorting Tool to sort the equation data base and choose those equations that might be relevant for solving the problem. The members of the textbook group (T group) were provided with a copy of the textbook that they had used in their particular physics course, and were allowed to use it as they wished to help them solve the problems.

Session 9 and 10 consisted of three post-treatment tasks: 1) the problem solving post-test, constructed to be equivalent in difficulty to the problem solving pre-test, 2) a repeat of the similarity judgment task, and 3) a second form of the explanations task.

EXPERIMENT 1: SIMILARITY JUDGMENT TASK

The similarity judgment task requires that subjects decide when two problems would be solved similarly. A variety of conditions can be constructed (described in the introduction), which require that subjects distinguish between essential (deep structure) and nonessential (surface features) characteristics of similarity among problems. This task was employed to determine whether subjects would more likely focus on a problem's deep structure after engaging in one of the three types of problem solving treatments. Since the Hierarchical Analysis Tool constrains the subject to an initial decision concerning the principle to be applied, we hypothesized that the HAT group would be more likely to focus on a problem's deep structure after treatment than either of the other two groups. The improvement should be particularly noticeable for the S-D items, where the the surface feature and deep structure are in direct competition.

The task contained 32 items. Each task item was composed of three elementary mechanics problems that were similar in type and level of difficulty to problems in the commonly used textbook written by Resnick and Halliday (1977). The problems were each three to five lines long and contained only text (no pictures or diagrams). For each item, one of the three problems was identified as the model problem, while the other two were the comparison problems. The subjects were to indicate which of the two comparison problems they believed "would be solved most similarly" to the model problem.

A comparison problem could share different numbers of and types of characteristics with its model problem. Four types of comparison problems were designed, each corresponding to one of the following matching characteristics: 1) surface features, meaning that the objects and descriptor terms that occur in both problems are similar, 2) deep structure, meaning that the physical principle that could be applied to solve both problems is the same, 3) both surface features and deep structure are the same for both problems, or 4) neither surface features nor deep structure are the same. These four types of comparison problems were termed S, D, SD, and N, respectively. The following is a sample model problem and the four comparison problems that were constructed to accompany it:

Model Problem

> A 2.5 kg ball of radius 4 cm is traveling at 7 m/s on a rough horizontal surface, but not spinning. Some distance later, the ball is rolling without slipping at 5m/s. How much work was done by friction?
>
> S Alternative
>
> > A 3 kg soccer ball of radius 15 cm is initially sliding at 10 m/s without spinning. The ball travels on a rough horizontal surface and eventually rolls without slipping. Find the ball's final velocity.
>
> D Alternative
>
> > A small rock of mass 10 g falling vertically hits a very thick layer of snow and penetrates 2 meters before coming to rest. If the rock's speed was 25 m/s just prior to hitting the snow, find the average force exerted on the rock by the snow.
>
> SD Alternative
>
> > A 0.5 kg billiard ball of radius 2 cm rolls without slipping down an inclined plane. If the billiard ball is initially at rest, what is its speed after it has moved through a vertical distance of .5 m?
>
> N Alternative
>
> > A 2 kg projectile is fired with an initial velocity of 1500 m/sec at an angle of 30 degrees above the horizontal and height 100 m above the ground. Find the time needed for the projectile to reach the ground.

In constructing allowable pairs of comparison problems to accompany a model problem, two comparison problems of different types were paired together, such that one of the two comparison problems matched the model problem in deep structure, while the other did not. This constraint led to four types of comparison problem pairs: 1) S-D, 2) S-SD, 3) N-D, and 4) N-SD.

In addition to varying the types of comparison sets, the deep structure and surface features of the model problems were also varied. There were initially eight model problems, later narrowed to five. Each model problem used in the study appeared four times, once with each of the four types of comparison sets. The deep structure and surface features of the eight model problems were as follows:

Model Prob.	Deep Structure	Surface Structure
1	Forces (Statics)	Spring, Friction
2	Energy	Spring
3	Linear Momentum	Two Blocks, Spring
4	Work-Energy	Rolling Ball, Friction
5	Angular Momentum	Rolling Ball, Friction*
6	Angular Momentum	Spinning Stick, Collision*
7	Linear Momentum	Collision*
8	Forces (Dynamics)	Friction, Motion

* Not used in the final analyses

Preliminary analysis of the data from a group of eight experts indicated that the eight model problems were not of equal difficulty, F(7,49) = 6.04, p<.0001. The experts' mean performance for the eight model problems ranged from 100% to 59% of the deep structure choices. Since it was important to establish a baseline of problems in which experts consistently agreed on the answer, the three model problems in which the lowest mean performance was observed were eliminated. These were model problem 5 (72% correct), model problem 6 (69% correct), and model problem 7 (59%

correct). After eliminating the 12 items that used model problems 5, 6, and 7, there was no longer a model problem main effect in the expert data, F(4,28) = 1.65, p=.1885. Eliminating these items from the novice analyses did not significantly alter any results.

The task was presented on IBM compatible PC's, to which a three-key response unit was attached. The subject was told to read carefully the model problem and the two comparison problems which would appear below it. Then they were to decide whether comparison problem A or comparison problem B would be solved most like the model problem, and press either the button labeled A or the button labeled B on the response unit to indicate their decision. The items were presented in random order, with no limit imposed on time to respond. After every 5 items, the subject was given the opportunity to take a brief rest. Most subjects completed the task within 45 minutes.

Results

The performances of the 42 subjects were compared in a 3 (Treatment Groups) x 14 (Subjects/Groups) x 2 (Times) x 4 (Comparison Types) x 5 (Model Problems) analysis of variance. Before considering the effects most relevant to this experiment, i.e., time and treatment groups, we will discuss the influence of Comparison Types and Model Problems. There were differences among the four types of comparison problem pairs, F(3,6) = 482.55, p<.0001, which followed the general pattern predicted by our hypothesis that novices would select comparison problems matching the model problem on surface features. Performance on each of the four types was (averaged over the pre- and post-task): 1) S-D, 27% (predicted 0%), 2) S-SD, 54% (predicted 50%), 3) N-D, 70% (predicted 50%), and 4) N-SD, 92% (predicted 100%). There were also differences among the five Model Problems, F(4,8) = 24.53, p=.0002. Mean performance ranged from 78% correct on the Forces/Spring-Friction problem to

42% correct on the Energy/Spring problem, with a mode of 61% correct. However, there were no significant interactions of either Comparison Type or Model Problem with Treatment Group or Time. Since these differences among Comparison Types and Model Problems are characteristic of novices in general, and not of the treatment per se, we refer the interested reader to Hardiman, Dufresne and Mestre (1987, these Proceedings) for further treatment of these topics.

The main question under investigation in this experiment was whether experience with the Hierarchical Analysis Tool would promote a shift toward reliance on deep structure rather than on surface features. The results indicate an affirmative response to this question. Initially, there were no differences between the groups, as can be seen in Table 1. Overall, there was a minor, insignificant improvement in performance after treatment for all subjects, from 60% on the pre-treatment task to 62% on the post-treatment task, suggesting that it takes more than a minor amount of practice to accomplish this shift toward reliance on deep structure. Hence, the fact that there were significant differences among the three groups in the amount of improvement from pre- to post-treatment on the judgment task (see Table 1), F(2,39) = 4.28, p=.02, is of considerable interest.

Table 1: Pre- and Post-treatment Percent Correct for the 3 Groups

Group	pre-treatment	post-treatment
HAT	56%	66%
EST	61%	61%
T	62%	58%
------	-------	------
Total	60%	62%

The HAT group showed a significant improvement in performance from the pre- to the post-treatment session, F(1,13) = 5.20, p=.04. Further-more the HAT group was the only group to show any indications of improvement; the mean performance of the EST group remained the same, while the performance of the T group declined. This result suggests that the Hierarchical Analysis Tool does promote a shift toward the use of deep structure, while the two control treatments do not. This shift was consistent across Comparison Types, as can be seen by improvements of at least 6 percentage points in each of the four categories (see Table 2). This improvement was significant for the S-D comparison type, t(13) = 3.12, p=.0324 (adjusted for four tests), which is encouraging given that the S-D items, where surface features and deep structure are in direct competition, present the most difficulty for novices.

Table 2: Pre- and Post-treatment Performance of the HA Group on the Four Comparison Types

Comparison Type	Pre-treatment	Post-treatment
S-D	17%	34%
S-SD	53%	61%
N-D	67%	76%
N-SD	88%	94%

Conclusions

The results of Experiment 1 indicate that the hierarchical approach to problem solving, exemplified by the Hierarchical Analysis Tool, does help students to shift their decision making criteria for problem categorization from one based on surface features toward one based on deep structure. We speculate that use of the Hierarchical Analysis Tool promotes this shift because it highlights the importance of applying principles to solve problems by asking subjects to select the applicable principle in the very first menu they encounter. Even if the user is not able to answer all of the subsequent questions correctly, the principle to be applied to obtain a solution may still be recognized as primary.

EXPERIMENT 2: PROBLEM SOLVING TEST

The problem solving pre- and post-tests were intended to assess changes in problem solving as a function of treatment. We expected that all subjects would improve in performance from the pre- to the post-test, since all subjects would have spent the same amount of time in problem solving activities during the treatment phase. However, we hoped that the HAT group would outperform the EST and T groups in the post-test for the reasons mentioned earlier.

The tests were composed of seven elementary mechanics problems that were similar in type and difficulty to both the problems used in the treatment sessions, as well as to problems that appear in the freshman level text by Resnick and Halliday (1977). Four of the problems involved the application of a single physical principle, while the remaining three problems each required the application of two physical principles. Two forms of the test were constructed (Forms A and B), such that corresponding problems could be solved by applying the same principle(s) using approximately the same number of steps. Half the subjects received Form A first and Form B second, while the other half took the tests in the opposite order.

The principles used to solve the seven test problems were:

Problem Number	Principle
1	Forces (Statics)
2	Conservation of Linear Momentum
3	Conservation of Energy
4	Conservation of Angular Momentum
5	Energy (applied twice) and Linear Momentum
6	Energy and Angular Momentum
7	Forces (Dynamics) and Torques (Dynamics)

The subjects were given approximately one hour to solve all seven test items. They were told to indicate as much of the problem solution as they could, even if they were not able to reach a final solution. In addition, they were asked to attempt to solve all of the problems before spending inordinate amounts of time trying to solve a problem that they had no idea how to approach.

For the pre-test and the post-test two scores were given for each problem: 1) a score of 1 was given for correctness of the principle chosen for solution, and 2) a score of 1 was given for the correctness of the final answer. The tests were graded by two physics experts. Whenever the score for a test item differed between the two graders, the solution was reevaluated and a grade determined by consensus.

In addition, performance on the 25 problems given during the treatment sessions (five problems in each of five sessions) was also analyzed. The scoring method was identical to that used for test items.

Results

The data was initially analyzed using a 3 (Treatment Group) x 14 (Subjects/Groups) x 2 (Forms) x 2 (Times) x 7 (Items) analysis of variance. The two forms of the test were combined for all the analyses to be reported here, as there was no main effect of Form and no interactions involving Form. The level of performance on the pre-test was quite variable for both the correctness of principle measure and the correctness of final answer measure. For the correctness of principle measure, scores ranged from 0% correct to 100% correct, with a mean of 37% correct. The pre-test means for the HAT, EST and T groups were 36%, 39%, and 35% correct, respectively, and did not differ significantly. For the correctness of final answer measure, scores ranged from 0% correct to 86% correct, with a mean of 26% correct. The means for the HAT, EST and T groups were respectively 19%, 32%, and 27% correct, and again, did not differ significantly.

The difference between mean percent correct principle and the mean percent correct answer was significant, $F(1,39) = 29.12$, $p<.0001$, indicating that correct computation of the final answer did not always follow a correct determination of the principle(s) needed in solving the problem. However, the probability of computing the correct answer, given that the subject had identified the correct principle, was high (70%), and no subject computed the correct final answer without first correctly identifying the principle for solution. These last two facts suggest the relative importance of an appropriate initial problem categorization in reaching a correct answer.

Identifying the principle needed for solution was not equally difficult for all items, $F(6,234) = 30.73$, $p<.0001$, and neither was correct solution, $F(6,234) = 25.45$, $p<.0001$. In general, subjects were able both to identify the principle of solution, $t(41) = 10.91$, $p<.0001$, and to solve, $t(41) = 8.99$, $p<.0001$, the one-step problems more easily than the two-step problems. The one exception to this trend is the one-step angular momentum problem, as an examination of Table 3 indicates. It is possible that the limited amount of experience students have with angular momentum influences their ability to recognize the solution principle involved. The substantial improvement in performance on the post-test after treatment on the angular momentum problems supports this argument.

Table 3: Correctness of Principle and Answer for Pre-test and Post-test

		Pre-test		Post-test	
Item	Principle	Principle	Answer	Principle	Answer
1	Forces	67%	36%	85%	50%
2	Linear Mom.	79%	74%	88%	81%
3	Energy	52%	31%	74%	62%
4	Angular Mom.	19%	9%	55%	50%
5	Energy & Linear Mom.	26%	26%	28%	26%
6	Energy & Angular Mom.	2%	2%	7%	4%
7	Forces & Torques	12%	4%	9%	7%

Performance on the post-test showed significant improvement in both identifying the correct principle needed for solution, $F(1,39) = 21.48$, $p<.0001$, and computing the correct answer, $F(1,39) = 23.32$, $p<.0001$. However, for neither measure was the actual amount of improvement overwhelming; on the average, subjects obtained one more question correct on the post-test than on the pre-test (a difference of about 14%). An examination of Table 4 shows that all three groups improved by about the same number of points.

Table 4: Pre-test and Post-test Performance by Groups

	Principle		Answer	
Group	Pre-test	Post-test	Pre-test	Post-test
HAT	36%	45%	19%	34%
EST	40%	51%	32%	46%
T	35%	53%	27%	41%
	-----	-----	-----	-----
Overall	37%	50%	26%	40%

The significant improvement in performance was due to improvements on the single-principle problems only, for both correctness of concept, F(1,39)=27.89, p<.0001, and correctness of answer, F(1,39)=28.56, p<.0001. Hence, although the experience gained in each of the treatments helped subjects achieve higher problem solving scores, no treatment improved subjects' ability to solve problems involving two principles. This is not surprising for the EST and T groups. However, the Hierarchical Analysis Tool had a specific provision for handling multiple principle problems: subjects could return to the first menu in the hierarchy and continue the analysis after invoking the second concept. Since even the HAT subjects did not improve in performance on the two-principle problems, we can conclude that the logic of the Hierarchical Analysis Tool alone is not sufficient to provide the insight that more than one concept must be considered in constructing the solution.

Clearly, our prediction that using the Hierarchical Analysis Tool would benefit problem solving performance more than using the Equation Sorting Tool or a textbook was not realized. Examining performance on the 25 problems solved during the five problem solving sessions provides some clues as to why the HAT group did not make larger gains in performance.

It was possible to compare the progress of subjects in all three groups through the treatment sessions because the subjects were required to write down a final solution to each problem when they had completed their analysis. These problem solutions were scored for correctness of the principle chosen and for correctness of the final answer. The scores for each session are presented in Table 5.

Table 5: Performance on the 5 Problem Solving Sessions

	HAT		EST		T	
	Principle	Answer	Principle	Answer	Principle	Answer
Session						
1	51%	30%	51%	36%	51%	33%
2	49%	27%	56%	44%	67%	37%
3	60%	57%	56%	67%	56%	49%
4	60%	39%	67%	53%	61%	44%
5	53%	23%	66%	46%	61%	46%
	-----	-----	-----	-----	-----	-----
Overall	55%	32%	59%	44%	59%	42%

Although the differences between the groups are not significant, the mean percent correct performance of the HAT group is lower than that of the other two groups. In addition, if performance on sessions 1 and 2 is compared with performance on sessions 4 and 5, the HAT group does not appear to have improved over time, while the EST and T groups show a modest improvement. The HAT group also had the lowest overall score on the correct principle measure, although the means of the three groups were much closer on this measure.

Of some interest is the finding that the Hierarchical Analysis Tool users compute the correct solution after identifying the correct principle only 58% of the time. This is to be contrasted with 74% for the Equation Sorting Tool users, 71% for the textbook users, and 70% for the combined groups on the pre-test. This result suggests that the Hierarchical Analysis Tool was not being used effectively and/or interfered with the process of determining the answer. If the analysis tool was being used effectively, the HAT users should have been able to compute the correct solution, after identifying the correct principle, well over 70% of the time; the program constructed the exact equations needed for solution while the user responded to the menu options.

Conclusions

The results of Experiment 2, like those of other studies (Chi, et al., 1981; Schoenfeld & Herrmann, 1982), indicate that novices have considerable difficulty identifying the principle needed to solve a problem. Novices had particular difficulty when a problem required the application of more than one principle; they often could not decipher the multi-principle problems well enough to identify even one of the principles needed to construct a solution. The results of both the pre-test and the post-test indicate that correct identification of the applicable principle is a necessary, and substantial step in developing the correct solution strategy.

Experience in solving problems is one way to improve proficiency in identifying the correct principle and computing the final answer. Subjects in all three treatment groups improved on both of these measures. It was our hope that the Hierarchical Analysis Tool would enhance the effect of this experience. However, it did not appear to do so. The apparent difficulty subjects experienced in effectively using the Hierarchical Analysis Tool may provide an explanation for why the HAT group did not improve more on the post-test. It would not have been possible for subjects to appropriately internalize the logic implicit in the analysis tool if they had never been able to use the tool effectively.

SUMMARY

The problem solving activities of novices were structured in the domain of elementary mechanics, in a manner consistent with the hierarchical approaches used by experts. This was accomplished with a computer-based system called the Hierarchical Analysis Tool. We found that novices who solved problems using the analysis tool: 1) shifted their decision making criteria for problem categorization from one based on surface features to one based on deep structure, and 2) improved their overall problem solving performance.

The effect of the Hierarchical Analysis Tool on problem categorization is particularly interesting since it was the only problem solving treatment that promoted a shift toward expert-like problem categorization. Neither solving problems in the traditional way using a mechanics textbook, nor solving problems with an unstructured computer-based system (Equation Sorting Tool) significantly changed subjects' problem categorization - as measured by the Similarity Judgment Task. In contrast the HAT group exhibited shifts toward expert-like categorization for all model problems and all types of comparisons.

The effect of the Hierarchical Analysis Tool on problem solving performance was less dramatic. While the analysis tool did improve novices' skill for identifying the solution principle and computing the correct answer, the improvement was comparable to what occurs when novices practice problem solving using a textbook as a resource. There may be several reasons why solving problems with the analysis tool did not improve performance beyond this more traditional approach. We list three:

1) Subjects could not appropriately internalize the logic implicit in the analysis tool because the tool was not used effectively, and/or the duration of treatment was too short.

2) Subjects did internalize the logic of the analysis tool to some degree but perhaps some other ingredient is needed to make it useful in solving problems. For example, recent research has identified feedback and reflection as two characteristics in succesful models of cognitive apprenticeship (Collins, Seely Brown and Newman, in press).

3) The textbook has certain pedagogical advantages over the Hierarchical Analysis Tool in that it provides the subject with organized information, motivation, explanations, examples, etc.

Perhaps one of the more surprising results of this study is that subjects who solved problems using the unstructured and nonpedagogical Equation Sorting Tool, also improved in their problem solving performance and by about the same amount as the other two groups. If we conclude from this that the increase in performance for each group was merely the result of practice, it raises a question as to why subjects in the HAT, and T groups could not take better advantage of the positive aspects of their treatment. If the improvement in performance for the EST group resulted from some quality of the sorting tool itself, it is neccessary to determine the precise nature of its success. To answer these questions requires looking at how the computer environments were used to solve problems. Currently we are investigating in detail how both novices and experts used the two computer-based tools.

In conclusion, it is our view that structuring novices' problem solving activities will be a powerful tool for conveying to novices a sense for how experts approach problems. Such structure serves to highlight the importance of certain concepts or principles, the relationships between different concepts, and the usefulness of various methods

and procedures. However, to enhance the effectiveness of structuring novices' problem solving, some pedagogical materials based on a hierarchical approach must be developed.

ACKNOWLEDGEMENTS

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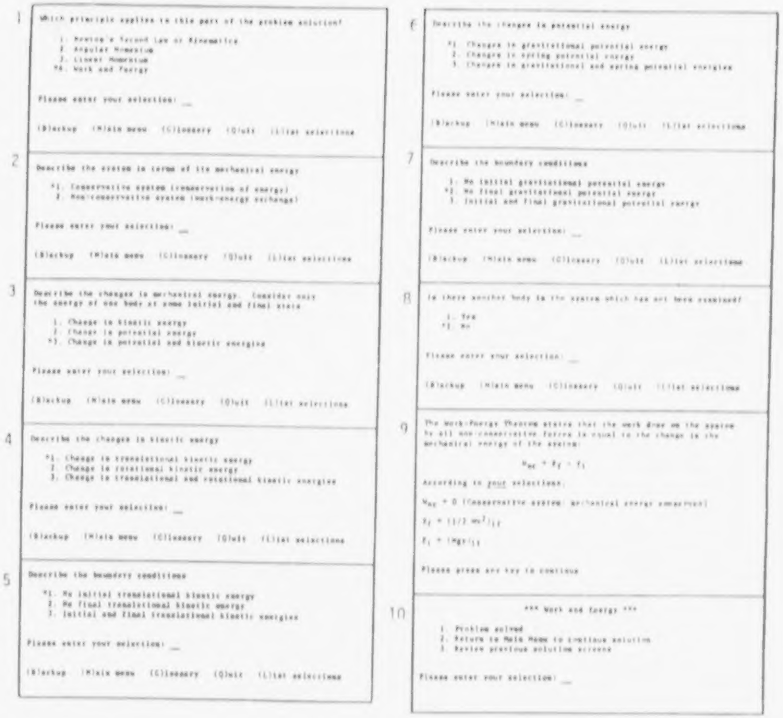
FIGURE 1



1
```
Which principle applies to this part of the problem solution?

    1. Newton's Second Law or Kinematics
    2. Angular Momentum
    3. Linear Momentum
   *4. Work and Energy

Please enter your selection: _

(B)ackup   (M)ain menu   (G)lossary   (Q)uit   (L)ist selections
```

2
```
Describe the system in terms of its mechanical energy

   *1. Conservative system (conservation of energy)
    2. Non-conservative system (work-energy exchange)

Please enter your selection: _

(B)ackup   (M)ain menu   (G)lossary   (Q)uit   (L)ist selections
```

3
```
Describe the changes in mechanical energy.  Consider only
the energy of one body at some initial and final state

    1. Change in kinetic energy
    2. Change in potential energy
   *3. Change in potential and kinetic energies

Please enter your selection: _

(B)ackup   (M)ain menu   (G)lossary   (Q)uit   (L)ist selections
```

4
```
Describe the changes in kinetic energy

   *1. Change in translational kinetic energy
    2. Change in rotational kinetic energy
    3. Change in translational and rotational kinetic energies

Please enter your selection: _

(B)ackup   (M)ain menu   (G)lossary   (Q)uit   (L)ist selections
```

5
```
Describe the boundary conditions

   *1. No initial translational kinetic energy
    2. No final translational kinetic energy
    3. Initial and final translational kinetic energies

Please enter your selection: _

(B)ackup   (M)ain menu   (G)lossary   (Q)uit   (L)ist selections
```

6
```
Describe the changes in potential energy

   *1. Changes in gravitational potential energy
    2. Changes in spring potential energy
    3. Changes in gravitational and spring potential energies

Please enter your selection: _

(B)ackup   (M)ain menu   (G)lossary   (Q)uit   (L)ist selections
```

7
```
Describe the boundary conditions

    1. No initial gravitational potential energy
   *2. No final gravitational potential energy
    3. Initial and final gravitational potential energy

Please enter your selection: _

(B)ackup   (M)ain menu   (G)lossary   (Q)uit   (L)ist selections
```

8
```
Is there another body in the system which has not been examined?

    1. Yes
   *2. No

Please enter your selection: _

(B)ackup   (M)ain menu   (G)lossary   (Q)uit   (L)ist selections
```

9
```
The Work-Energy Theorem states that the work done on the system
by all non-conservative forces is equal to the change in the
mechanical energy of the system:

                    W_nc = E_f - E_i

According to your selections:

W_nc = 0 (Conservative system: mechanical energy conserved)

E_f = (1/2 mv^2)_1f

E_i = (mgy)_1i

Please press any key to continue
```

10
```
                  *** Work and Energy ***

    1. Problem solved
    2. Return to Main Menu to continue solution
    3. Review previous solution screens

Please enter your selection: _
```

Mathematics and Chemistry Problems Created by Students

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Introduction:

Understanding in mathematics and science has traditionally been assessed through tests and exercises. These methods, however, reveal little of students' perception of mathematics or of science, nor of their use of mathematical and scientific language in other contexts. Although recent teaching in these areas has focused more on pupil interest and involvement than has been the case in the past,
research in mathematics and science education has still tended to utilise or examine student responses to solving problems chosen by the teacher or researcher, rather than incorporate student-generated ideas or problems.

This is, perhaps, surprising in view of the emphasis being given to accepting and building on children's ideas. Perhaps this reflects the extent to which we value the problems we create for our students, and how we still tend to feel 'the expert' even though we 'listen to' and 'build on' the understandings students bring to the classroom.

When students make up problems, their view of the content areas is inextricably embodied in their creation. By superimposing the idea of a *difficult* problem, students' responses reflect their perception of difficulty in a form that involves their repertoire of concepts.

In mathematics education, several recent research studies have referred to the use of students' made-up problems. Krutetskii (1976), for example, cited examples of gifted children who enjoyed making up their own mathematics problems and solving them. Ellerton (1980), in a pilot study investigating how children develop abstract ideas, asked children to make up a problem that would be difficult for a friend to solve.

A research study conducted by Hart (1981) used children's made-up mathematics problems to investigate how children draw on concrete situations to describe symbolic expressions, and Bell *et al* (1984) used mathematical problems created by students to investigate how numerical and operational misconceptions can be identified. In both of these studies, a symbolic expression was given and students were asked to make up a story that fitted. Ellerton (1986a) described contrasting features of mathematics problems made up by talented and by less able students.

The tasks:

As part of a large scale study of the development of abstract reasoning (Ellerton, 1985), students were asked to make up a mathematics problem that would be quite difficult for a friend to solve. Suggesting a problem *for a friend* was intended to help the students project their thinking beyond themselves, while keeping the context in an environment familiar to themselves and their peers. In addition, the inclusion of *for a friend* was designed to stretch the students to the limit of their concept development; students were also asked to provide the answer, thus giving the researcher an inbuilt check on the extent of each student's level of concept attainment. The sample incorporated a total of eight secondary schools in South Australia, and ten secondary schools in New Zealand. Year 4 children were simply asked to make up a mathematics question or problem that would be hard to answer, and were reminded 'don't forget to work out the answer'. Years 4, 6 and 7 pupils were drawn from primary and secondary schools in the Geelong district of Victoria. As there was no overlap between the samples used for the made-up problems in mathematics and in chemistry, mathematics problems have been selected from students attending schools set in socio-economic areas similar to that of the school which the science students attended.

A sample of 66 science students from Years 8 to 11 (ages 13 to 16 years) from a state secondary school set in a middle class suburb of Geelong, Victoria was asked to respond to the following:

Write a chemistry question that you think would be difficult to answer.
If this question was in a test, how would you respond?

In this case, students were asked only for a *difficult* problem, without the additional restriction of *for a friend*. This approach was taken because many of the students concerned were only starting to formulate their ideas of what *chemistry* incorporates, whereas students of this age have been accustomed to *mathematics* for many years.

To assess their interest in chemistry, a semantic differential list incorporating sixteen adjectives (see Appendix) was given to each of the students. By assigning numerical values from -3 to +3 (including 0 for a neutral reply), a total attitude rating could be given to each student ($-24 \leq$ possible rating ≤ 24).

Results:

Part I: Mathematics problems

Problems have been chosen for inclusion here to illustrate the type and style of those written by the students in this sample.

Year 4:

$$\begin{array}{rcccccccccc} & ^{1} & ^{1} & ^{2} & ^{3} & ^{2} & ^{2} & ^{2} & ^{2} & \\ & 7 & 6 & 1 & 2 & 5 & 8 & 9 & 4 & 8 \\ & 2 & 1 & 3 & 6 & 9 & 5 & 6 & 3 & 4 \\ & 4 & 3 & 1 & 8 & 9 & 4 & 1 & 8 & 7 \\ & 8 & 0 & 4 & 3 & 1 & 1 & 6 & 5 & 4 \\ + & 1 & 1 & 2 & 3 & 4 & 5 & 3 & 2 & 1 \\ \hline 2 & 2 & 2 & 3 & 5 & 0 & 5 & 7 & 4 & 7 \end{array}$$

(Girl, aged 9 years 4 months)

$3 \times 7 - 5 \times 1 - 1 + 3 = 16$

$7 \times 3 - 7 + 23 - 15 = 22$

(Boy, aged 9 years 2 months)

If I went to a toy shop with $100.00 and bought one cabbage patch Kid which costs $40.00 and a rubber which costs $1.00 and then I go over the road to the milkbar and get a milkshake which costs $2.00 how much would I have left? $57.00 dollars left

(Girl, aged 9 years 7 months)

Year 6:

$$\frac{7}{8} \times 1\frac{5}{6} \times \frac{7}{9} \div \frac{2}{3} =$$

$$\frac{7}{8} \times \frac{11}{6} \times \frac{7}{\cancel{9}_{3}} \times \frac{\cancel{8}^{1}}{2} = \frac{539}{288} = 1$$

(Girl, aged 11 years 8 months)

$$\frac{\cancel{27}^{3}}{{}_{24}\cancel{28}} \times \frac{\cancel{8}^{1}}{\cancel{9}_{1}} - 3\frac{3\cdot 9}{\cancel{9}\,\cancel{10}_{210}} \times \frac{\cancel{2}^{1}}{6} \times \frac{\cancel{5}^{1}}{6} = \frac{9}{144}$$

(Boy, aged 12 years 4 months)

Year 7:

$3\underline{)401.3200}$
133.7733

(Girl, aged 12 years 11 months)

$8 + 4 \div 6(1 - \overset{0}{1}) \times 12 = \cancel{24}^{12}$

(Boy, aged 12 years 5 months)

Year 8: If Peter was to run a 20 mile race but he only ran 30% of it how far did he run?

6 miles is the answer

$\frac{30}{100} = \frac{3}{10} = \frac{6}{20}$

(Girl, aged 12 years 5 months)

(Boy, aged 12 years 3 months)

$$\begin{array}{r} 46799741 \\ \times\ 22 \\ \hline 93499482 \\ 934994820 \\ \hline 1028494302 \end{array}$$

Year 9:

The cardinal no of the N set
∈ N = multiplys of 373 up to 1000
I would Find out how many 373 in 1000 and that would = C no of N set

(Girl, aged 12 years 11 months)

$$\underline{4^3+7^2+14^3+3^2=?}$$
$$64+49+2744+9=2866$$

$$\begin{array}{r} 2744 \\ 64 \\ 49 \\ 9 \\ \hline 2866 \end{array} \qquad \begin{array}{r} 14 \\ 14 \\ 56 \\ 140 \\ 196 \\ 14 \\ 784 \\ 1960 \\ 2744 \end{array}$$

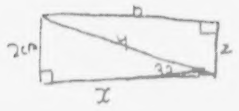
(Boy, aged 13 years 5 months)

$$\begin{array}{r} 45 \\ 8 \\ 360 \end{array}$$

$$\underline{18\div 2+100\times 2-18\times 3\div 6-55\times 8-3}=?$$
$$9+100\times 2-18\times 3\div 6-55\times 8-3$$
$$109\times 2=218-18\times 3\div 6-55\times 8-3$$
$$200\times 3=600\div 6=\cancel{2}\,100-55=45\times 8=360$$
$$360-3=\boxed{357}$$

(Boy, aged 13 years 3 months)

Year 10:



(Girl, aged 14 years 11 months)

$$\left(\frac{\frac{1}{7}\times \pi^3}{\log(\sin 42^\circ)}\right)-\left(\frac{\frac{17}{42}\times\sqrt{2}}{\frac{4}{3}\times \pi^2}\right)$$

(Boy, aged 14 years 8 months)

Ques $(3a+2b)^2$

Ans $3a\times 3a+2b\times 2b$

$=3a^2+2b^2$

(Girl, aged 14 years 6 months)

EXPAND :- $(2x-2)(x+3)=0$

Solution :- $2x^2+6x-2x-6=0$

$2x^2-4x-6=0$

(Boy, aged 15 years 1 month)

Year 11:

if $x=2+\frac{1}{\sqrt{3}}$ find $x+\frac{2}{x}$

$$2+\frac{1}{\sqrt{3}}+\frac{2}{2+\frac{1}{\sqrt{3}}}=2+\frac{1}{\sqrt{3}}+\left(2\times\frac{1}{2}+\sqrt{3}\right)=2+\frac{1}{\sqrt{3}}+\left(2+\frac{2\sqrt{3}}{2}\right)$$

(Girl, aged 15 years 9 months)

$$\frac{x^2-a^2}{x-a}\Rightarrow\frac{(x-a)(x+a)}{(x-a)}\Rightarrow x+a.$$

(Boy, aged 15 years 9 months)

Year 12:

$$\underline{3^{x+1}=9^{x-2}\qquad \text{find } x.}$$
$$\Rightarrow 3^{x+1}=3^{2(x-2)}$$
$$\Rightarrow 3^{x+1}=3^{2x-4}$$
$$\Rightarrow x+1=2x-4$$
$$\Rightarrow 5=x$$

(Girl, 16 years 8 months)

$\frac{1}{4}, -\frac{1}{16}, \frac{1}{64}\cdots$ find 5th Term

Ans - find common ratio and use formula $T_n=ar^{n-1}$

$\Rightarrow T_5=\frac{1}{4}\times\left(-\frac{1}{4}\right)^{5-1}$

$$\begin{array}{r} 216 \\ 4 \\ \hline 64 \end{array}$$

(Girl, aged 16 years 5 months)

find $\frac{d^2y}{dx^2}$ of $16x^3+5x^2+\frac{1}{2}\sqrt{x}+3$

$$\frac{dy}{dx}=48x^2+10x+\frac{1}{4}x^{-\frac{1}{2}}$$
$$\frac{d^2y}{dx^2}=96x+10+-\frac{1}{8}x^{-1\frac{1}{2}}$$

(Boy, aged 16 years 2 months)

The view of mathematics presented by the students whose problems have been reproduced above was predominantly a formal one which used very few words and relied heavily on symbolic mathematical language. Word problems were rare and made up less than ten percent of the total.

In some cases it was evident that the students had recently been working on the topic chosen by a large proportion of the class for their made-up problem - the fractions for Year 6 was one such example. During an interview with one of the students from this class, one of the boys was able to recite, fluently, the formal language associated with the fractions he had written. When asked if he would have written the same (quite complex) problem if he had been asked to make up one that would have been difficult for *himself* to solve (instead of a friend) he said no, and described a practical shopping problem that involved a very simple fraction. Inspite of his apparent fluency with the language, he had had few *practical* experiences (Ellerton, 1986b).

Part II: Science problems

Problems have been chosen for inclusion here to illustrate the type and style of those written by the students in this sample.

Year 8:

What is a compound?

I would respond by trying to answer it. If I didn't know it I would leave it out and come back to it.

(Girl, aged 13 years 10 months; attitude rating: 1)

Year 8:

What is a substance?

I find it difficult to answer.

(Boy, aged 13 years 8 months; attitude rating: -21)

Year 8:

What is a chemical reaction?

I would respond by not knowing what it is and leaving it blank.

(Girl, aged 13 years 8 months; attitude rating: -24)

Year 9:

Why the planet Earth is the only one with an atmosphere eg with oxygen, carbon dioxide, etc.

(Girl, aged 14 years; attitude rating: 8)

Year 10:

Write the solubility rules of Nitrate, Chloride, Sulfate and Carbonate.

Answer: All nitrates are soluble.

All chlorides are soluble except silver and lead.

All sulfates are soluble except lead and barium.

All carbonates are soluble except sodium, potassium and ammonium.

(Boy, aged 15 years 4 months; attitude rating: 3)

Year 10:

What is formed when iron (II) nitrate solution and sodium hydroxide solution are mixed together?

I wouldn't respond because I couldn't. This is a difficult question.

(Boy, aged 15 years 2 months; attitude rating: 7)

Year 10:

sodium carbonate + sodium nitrate ---> ?

If this question came up in a test I'd panic, then leave it and go on to the next question.

(Girl, aged 15 years 10 months; attitude rating: 15)

Year 10:

What are the steps involved in writing correct equations?

(Boy, aged 15 years 8 months; attitude rating: -3)

Year 10:

Balance equations.

$Ca\,(Cl) + Na\,(PO_4) \dashrightarrow Na\,Cl + Ca\,(PO_4)$

I would tackle it and give it a go but would probably get it wrong.

(Girl, aged 15 years 11 months; attitude rating: 2)

Year 10:

Why do we write chemical equations?

(Girl, aged 15 years 9 months; attitude rating: -12)

Year 10:

What is the test for oxygen?

Pour something into a test tube. Heat. Move glowing splint over open end. If splint flares oxygen is present.

What is the test for carbon dioxide?

Pour some chemical into a test tube. Heat. Move lit splint over open end. If splint goes out, carbon dioxide is present.

(Boy, aged 15 years 9 months; attitude rating: 6)

Year 10:

Name all the compounds and elements in symbol form and also showing how many positives and negitive.

(Girl, aged 15 years 6 months; attitude rating: -6)

Year 11:

Give the atomic numbers of the elements.

Panic, trial and error until one looks right or I remember what it is.

Look back to see if it's on a previous page.

(Girl, aged 16 years 6 months; attitude rating: -2)

Year 11:

List the emf series in correct order.

My Answer: I'd leave it out.

(Girl, aged 16 years; attitude rating: -13)

Year 11:

State the atomic number of all the elements on the periodic table.

"Would fail chemistry"

(Boy, aged 16 years 3 months; attitude rating: 10)

Year 11:

Explain how metal reflects light.

If I didn't know the answer, I would not do it.

(Girl, aged 16 years 10 months; attitude rating: 10)

Year 11:

Why do electricians say that the electrons move from + to - when we are taught from - to +

(Boy, aged 16 years; attitude rating: 5)

Table 1: Numbers of students making up problems in particular content areas.

Year	equations	periodic table; naming compounds	emf, pKa	chemical reaction	explanation; rules	no problem made up	mean attitude rating
10	5	3	-	8	1	4	-4.0
11	3	5	2	-	4	6	1.1

Table 1 summarizes the content areas covered by the chemistry questions made up by Years 10 and 11 students in the sample. The mean attitude rating for students was found to increase in going from Year 10 to Year 11, although the value of 1.1 for the rating suggests only a very slightly positive attitude towards chemistry. The range of attitude rating values was from -24 to +22, with only four students showing an attitude rating of +16 or more.

Years 8 and 9 students have not been included in Table 1 because all of those who made up problems concerned themselves with definitions such as those given as examples. Mean attitude ratings for Years 8 and 9 were -14.6 and -2.4 respectively, with the highest ratings being 7 and 8. Seven of the nine Year 8 students with attitude ratings of less than -20 did not make up a chemistry problem at all.

Discussion:

The students' responses suggest that, not only did many students in this sample (particularly at the more junior level) find chemistry confusing and uninteresting, but they perceived it as composed largely of facts, figures, definitions and symbols. Very few students alluded to the investigative side of chemistry.

The problems presented as *difficult* seemed to achieve their level of difficulty by placing a larger load than normal on students' memory span - difficulty was seen to parallel the need to memorize more facts. There was a clear response that if a problem was too difficult, it would simply not be attempted. Resignation to an inability to answer difficult problems was also evident.

The perception of *difficulty* of particular concepts in chemistry has been explored by Johnstone, Morrison and Sharp (1971) and more recently by Butts and Smith (1987). In the latter study, students rated fifty concepts on a four point scale from easy to extremely difficult. The survey showed that many students report experiencing difficulty with some of the fundamental concepts related to atomic, ionic and molecular structure. Some of the results suggested that the interpretation of macroscopic observations in terms of atomic and molecular properties may be more difficult for many students than teachers anticipate. This is consistent with the made-up problems included above, where students seemed to have difficulty in writing down equations to represent simple practical experiments.

Conclusions:

The problems made up by the students reflected the subject areas they have come to know as mathematics and chemistry. Thus both were perceived as formal, factual and literal areas with little room for imagination and investigation. An element of mystique seemed to dominate some responses *('pour something into a test tube')*. Solutions to the problems were perceived as either unattainable or difficult to attain.

Thus students in the samples perceived mathematics and chemistry in a very restricted light; presumably the set of experiences that they had come to associate with the terms *mathematics* and *chemistry* at school reflected this perspective.

If, at this point, we decided to analyse some of the difficulties that many of these students would have been experiencing in the classroom,we may be able to identify some misconceptions concerning particular concepts. To do so, however, would be to neglect the basic observation that these students carry with them misconceptions about the very nature of mathematics or of chemistry. Thus any attempt to identify misconceptions about specific concepts needs to take account of each student's perception of the subject area. A student, for example, who views chemistry only as a set of unrelated facts and figures may have great difficulty in balancing a simple but unfamiliar equation. The Year 11 student who made up the following problem, for example, could be described as having an inadequate understanding of the concept of *equations* - but, viewed from the perspective outlined here, this student appears to see *chemistry* as a jumble of unrelated symbols and facts. The latter misconception, in fact, can account for the former.

$$PbO + H_2 + Cl_2 + Mn^{++} + NO_3^- + Mg + Cu^{++} + Zn^{++}_{(aq)} \longrightarrow$$

$$CO + Cr_2O_2^{2-} + 14H^+ \; 6Fe^{++}_{(aq)} \cdot CrO_7^{=} + SO_3^{=} + ClO_4^-$$

BALANCE THIS EQUATION AND FIND THE VOLTAGE (EMF)

Thus it is the set of experiences that define *mathematics* and *chemistry* for students that needs to be addressed in the first instance. Until this is done, the source of misconceptions of particular concept areas will often remain ambiguous. Perhaps the most pertinent comment of all was made by the Year 10 student (attitude rating -15) who wrote:

I would not be able to wright a question because
I cannot understand chemestry at all

Surely this represents the greatest misconception of all.

Appendix:

Semantic differential items used with science students in this study.

The following section shows pairs of adjectives. Look at each pair and think about how the words describe chemistry. For each pair of adjectives put a tick in the box which shows how you think chemistry is best described.

Boring	☐	☐	☐	☐	☐	☐	☐ Fun
Easy	☐	☐	☐	☐	☐	☐	☐ Hard
Complicated	☐	☐	☐	☐	☐	☐	☐ Simple
Uninteresting	☐	☐	☐	☐	☐	☐	☐ Interesting
Important	☐	☐	☐	☐	☐	☐	☐ Unimportant
Confusing	☐	☐	☐	☐	☐	☐	☐ Understandable
Disorganized	☐	☐	☐	☐	☐	☐	☐ Organised
Useful	☐	☐	☐	☐	☐	☐	☐ Useless

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PRAGMATICAL CONCEPTIONS IN THE ATOMIC DOMAIN

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INTRODUCTION

It is well known that, in the area of classical physics, students develop naive conceptions which may be in conflict with the accepted views of science. For example, noticing that in ordinary life moving objects will stop unless a force is applied, they will believe in a $F \propto v$ law instead of the Newtonian one, $F \propto a$. Even at the university level many still think that a ball thrown upwards does not feel a force at the top of its trajectory, since $v = 0$ at that point (Viennot 1979).

Sometimes the spontaneous reasoning of students is more sophisticated and based less directly on observations. With a circuit consisting of a battery and 2 resistors (or 2 bulbs), the "milkman delivery" model states that the current will be higher in the resistor nearest to the battery and smaller in the other one: after passing through the first resistor the current is used up and delivers less. The current behaves sequentially (Shipstone 1983). Students have difficulties in considering the circuit as a system (Closset 1983).

Alternative frameworks exist also in other fields, especially optics and thermodynamics. They are very robust and resistant to change (Champagne 1983). The common factor of these naive beliefs in classical physics is that they are more or less extracted from experiences in daily life or from analogies with it. This factor should be absent in the area of modern physics where there is no equivalent in daily life. It has been argued by Bachelard (1983) and others that, then, the obstacles are purely epistemological.

The difficulty encountered in conceptualizing wave-corpusule duality is quite typical. In the macroscopic world we deal with particles which are localized and with waves which extend in space. When the electron is introduced, it is classified as a particle (even as a point particle, most of the times). The idea that the electron might be also a wave seems absurd from the macroscopic point of view. The dilemma will be solved when we accept that the macroscopic entities, wave and corpuscule, cannot be transposed to the microscopic domain. We refer the interested reader to Bachelard for an interesting discussion on epistemological obstacles.

Though we accept the Bachelardian thesis - epistemological obstacles are of primary importance, especially in modern physics - we describe here the frequent attitude of pragmatical conceptualization which we have observed among university students. After defining that attitude and describing some of its features, we will show that it originates from a strictly empirical view of science held by those students. Our picture will be illustrated by misconceptions picked up in a quantum mechanics course after a few years; the students had previously followed a modern physics course. Only misconceptions held by a sizable fraction (at least 1/3) of each class will be used and most of them will be taken from the atomic domain.

PRAGMATICAL CONCEPTIONS (PC)

The students developing pragmatical conceptions (PC) usually do not question accepted theories; they accept them as *facts* very easily after a short period of incubation, where doubt is allowed. But theories are reduced to their verifiable parts. Rutherford's nuclear atom model *is* Rutherford's scattering formula. No or small attention is given to the assumptions (point particles, no nuclear recoil, electrostatic force) on which it is based. An accepted theory is a successful theory: its conclusions have been verified, the calculations (or the thinking) leading to them has been checked by some people; whatever it says must be true and must be accepted. It has been said jokingly that psychoanalysis succeed mainly with young, aggressive, verbal, intelligent and successful patients: they pick up those patients which are more prone to be cured. This Yavis effect is found among PCers: they pick up successful theories and find that their conclusions are indeed verified.

The examples of misconceptions in classical physics given in the introduction describe the learner as a constructivist; to make intelligible some phenomenon, he builds his own explanation which may be superseded by a more convenient one. The PCer is essentially *non constructivist*; the construction, one might say, has been done by someone else and nothing needs to be added: truth has been reached in one step. However, the PCer is also mildly a *reconstructivist* in the following way: some parts of a theory will be adapted by him; he will reconstruct some concepts but will not touch to the conclusions. Truth is preserved (the verifiable is not affected), but some concepts, which are too intellectual, "too conceptual", are sweetened to his taste.

CHARACTERISTICS OF PC

We have defined the PCer as a non constructivist (theories are definitive) and a realist (theories are not models of reality but descriptions of reality). We will now illuminate these two points with some characteristic features. From what has been said in the preceding paragraph, one can deduce that the PCer short-circuits the conceptualization period; he does not bother about concepts, definitions and assumptions, which are the ingredients for cooking a model. Asked if nuclei are solids, liquids or gases, *(question #1)*, some students found that they indeed are solids (since they are compact) or liquids (because of the liquid-drop model). They could not see that nuclei (and electrons) are constituents; being a solid (or a liquid) involves a relation between the constituents, like the individuals which are united in a family. Concepts and definitions are involved frequently by the PCer without apparent understanding, in a kind of jargon. Considering wavelength as a proper length shows a poor comprehension of what is a wavelength and of what is a proper length; this misconception appeared in a problem where one had to derive the Compton effect from the Doppler effect *(question #2)*.

Part of the trouble of the PCer has in modern physics comes from his poor comprehension of classical physics. He may remember a lot of facts (he is an amateur of facts) but he has difficulties in having an organized view of them. He has, for example, studied the laws of electromagnetism, but fails to see the synthesis introduced by Maxwell. During one of my lectures, some students tried to explain superconductivity *(question #3)* in the following way: instead of being distributed in the whole volume of a metal, electrons lie only at the (exterior) surface; therefore they can move without bouncing on atoms and on each other. This misconception, which has some constructivist touch, originates from an earlier one: the net charge of a conductor at electrostatic equilibrium lies at the (exterior) surface and not at the interior surface (within a few atomic radii). As Ausubel (1978) said *"the most important factor influencing learning is what the learner already knows"*.

Even those PCers who bring a good knowledge of classical physics have difficulty in reconciliating modern physics and classical physics. They hate Bohr's correspondence principle and semi-classical arguments. Since a successful theory (modern physics) describes reality, a second theory (classical physics) cannot be right too. There is place for just one truth. Instead of making the complete and long quantum-mechanical calculation of the electron magnetic moment, one often prefers the semi-classical one with Bohr orbits. The PCers seem to think that this approximation is wrong.

Prone to consider theories as factual and definitive, the PCer does not like to face news facts. *Question #4*: what is the result of the Stern-Gerlach experiment with ions? In the original experiment with neutral silver atoms, the net spin interacted with the inhomogeneous magnetic field. If, moreover, the atom is charged, there will be also a variable magnetic force $\vec{F} = q\,\vec{v} \times \vec{B}$. More than half of the students tried to find reasons why ions would be inoperative in a Stern-Gerlach apparatus, the most far-fetched being that ions have no spin. They think that the Stern-Gerlach experiment works only with electrons. The PCer does not react well if different calculations are required of him. Most of the problems in quantum mechanics end with symbolic answers and involve therefore no units. *Question #5*: for hydrogen in the ground state, calculate the maximum value of the wave function, the probability density and the radial probability density; give numerical values. Less than half of the students could give the right units, some even saying that ψ has no physical meaning and therefore is dimensionless. The PCer does not like to make generalizations. Often special relativity is introduced at the end of the mechanics course. Therefore students tend to believe that relativity is needed only in particle accelerators. *Question #6*: does special relativity apply to electricity and magnetism? The following answer is much too frequent: if charges do not move fast, no. And the fact that Maxwell's equations are Lorentz invariant is swept under the rug, as well as the reference to Faraday's law in the 1905 paper of Einstein.

EMPIRICAL VIEW OF SCIENCE

We have described the main features of the students who hold pragmatical conceptions while learning modern physics: poor conceptualization of phenomena, weak comprehension of basic classical physics, inability in matching classical and modern physics, inaptitude to face new facts and to make generalizations. One possible explanation for that attitude is that these students hold a purely empirical view of science.

If one believes that discoveries are made only experimentally, one will attach less importance to assumptions and concepts in theories. The important things will be the facts and the verifiable conclusions, which are considered more or less equivalent. This is an awful mistake, because when we make an observation, we must have at least some a priori idea of what to observe. Experiment does not tell us which concepts must be used to explain a phenomenon. Experiment may indicate which one of two or more concepts is more appropriate, but we must create them ourselves. The whole area of conceptualization is reduced to a minimum in a quite restricted view of science.

Empiricists believe that the ultimate truth may be achieved. Modern physics coming after classical physics, the latter must be wrong. That is why the PCer usually neglects classical physics. Since he needs a minimum of concepts in modern physics, the concepts of classical physics are seen as ridicule and out-of-date. The problem of matching both theories is not a real one and Bohr's correspondence principle is purely academic.

If science is limited to making experimental discoveries, a new theory must originate from a new fact. That is why the PCer does not accept paradoxically new facts unless he has the theory proving them at hand. That is also why he does not like to make generalizations, to see the link between apparently divergent phenomena.

The role of logic in science is reduced to a minimum by the PCer. That is why he frequently shows incorrect logic, like contradictory explanations. This is not important for him, since experiment is superior to logic. He does not accept the *gedanken* experiments of Einstein, Bohr and Heisenberg. He likes real things not imaginary ones.

My *question #7* is a perfect example of what has been described up to now: do a copper wire and an atom evaporated from it have the same properties? Discuss electrical conductivity and malleability. This question is related to the first one. More than 30% of the students answered that both the wire and the atom had the same electrical conductivity. This result is stupendous for students who can make quite lengthy calculations involving Fourier transforms or confluent hypergeometric functions. It is however better than the one of Ben-Zvi (1986) on 10th-grade students: only 15% thought that the wire and the extracted atom had different properties.

CONCLUSION

It is quite distressing that in the study of modern physics, an area where concepts are so important, so many students regress to (or stay at) a level of low conceptualization, where only the verification of formulas is important. It seems that an important part of the course should be devoted to explaining what science really is - that is from a constructivist point of view (Novak 1984).

Modern physics being based on classical physics, the introductory courses of mechanics and electricity and magnetism should carefully develop the mental structure necessary for later integration. Unfortunately this is not the case. Teachers wishfully think that students will catch up in more advanced courses the concepts they missed in introductory courses. However, as we have shown (Faucher 1983), advanced physics courses do not provide students with the expected conceptual change.

We now have to pay the price for our haste to forward our students to more interesting, high-tech subjects. They run hastily from basics to sophisticated fields (Newton's laws vs high-temperature superconductivity). When we detect deficiencies in advanced courses, we invent reasons like a poor background in mathematics. We should rather say: that student did not spend enough time in basics.

Posner (1982) has described the conditions that must be met for a conceptual change: people must be dissatisfied with the old concept and find the new one more acceptable; the rising notion must be intelligible, plausible and fruitful. The problem with PCers is to induce them to a state of dissatisfaction towards their aconceptual attitude, so that they try to build a new one.

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THE PSYCHOLOGICAL STRUCTURE OF NAIVE IMPETUS CONCEPTIONS

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What does preserve the motion of a launched object after losing contact with the mover? The belief of laymen is, usually, that motion continues as the effect of an internal force - an impetus - imparted on it by the mover (Viennot, 1979; Clement, 1982; McCloskey, 1983; McCloskey, Washburn and Felch, 1983; Green, McCloskey and Caramazza, 1985).

The present research has been devised having in mind the following aims:

1. We wanted to check the hypothesis that naive people indeed interpret the free motion of an object (if no contrary forces intervene) as the effect of an imparted force - and not as the effect of inertia.

2. We intended to analyze the psychological nature of the naive impetus theory.

3. We considered that the impetus interpretation has its roots in the practical, terrestrial life experience. We then assumed that the predictions of the naive subjects referring to the continuation of motion of an object - after losing contact with the mover - will depend on some properties of the object, such as shape or weight, even in the absence of friction.

   For instance, we have assumed that the number of naive subjects who predict the infinity of motion of a launched ball (in the absence of friction) will be greater than the number of those who predict the same about a box; or that the number of subjects who predict that a heavy chest will fall straight down, will be greater than the number of subjects who predict the same about a box.

4. A fourth question addressed by us referred to the effect of instruction. Does the teaching of mechanics modify structurally the views of students with respect to the free motion of a launched object? The investigations of Viennot (1981); Caramazza et al (1980); Clement (1982) etc., have indicated that many students, even after taking courses in meachnics, continue to present the same types of misconceptions referring to the motion of objects.

5. A fifth problem referred to was the effect of the way in which an object is set in motion by an external mover. It has been supposed (McCloskey, Washburn and Felch, 1983) that, according to naive subjects, carried objects do not get any impetus from the mover, while launched objects or objects "moving on their own" get such an impetus. The absence or presence of the supposed impetus should influence, in the novice's opinion, the continuation of

The Method

The subjects were 45 students enrolled in 4 classes grade 10 and 44 students enrolled in 4 classes grade 11. The schools were situated in an urban area - Tel Aviv. The 10th graders did not possess any systematic knowledge in mechanics, while the 11th grade students had already taken a course in mechanics.

We used questionnaires and interviews. The aim of the questionnaires was mainly to determine the influence of the shape and weight of objects on the subjects' predictions concerning the motion of these objects in the absence of fritcion, after losing contact with the mover. The interviews were especially aimed to elucidate the psyuchological structure of the impetus conceptions, but also to deepen the understanding of the other aspects.

The basic concern of our research was, in fact, the nature of the intuitive, naive interpretation of the motion of a body after losing contact with the mover.

The Questionnaires

Two questionnaires were administered which took into account the following aspects:

a) The conditions in which the motion takes place. Air: objects carried by an airplane or the motion of an object launched over a precipice; Ground: a car carrying objects which are subsequently released; An expanding spring launching an object on a smooth surface.

b) Objects with various shapes, weights and functions were considered: a box, a ball, a glider, a chest containing equipment, etc.

c) Two types of motion were considered: (1) Passive motion: the object is carried by a vehicle (car, airplane) and subsequently released; (2) Active motion: the object is pushed suddenly by an expanding spring; an object is moving down a slope, gets acceleration and continues its motion over a precipice. Objectively, both types of motion are, in the absence of friction,the effect of inertia, or inertia and gravitation. But it has been assumed that subjects who think in terms of impetus will predict differently the continuation of free motions in the two circumstances (for instance that a carried object will be considered more frequently to fall straight down than a launched object).

The interviews were based on the same type of questions as above (with some additional items) but trying to obtain more information about how students explain the continuation of motion after losing contact with the mover.

Procedure:

The questionnaires were administrated collectively in the usual conditions of a classroom activity. The questions of each questionnaire were typed in two different orders and consequently four formats were produced. The aim of this technique was to neutralize the effect of order. The four obtained formats were administrated simultaneously to all subjects present in the classroom at the time of testing, one format per individual.

The Results:

The shape and weight of the moving objects.

Inspecting Tables 1 to 4 it becomes evident that there are differences between the motions of objects according to their shape, despite the fact that the questionnaires were insisting on the absence of friction. But these differences appear only in grade 10 students, i.e. in students who have never before taken a systematic course in mechanics. Let us be more specific:

a. While a glider is supposed to move forward (as answered by 70.5% of the subjects) after being released, only about 40% or less supposed the same about a box, a ball or a wheel. The smallest number of "forward" responses was given to the "chest" question (39.5%). Sixty percent of the subjects predicted that the chest will fall straight down.

b. In the car and spring problems, it is the ball which is supposed most frequently to move forever after losing contact with the mover (i.e. the car or the spring). The explanation given by the subjects is that the ball is rolling (on a supportive surface: a table, the ground). Being round, the ball rolls, gets impetus, and thus the impetus continues forever.

TABLE 1

The airplane* (passive motions)

		A1	A2	A3	A4	A5
Type of answer	Grade	The plane pulls a box	A chest is dropped from the plane	The plane pulls a ball	The plane looses a wheel	The plane pulls glider
Forward	10	45.4	29.5	36.4	40.9	70.5
	11	93.1	90.0	93.2	91.0	84.1
Straight down	10	43.2	59.1	50.0	43.2	22.7
	11	2.3	4.5	--	--	--
Backward	10	11.4	9.1	13.6	15.9	2.3
	11	2.3	2.3	2.3	4.5	2.3
No answer	10	--	2.3	--	--	4.5
	11	2.3	2.3	4.5	4.5	13.6

* The numbers represent percentages

TABLE 2

The car (passive motions)

		B1	B2	B3
Type of answer	Grade	The car pulls a chest	The car pulls a ball	The car pulls a wheel
Infinite	10	2.2	31.0	8.9
	11	55.8	58.2	67.4
Short distance	10	64.4	53.4	77.8
	11	39.5	37.2	25.6
Forward: infinite & short distance	10	66.6	84.5	86.7
	11	95.3	95.4	93.0
Stops immediately	10	29.0	4.4	4.4
	11	---	---	4.7
Backward	10	4.4	8.9	8.9
	11	4.7	2.3	---
No answer	10	---	2.3	---
	11	---	2.3	2.3

TABLE 3

The spring (active motion)

Type of answer	Grade	The spring pushes a box	The spring pushes a ball
Infinite	10	22.5	28.9
	11	60.5	55.8
It moves a short distance	10	71.1	64.4
	11	18.6	23.3
Forward: infinite & short distance	10	93.6	93.3
	11	79.1	78.8
Harmonic motion	10	4.5	6.7
	11	16.3	18.6
No answer	10	2.2	---
	11	4.6	2.3

TABLE 4

The precipice (active motion)

Type of answer	Grade	A6 The box	A7 The ball
Forward	10	68.2	81.8
	11	91.0	97.7
Straight down	10	27.3	13.6
	11	4.5	---
No answer	10	4.5	4.6
	11	4.5	2.3

c. In the precipice problem, it is again the ball which is most frequently supposed to move forward (but not forever; because of gravitation, the ball will finally fall in the precipice). Let us remember that in each problem it was emphasized that the object is supposed to move freely in ideal conditions in which there is no friction.

These findings have then proven that the shape or the weight of an object influence the subjects' predictions about the continuation of motion in the absence of external forces.

Carried versus launched (or "moving on their own") objects

In order to compare the reactions of the subjects to questions referring to passive motions (carried objects) versus questions referring to active motions (launched or pushed objects), two pairs of items have to be considered.

1. The plane pulling or carrying an object (passive motion), and the shape-precipice problem in which the objects get an acceleration (active motion). In both circumstances the objects (a box and a ball) are moving in the air after being released.

By comparing Table 1 with Table 4 it is evident that much more 10th grade students give the "forward" answer to the slope-precipice problems (about 70-80%) than to the airplane problems (30-40%).

Some of the 11th graders refer to an imaginary harmonic motion and thus the respective frequencies become inconclusive.

2. A second comparison may be made between the findings referring to the car problems (passive motion) with those referring to the spring problems.

   Here too, the frequencies of the "forward" type of answers are higher in the case of the active motion than in the case of the passive motion. The difference is striking for the box questions (93.3% vs. 66.6% of "forward" answers), and less important for the ball problem (93.3% vs. 84.4% of "forward" answers by grade 10 subjects).

The above findings corroborate those of McCloskey et al (1983, pp.638-639). They seem to indicate that in the case of an active motion, the subject is more inclined to believe in the effect of an impetus (which would keep the object in a forward motion) then in the case of a passive motion. In the second case, the object, not moving "on its own", does not seem to possess an impetus.

The effect of instruction

In our previous analysis we have referred almost exclusively to 10th grade students. The reason is that almost all the subjects belonging to grade 11 gave "forward" answers to all our questions. These subjects have learned mechanics shorlty before the questionnaires were administrated. It would

seem that instruction has radically changed the views of the students in the respective matter. Our assumption - based on the subjects' explanations - is that many of the 11th grade subjects maintain their impetus intuitive interpretations. Instruction only enhanced the "forward" reactions without, in fact, improving the corresponding theoretical explanation. A very few eleventh-grade students referred to inertia. Most of these eleventh grade students referred to energy for explaining the continuation of motion. A few ones also mentioned speed and impetus (see also Whitaker, 1983; and McCloskey, 1983).

"These students can give a correct statement of the law of inertia, but they have no real understanding of its meaning. The idea is subtle; it takes time to assimilate from a sufficiently wide context of laboratory experience and thought experiments, but students are rarely afforded this luxury"...(A. Arons, cf. Whitaker, 1983: p.356).

The Interviews

In order to get a better understanding of the psychological structure of the impetus conception, a number of inteviews have been carried out. The following types of interpretations have been identified:

1. The Marchian type (Franciscus de Marchia, ca. 1320, cf. Franklin, 1978, p.203):

   The subject considers that the mover impresses an impetus on the object which continues its motion until the imetus dissipates and the object stops. Here is a first example:

S: After being pushed by the spring, the box continues its motion and will stop gradually.

R: Why does the box continue to move?

S: Because the spring has pushed it and has give it an impetus.

R: What is impetus?

S: Impetus...the object may continue to move despite the fact that there is no force to push it. I myself do not understand what it is.

R: And if there is no friction at all?

S: The distance will be longer, but the box will finally stop.

(Aviad - grade 10)

A second example:

S: "The box will continue its motion until it will not be any more that (cause) what gives it the force to move. It has received force from the spring...At the beginning it had a certain force which gave it the impetus. It has not its own force. It has got it from the spring corresponding to the capacity of the spring." (Alon - grade 10)

Another quotation taken from the same protocol:

S: The ball will continue the motion until its force is finished.

R: Why?

S: Because it will not have any more force to continue and will stop.

R: What is pushing it forward?

S: There is an internal force pushing it.

(Alon - grade 10)

In the above interpretation the subject considers that the body continues to move because of an impressed force. It stops when the impressed force dissipates.

2. The Buridan type of interpretation (Buridan, 1300-1358, Cf. Franklin, 1978, p.204).

S: "In principle, the motion of the ball will stop because of friction. If there is no friction, the motion will continue because it is rolling all the time...till infinity...because it is rolling, it is gaining speed"... (Aviad - grade 10)

The subject is not aware about the fact that in the absence of friction, the ball does not roll. It rolls forever. Aviad does not mention explicitly an internal force which maintains the motion of the ball. But it is evident that the subject considers a certain cause which maintains the motion, expressed in the rolling of the ball. This cause is of permanent nature as in the conception of Buridan. Let us remark that the same subject, Aviad, referring previously to a box, has considered that the object will stop to move (also in the absence of friction!) because the impetus dissipates with time (a Marchian type of impetus). Such contradictory attitudes may be explained by the fact that the subject mixes in his reactions, empirical with ideal - logically based - representations. The object will stop moving after a certain interval, as it happens in real, terrestrial conditions; the object will continue its motion to infinity as it should happen in ideal conditions (absence of friction).

3. A third type of interpretation identifiable in the protocols represents a transitory level towards the motion of inertia. The subject admits that the object continues its motion indefinitely after losing contact with the mover, despite the fact that there is no force (impressed on the object) to maintain the motion.

> R: What happens to the box after being pushed by the spring?
>
> S: It depends on the weight.
>
> R: We suppose there is no friction.
>
> S: If there is no friction, it will continue to move until a force will stop it. There is no reason that the motion should stop. Since there is no friction,there is no other force. (Ron - grade 10)

This student has not learned about inertia, he does not know the first law of Newton and, nevertheless, intuitively, he knows that an object continues its motion if no force interferes to stop it. In contrast to Buridan's conception, Ron does not mention any internal agent which continues to support the motion. What distinguishes the student's conception from that of Newton is the absence of a theoretical framework which would enable him to identify the state of rest with that of uniform rectilinear motion.

4. The Newtonian conception

We have found an explicit Newtonian conception of motion and rest in some eleventh-grade students, i.e. students who have already taken a systematic course in mechanics.

> S: After the connection of the box with the plane is cut off, the box continues its motion on a parabolic trajectory. It has its constant horizontal speed on the X-axis and in addition it is the influence of gravitation exerted by the earth.
>
> R: Why does the box keep its constant speed on the X-axis?
>
> S: Because there is no reason not to continue its motion.
>
> R: If gravitation have not had influenced the motion?
>
> S: The object would continue its motion on a straight line with constant speed to infinity according to the first law of Newton, the law of inertia.
>
> R: Let us imagine that you launch a ball straight upwards. Which forces are acting on the ball?
>
> S: It is only the force of gravitation. When the upward speed becomes zero, the ball begins to fall.
>
> (Ordat - grade 11)

Do such answers express only a system of formal, acquired notions, or the student realizes also internally, intuitively (as an intrinsic necessity) their validity? It is difficult to give an answer to that question. But our belief is that, at least in some cases, the students have really grasped the Newtonian law as an intuitively imposed idea.

S: The box will keep its motion with constant speed after being released.

R: How long?

S: There is no limit. There is no force to stop it.

R: Why?

S: Simply, there is no force to stop the motion of the object.

R: Is there any force which pushes the box?

S: No.

(Ronen - grade 11)

The sentence: "Simply, there is no force to stop it" indicates, in our opinion, that the student realizes the role of inertia as an 'a priori' necessity and this is a fundamental charcteristic of an intuitive acceptance.

Terminology

Let us briefly come back to the notions used by naive subjects to explain the continuation of motion of an object after it loses contact with the mover. Some subjects use the term impetus, some others use the term force. But also terms such as energy, speed and acceleration are employed. The terms acceleration and energy are frequently used with about the same meaning as impetus.

Generally, it was impossible to obtain from the subjects a clear definition of their own acceptance of these terms. Therefore, when referring to the various types of naive impetus conceptions one has to take into account that, generally, the subjects do not have a clearly expressible representation of the nature of the impetus. The fact that they confuse the terminology prevents the researcher from getting a clear understanding of a novice's naive conception. The only idea which can be clearly stated is that naive subjects usually consider that the continuation of motion of a body, not subject to external forces, is determined by a certain agent, an active cause, in contrast to the Newtonian machanics. In our opinion, the subjacent, tacit model of that agent is a kind of fuel, similar to that which keeps working an engine.

Discussion

Inhelder and Piaget (1958, p.123-132) have stipulated that the notion of uniform rectilinear motion represents a form of conservation which goes beyond direct empirical verification. It develops, according to these authors, as an operational schema during the formal operational stage. In other terms, according to Inhelder and Piaget, the principle of inertia is acquired naturally during the formal operational stage together with other operational schemata.

In reality, things are much more complex. No one of our subjects has reached naturally (i.e. before taking a course in mechanics) the concept of inertia. Most of the 10th grade subjects affirm that a body set in motion by a mover stops after a certain interval (even in the absence of friction). Others - a few ones - admit that the motion may go on forever because the impetus it has got is inexhaustible (for instance, a rolling ball).

One may suppose that many naive subjects mixed in fact empirical with ideal representations of motion. The Newtonian concept of inertia is the conclusion of a mental ideal experimentation. "Thus, say Inhelder and Piaget, the subject is proceeding on the basis of pure implications and no longer on the basis of transormations which can actually be effected" (Inhelder and Piaget, 1958: pp.131-132). As a matter of fact, naive subjects are generally not able to detach completely these sequences of pure implications from empirical constraints. In order to genuinely understand the notion of inertia, the subject must attain a highly sophisticated conceptual structure which could lead to the absolute equivalence of rest and uniform rectilinear motion.

These considerations may lead to the following didactical advise:

One should endeavor to develop in students the ability to perform ideal mental experimentations. There are many such opportunities in physics. One may, theoretically, assume that the logical basis for that ability develops virtually in adolescence. But our findings have shown that the capacity for mental experimentation is, in fact, hindered by the empirical habits of the students' mental behavior.

The ability of mental experimentation referring to ideal, purely logically governed conditions, is an essential prerequisite of the scientific mind. Consequently it should not be neglected in the process of science education.

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DO WE FEEL FORCES?

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1. INTRODUCTION

We commonly start teaching physics by introducing mechanics. It is argued that we have a feeling for motion and for forces which should facilitate entering an abstract field like physics. However, if we take serious recent investigations which demonstrate that students have considerable difficulties with the concept of force[1], we have to admit that the situation cannot be so simple.

Students (1) mix up force and momentum; (2) they employ an "aristotelian" view of forces (i.e. there always has to be a force for moving an object); and (3) they cannot identify forces correctly even in simple situations[1]. This is still the case after considerable time has been spent on instructing them. I will argue that part of this problem stems from the belief of students and teachers that we have a feeling for forces which, however, is not the case. What we feel is something different, namely stress or, equivalently, the flow of momentum through an object[2-5]. Another problem has to do with the motion of bodies. We have a feeling for the thrust (or momentum) of bodies and this we call a force which contradicts theory. The learning problems should be addressed by carfully distinguishing between the flow of momentum on the one hand, and forces on the other. Force turns out to be a much more abstract concept than momentum currents. In the following sections, I will render these statements precise.

The central point is the distinction between three different types of momentum transfer. I have been speeking of momentum flow *through* bodies which is associated with stress. This type of flow is called *conductive*. The interaction of bodies and fields, however, is of a different nature. It leads to sources (and sinks) of momentum in bodies (and fields). In analogy to thermal processes, we can speak of *radiative* transfer of momentum. Due to this interaction, momentum does not flow through bodies and therefore does not lead to stress. In other words, we cannot feel the force of gravity. Finally, there are *convective* momentum currents associated with the transport of matter.

Forces, it turns out, are extremely abstract objects. In short, they are the momentum fluxes of two of the three kinds of momentum transfer. Forces are associated with conductive and radiative momentum flow only, while stress has to do with the conductive type only. No wonder that identifying forces is a nontrivial task. Especially the fact that we exclude convective momentum currents from what we call forces is confusing. It is responsible for the trouble we have with variable mass systems[3,6]. Incidentally, the convective momentum current might well be the kind of "force of motion" which students introduce in the direction of a moving object. Naturally, approaching mechanics from this angle does not make it an "easy" subject. In fact, the road to an understanding of the concept of force is long and steep. It pays, however, to point out clearly that we do not feel forces, and that we have to reach quite a level of abstraction before we arrive at a precise understanding of their role.

In this alternative presentation of mechanics and of forces[2-5], the role of misconceptions is a different one. This does not mean that there could not be misunderstandings. However, it seems that the alternative conceptions held by students can be used to advantage in this new approach to mechanics. We can employ our feeling for introducing the fundamental mechanical quantity: stress.

II. STRESS AND CONDUCTIVE MOMENTUM CURRENTS

We can learn a lot by observing students at work when they are given the task of identifying forces. Take the example of a crate being moved across the floor at constant speed. We intuitively know that if we were in the crate´s place we would be feeling "something". Something, which cannot be zero, is needed for moving the crate. Usually we call this a "force". Therefore it is quite natural to assume that a (net) force is necessary to keep the body moving (this is the aristotelian view of motion). When we study mechanics we are confused by the statement that the force on the crate is zero in this case. The same problem occurs in statics as well. I often observed students drawing only one (horizontal) force acting on a block which we press against a wall. The rationale is clear: we (and the block) feel a force; therefore, there can only be one. In general, I found that students correctly identify cases of mechanical stress. They know when and where "something is happening". Still, very often they are incapable of translating this knowledge into a sure identification of forces.

If we present mechanics using the notions of momentum (amount of motion: a substancelike quantity analogous to charge in electricity[2]) and its flow, we can address some of these problems on both an intuitive and a rational basis. We can identify the physical quantity associated with what we feel in mechanical processes, namely momentum currents through matter; these do not have to be zero even if the net force vanishes. This feeling can be translated into pictures using stream lines representing the momentum currents and their distribution, before a mathematical formalism is developed.

A simple, and nonetheless confusing, example of mechanical stress is the streched rope (Fig.1). In the case of equilibrium, we commonly say that we pull on either end with equal but opposite forces. For many students it is not clear how large the force on the rope is. Is it zero, is it |F|, or is it 2|F|? Since it is well known that we could feel something if we were at the rope´s place, we tend towards the second or the third answer; however, neither is correct. The correct answer, namely zero, causes problems because it contradicts our feeling.

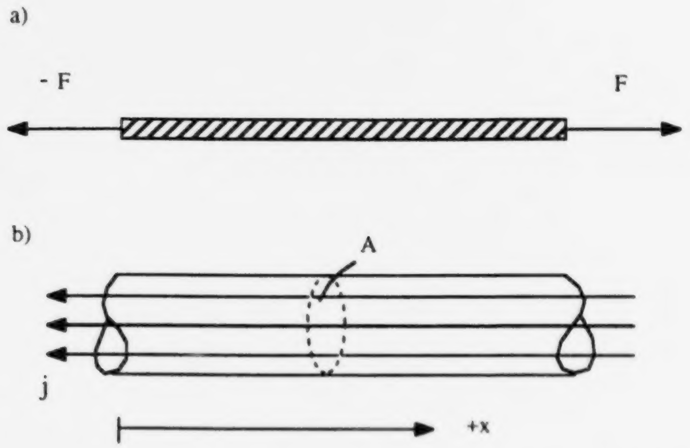

Fig.1: A rope under tension. Momentum is flowing through the rope in the negative x-direction. The momentum current per cross section (A) is the measure for the tension in the rope. j is the momentum current density.

There is a simple solution to this dilemma. The rope is under stress, and this is what we feel. We only have to make this a physically useful statement. There are ways of representing correctly what we mean with the help of the momentum current picture[3,5]. If we introduce mechanics on the basis of the concept of momentum, it is clear also for the beginner that momentum must be flowing through a streched rope. I call the flow of momentum through the rope, which I can represent by stream lines (Fig.1 b), the cause of what the rope feels. Indeed, we can introduce a quantitative measure for our feeling, namely the amount of momentum which flows through the rope per time (the momentum current I_p) and per cross section (A). This is called the momentum current density j :

$$j = I_p/A \quad . \qquad (1)$$

This quantity is not equal to zero for the streched rope, even though the net force vanishes.

The dynamical case can be developed easily. A block is pulled by a rope across a frictionless surface (Fig.2). The block accelerates. Momentum must be supplied by the one pulling the block, and it will flow through the rope into the body. Without knowing much about mechanics, we can draw stream lines representing the distribution of momentum currents through the body. The entire momentum current enters trough the point where the rope is attached (telling the future engineer that this is a point to be watched since the momentum current density is largest there). From there it flows toward the back. Constantly, some of the momentum necessary for acceleration is deposited along the path. Far from the point where momentum enters the body (the "inlet"), the current density is expected to decrease linearly (Fig.2).

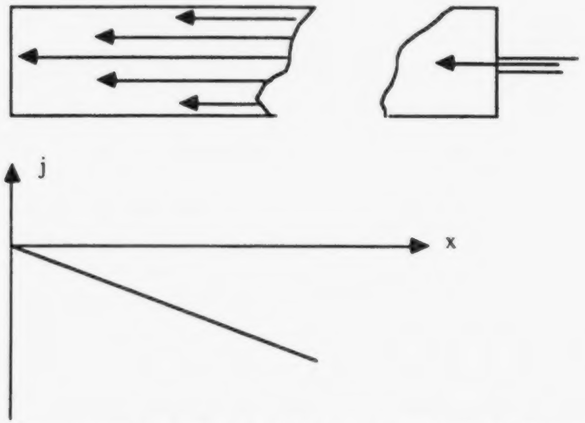

Fig.2: Momentum current distribution in a block which is accelerating to the right. Momentum accumulates in every part of the body.

This can be stated mathematically. Take a slice of the body being accelerated (Fig.3). Assume that we are far from any "inlets" and that therefore momentum is flowing horizontally. How much momentum has to be deposited in this slice to give it a certain acceleration? At the point $x+\Delta x$ (Fig.3), momentum enters the slice from the right. There, the momentum current density is given by $j(x) + \Delta j$. Momentum is entering the body at the rate $I_p(x+\Delta x) = -A(j(x)+\Delta j)$, where A is the surface area perpendicular to the flow, and I_p is the momentum current through the surface. The rate is counted negative because we take the orientation of the surface positive for flow out of the body. Some of this momentum will stay in the slice, the rest will flow out of it through the opposing surface at x. There, the rate at which momentum is leaving is $I_p(x) = Aj(x)$. This means that momentum is deposited in the slice at the rate $-A\Delta j$. On the other hand, this momentum is used for acceleration. The time rate of change of momentum of the body is $\Delta(\rho \cdot A \cdot \Delta x \cdot v)/\Delta t$. Here, ρ and v are the density and the velocity of the slice, respectively. Equating the two rates leads to:

$$\rho \frac{\Delta v}{\Delta t} + \frac{\Delta j}{\Delta x} = 0 \quad . \qquad (2)$$

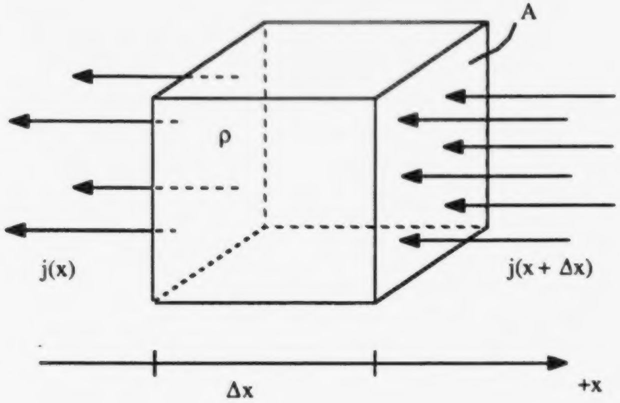

Fig.3: Momentum flows through a slice of matter of density ρ, thickness Δx, and cross section A. If the current density changes along x, momentum will be deposited in (or removed from) the slice, leading to its acceleration.

This is a simple form of the *equation of motion* of a (linear) continuum. If the acceleration is constant as in our second example (Fig. 2), the momentum current density decreases linearly from front to back (far from the "inlet"). The solution of (2) is $j(x) = -\rho a x$, where a is the acceleration, and x is counted from the back of the body (Fig.2). The complicated distribution of stream lines near the point where the rope is attached demonstrates that conditions must be more complicated around there. You see that qualitative ideas paired with simple mathematics allow us to treat basic continuum mechanics.

I have left out discussing the direction in which momentum flows. As in the case of electricity, we have to define the direction of flow of positive momentum. Simple rules have been given for identifying this direction[5]. In short, positive momentum flows in the positive x-direction through a compressed body, and it flows in the negative direction through matter under tension.

III. BODIES IN FIELDS: "RADIATIVE" TRANSFER OF MOMENTUM

I have introduced the momentum current density as the measure of what bodies feel when they are undergoing mechanical processes. However, the situation is not quite so simple; this is demonstrated by the action of gravity, inertia, and electricity, upon bodies. By studying bodies in the gravitational field we will be able to extend our treatment of continuum mechanics.

A) A first example: free fall

Take a freely falling object. It accelerates, which means that it is receiving momentum from somewhere. We know that this momentum must be coming from the Earth. Therefore, we have momentum flowing into, and possibly through, the body via the field. However, it is well known that a person falling freely does not feel any mechanical stress. Therefore, we now are dealing with a situation in which we cannot feel the momentum current.

We can solve the problem by postulating that the momentum which is deposited in the body via the field *does not flow through our object*. If momentum arrives through the field directly at every part of the body, we do not have a (surface) current density j. In other words, we have to assume that the action of a field constitutes a *source* (or a sink) of momentum for the body. It has been shown elsewhere that this interpretation of gravitational and electromagnetic interactions is correct also in a mathematical sense[4]. [The actual mechanism of momentum transport through fields and bodies in fields is much more complicated. However, the net result is the one stated here.] Therefore, it makes sense to divide momentum currents into two kinds, only one of which can be "felt" by matter, i.e. leads to stress. The second type associated with the interaction of bodies and fields we call *radiative transport of momentum*.

From the example of free fall we know that the gravitational field deposits momentum at every point of the body at a rate proportional to its local mass density ρ:

$$i_p = -g\cdot\rho \quad , \qquad (3)$$

is the expected source density of the supply of momentum[4] (summed over the body, this must be equal to the total net current due to the field, namely $g\cdot m$). Therefore, the equation of motion takes the form

$$\rho\frac{\Delta v}{\Delta t} + \frac{\Delta j}{\Delta x} + i_p = 0 \quad . \qquad (4)$$

Here, only $\Delta j/\Delta x$ has to do with a conductive momentum current which is associated with stresses in the body; i_p takes the role of a source term. In the example of free fall this means that

$$\rho g + \Delta j/\Delta x - \rho g = \Delta j/\Delta x = 0 \ .$$

Together with $j(0) = 0$, the solution is $j(x) = 0$. This is the expected result since we know that the body does not feel any stress in free fall.

B) A rope hanging from a tree

Consider this example which shows the action of both conductive and radiative momentum currents. A rope of length L, mass m, and cross section A, is hanging from a tree (Fig.4). Our feeling tells us that the magnitude of the momentum current density (stress) must increase upward along the rope. This I first describe qualitatively using the momentum stream lines. Later, a simple calculation will confirm the ideas.

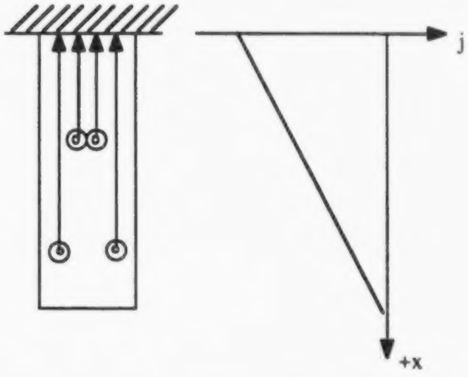

Fig.4: A rope hanging from a tree. Momentum appears in the rope via the field (circles indicate sources). It has to flow out of the rope via the point where it is attached (solid lines), leading to a linearly increasing magnitude of the conductive current density (stress). The current flows in the negative x-direction, indicating that the rope is under tension.

Momentum is deposited inside the rope via the field at the rate $-\rho g$. As I have pointed out, the body does not feel this current. However, the momentum deposited inside the rope cannot stay there; it has to flow out via the point where it is attached to the tree. This constitutes a surfacelike current through matter which is felt as stress. From the momentum stream lines we expect the current density to increase linearly upward along the rope.

This is borne out by mathematics. The equation of motion of a slice of the rope is given by (4). Since $\rho \Delta v/\Delta t = 0$, we see that

$$\Delta j/\Delta x + i_p = \Delta j/\Delta x - \rho g = 0 \quad ; \qquad (5)$$

together with $j(x{=}L) = 0$, the solution is $j(x) = \rho g(x-L)$, which we had expected. [The conductive current through the rope is in the negative direction which agrees with our rule concerning the direction of momentum currents: the rope is streched.]

C) The tides

A still more interesting example is that of a body falling in an inhomogeneous gravitational field (Fig.5). The body will experience tides. The momentum stream lines beautifully demonstrate the power of the approach taken here. Assume that the field strength g decreases linearly upward along the body. Due to the field, momentum is deposited at a higher rate in the lower portions of the body. The object accelerates at an average rate which means that the upper parts receive too little momentum while the lower parts get too much. Therefore, momentum will be redistributed throughout the body: a conductive current from the lower to the upper end is the result; since this current points in the negative x-direction (Fig.5), it tries to pull the body apart. We can also see that in the middle of our object the stress will be largest since all the momentum which is being rearranged must flow through that cross section. The conductive currents are zero at the lower and upper ends, leading us to expect a momentum current density as displayed in Fig.5.

Again, the calculation is straight forward. Assume that g increases downward according to $g(x) = g_o + bx$ (b=const.). The body's acceleration is $g_o + bL/2$, where L is the length of the object. The equation of motion (4) in this case is:

$$\rho(g_o + bL/2) + dj/dx - \rho g(x) = 0 \quad . \qquad (6)$$

Its solution is $j(x) = \frac{1}{2}\rho b(x^2 - Lx)$; as expected, the current is negative in our coordinate system.

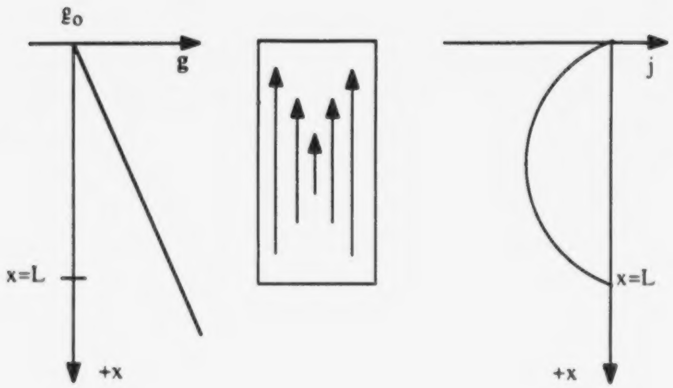

Fig.5: A body falling in an inhomogeneous field. The lower parts receive relatively too much momentum through the field. A conductive momentum current in the negative x-direction (stress) will be established so that momentum can be distributed evenly.

The approach can be extended without much difficulty to include hydrostatics and inertial fields[4].

IV. CONVECTIVE MOMENTUM CURRENTS

Convective transport of momentum is another phenomenon we have a sort of feeling for. It is a kind of thrust, and because of this, students are often led to introduce forces in the direction of motion of bodies. Just as in the case of stress, they give the phenomenon the name "force". It is of utmost importance to point out to the learner that we are dealing with two different things which both are called force in everyday life. The problem is that in mechanics we do not call this "thrust" a force.

From the point of view of momentum transport, the distinction between the two phenomena is simple. One represents the flow of momentum *through* matter (without matter moving), while in the second case momentum is carried along by moving matter. We have a concrete, yet vivid, image which lets us deal with the differences encountered.

Also, in order to understand mechanics, we definitely need to know about the roles of conductive and convective momentum transport, respectively. The trouble with variable mass systems can be traced to the fact that we usually do not introduce the distinction between these two modes of momentum flow. This keeps us from recognizing in a simple way that forces cannot change the mass of a body. Forces only are associated with conductive (and radiative) momentum transport, both of which cannot transport matter across system boundaries. Therefore, it is wrong to use Newton´s Second Law in any other form than $F=ma$. If we want to treat changing mass systems, we have to include in the momentum balance law (4) convective momentum currents as well. There will be another term in Newton´s Law[4,6].

V. WHAT ARE FORCES?

The identification of forces is anything but trivial. This is so because there are at least two different phenomena with which we intuitively associate forces, while the word force is reserved for still something different in physics (Table 1).

TABLE 1: Momentum transport and forces

Type of transport	Is stress associated with it?	Do we associate forces with it? intuitively	in physics
conductive	yes	yes [1]	yes
convective	no	yes [2]	no
radiative	no	no [3]	yes

1: Forces are associated with the stress, not with what is called force in physics.
2: Forces are associated with the thrust of bodies.
3: Forces are introduced for the stress experienced by a body resting at the Earth´s surface, and not for the force of gravity.

Now, what are forces in physics? There are two types. The first has to do with the conductive flow of momentum, and the second is associated with radiative momentum transport. Let me first introduce the former.

When momentum flows through matter, we describe the situation by the momentum current density (stress). Often, we are interested in the question of how much momentum flows per time through a given surface (which might be the entire surface of a body, or a part thereof, or a surface inside the body). We call this the *flux* of momentum. If we talk about one component of momentum only, the flux is a number which can be positive or negative, depending on the flow and the orientation of the surface. This *conductive momentum flux is called a (surface) force in continuum mechanics*. If the surface is the closed surface of the body, we speak of *the* force (or the net force) on the body. It is easily possible that the net flux is zero even though the conductive current is not zero at all. This is the source of the problem we have with the identification of forces acting on the stretched rope (Section II).

In the case of the interaction of bodies and fields we are interested in how much momentum is deposited in a body per time via the field. In order to calculate this quantity, we have to integrate the source density of momentum over the volume of the body. The resulting *radiative momentum flux is called a (body or volume) force in mechanics*. We cannot feel this type of force or interaction. Still, we are intuitively ready to introduce a force associated with it, especially if the body is resting at the surface of the Earth. Very often, students only introduce this force, the rationale being that we feel something which is not zero. Adding a second force which balances the first contradicts this feeling.

As mentioned in the preceeding section, students use the term "force" for the thrust they see in a moving body. This is the source of the forces introduced in the direction of motion of bodies. In physics, we cannot call this type of phenomenon a force.

VI. CONCLUSION

It has been shown that we do not have a feeling for what physicists call force. In everyday life, the term "force" is used for other processes. The phenomena can be squared with our feelings if we accept that we only experience momentum flow through matter; flow through fields is of a different type which does not lead to stress in matter; from the point of view of the body it constitutes a source (or a sink) of momentum. A natural measure of what we feel is the (conductive) momentum current density. It is possible to picture what is happening by momentum stream lines. The approach directly leads to a treatment of continua rather than mass points.

Forces are abstract objects. They serve to measure the strength of momentum flow through surfaces or from fields into bodies (flux). As such they are usefull for relatively simple situations known from the mechanics of mass points, and maybe that of rigid bodies. They should be introduced as what they are, namely means of accounting for momentum flow, after the role of momentum currents has been made clear. It should be kept in mind that convective momentum currents are not called forces. Introducing mechanics in such a way has considerable advantages which are worth the effort: we are given a tool for dealing with continua[3] which can be motivated directly on the basis of our everyday experience. The problems we can deal with go far beyond the mechanics of mass points.

Finally we have to ask ourselves what remains of the misconceptions which have been identified in mechanics?[1] I would say that *the main misconception is the belief that we can feel forces, or to be precise, that we can feel what physicists call force*. Texts on mechanics do not make it clear that such a feeling does not exist. *We have a feeling for stress and for thrust (of a moving body), and these are the phenomena we associate with force in everyday life*. We could circumvent some of the problems by using the word "force" for stress (we would then still have to make clear that thrust is something different). Actually, for purely didactic reasons, it would be best not to speak of forces at all. Even after my students have learned how to rationally approach the identification of forces, they return to their feelings if they are told to find those elusive arrows. And as usual, the results are catastrophic.

I can only hope that a clarification of mechanical concepts will come about by the use of the concept of momentum transport as we know it from continuum mechanics[2,3]. This should make teachers aware of the problem that misconceptions in mechanics might be of a different nature as hitherto assumed. What we identify as a misconception depends on the structure of theories taught in our classes. This holds particularly for thermodynamics[7].

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[2] The concept of the flow of momentum is well known in continuum mechanics and in the theory of fields. See Landau and Lifshitz: Fluid Mechanics, Pergamon, 1959, p.14, and Landau and Lifschitz: Klassische Feldtheorie (Lehrbuch der theoretischen Physik, Band II), Akademie-Verlag 1973, pp. 95-101. Recently, the notion of momentum currents has been introduced in the context of physics teaching based on substance-like quantities. See G.Falk, F.Herrmann eds.: Konzepte eines zeitgemässen Physikunterrichts, Vol. 5, Schroedel, 1982.

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THERMODYNAMICS: A "MISCONCEIVED" THEORY

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1. INTRODUCTION

M.Wagenschein[1] tells the story of how he observed a little girl sitting on a park bench in the sun. The girl placed her hand on the hot bench just to withdraw it quickly after some time and hold it in her other cooler hand. According to some recent investigations[2] we can safely assume how concepts about heat form in this little child (see also Section II). The heat of the sun goes into the bench and makes it hot. It is possible to withdraw some heat with a hand, and then let a part of the heat flow into the other hand. Heat is a "thermal fluid" which flows between objects and is stored in them.

Wagenschein then goes on to tell us how these ideas have to be given up to make way for the profounder knowledge gained by those physicists who developed thermodynamics more than a hundred years ago. Wagenschein, who places much emphasis on unifying prescientific and scientific knowledge[3], finds nothing wrong with this. And indeed, why should he? We all know beyond a shadow of a doubt that the idea of a thermal fluid is wrong[4]: heat is energy, or a form of energy, and not a kind of "fluid".

Obviously we are confronted with a new case of a misconception which provokes the usual reaction in teachers. While we might be sorry that there is another discrepancy between physics on the one hand, and concepts formed in everyday life on the other, we do not see how we could do anything else but exorcise the misguided notions. The suggestion that we comfort students by mentioning that scientists before Mayer, Joule, and Clausius, erred on the same count[5], does not change the fact: we are determined to undo what nature has put into students minds.

Here I shall propose a completely different solution to the problem of how to deal with the misconception. There is good reason why we should reject the usual form of thermodynamics rather than force our students to change their intuitive concepts. The reason is simple: *thermodynamics does not allow for heat to be contained in bodies* (Section III). This is not a matter we can pass over lightly. Texts on physics often create the impression that nothing is wrong with the concept of a "heat content"[6]. It seems that even we teachers desperately need to believe that heat can be stored in bodies. This is unacceptable. A theory which makes writers of textbooks succumb to a fundamental misconception should be rejected. We should search for a form of a theory which allows us to retain our intuitive notions regarding heat. If we achieve this goal it will become clear that the ususal version of thermodynamics is misconceived. It is not the concept of a thermal fluid which is at fault.

There are still other reasons for investigating the structure of a theory before deciding that, as usual, the intuitive notions are misconceived. Forcing students to give up concepts would certainly be alright if the theory of thermodynamics (as it is being taught) was worth it, and if we did not have a choice. However, a critical analysis of the theory shows that neither one of these possible reasons carries much weight.

(1) *Thermodynamics is not worth giving up intuitive concepts for.* Thermodynamics is considered to be one of the most difficult and abstract disciplines of the physical sciences. Usually we try to excuse this problem by claiming that we are paid for our efforts by one of the most beautiful and most general theories. I do not know whether thermodynamics is beautiful, but I do know that the theory we are taught in school is not as general as we are made to believe. Clausius´ famous words that the energy of the world stays constant while the entropy of the world can only increase, are quoted as proof of the breadth and depth of thermodynamics. In fact they only serve to cover up the weeknesses of the theory. Thermodynamics is little more than glorified thermostatics. A form of mathematics unknown to any other branch of the sciences is used and made to look profound. The theory does not provide us with the means of calculating general initial-boundary-value problems as we know them from mechanics and electromagnetism. And finally, entropy is not understood even though it is the fundamental thermal quantity (Section III).

All these reasons make thermodynamics the odd man out among physical theories. They do not make it worth the effort. We should look for a theory which prepares us for modern thermodynamics[7]. Only this expanded version of thermal physics would make it worth giving up intuitive ideas for.

(2) *It is not necessary to give up intuitive concepts formed in everyday life in order to create a theory of thermodynamics.* There is an alternative. It has been demonstrated[8,9] that thermodynamics can be built upon the notion of heat as we acquire it in everyday life (Section V). There is no need for giving up the intuitive concept of a "thermal fluid" which is contained in bodies and which can flow from one body to another. If it is rendered precise, this notion can be made the basis of a simple understanding of thermal phenomena: the heat of everyday experience is the thermodynamicist's entropy. Such a theory makes thermal physics structurally analogous to electricity and mechanics[10,11]. In this way it becomes the natural introduction to advanced modern thermodynamics of irreversible processes[7]. And what is most important for us: *there is no misconception on the part of the student* (and the teacher) who formed the image of the thermal fluid.

II. HEAT IN EVERYDAY LIFE

A good number of investigations[2] have demonstrated that students have their own ideas regarding the nature of heat. Here I would like to add some observations concerning concepts formed before much formal schooling in thermodynamics has occured. I shall use material gathered in my classes at Winterthur Polytechnic[12,13]. I let my first-year engineering students write a short essay on what they believe heat to be before I introduce thermal physics. I always stress that I would like to know what images they have formed of heat irrespective of what they have learned in school. All of them, however, have been told by previous teachers that heat is energy. It will be interesting to investigate the influence of this pre-college teaching. This study is complemented by a second one in which junior and senior engineering students answered some specific questions regarding heat[13].

It is clear that students believe that heat is contained in bodies, and that it can be created (Fig.1). Indeed, in the second study[13], 29 out of 31 students (who have had some college thermodynamics) answered in the affirmative the direct question of whether or not heat was contained in physical systems. Compared to this, the belief that heat and temperature are the same has been voiced rather infrequently. Indeed, at this level, students usually distinguish between an intensive and an extensive thermal quantity. The extensive quantity is called heat, and it is believed to be something like a "medium" which is stored in bodies, and which can flow from body to body. Again, the question of whether or not they picture heat to be a sort of invisible medium which is stored and which can flow (rather like electrical charge and water), was answered in the affirmative by 27 out of 31 juniors and seniors[13]. Explanations of the phenomenon of heat in terms of energy play a minor role (Fig.1). If at all, energy only has been mentioned in passing in most cases. More detailed results[12] show that the concept of energy is used incorrectly most of the time: students' knowledge does not extend beyond the superficial notion that heat is energy.

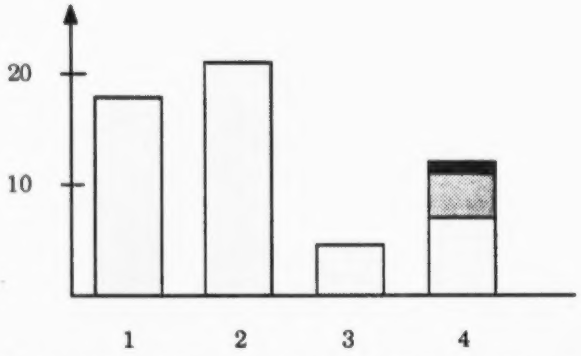

Fig.1: Results of the analysis of 42 essays on heat (ref.12). Different opinions are given. Numbers of answers are plotted vertically. 1: heat can be created; 2: heat can be stored; 3: heat and temperature are the same; 4: heat is energy (light: energy mentioned in passing; darker: energy mentioned more often; black: energy is an integral part of the explanation).

All in all, we can conclude that *students picture heat to be an extensive quantity with all its attributes: it can be stored, and it can flow.*

Often, we asociate with the term "extensive" a quantity which is conserved. The study shows that students do not spontaneously assume heat to be conserved. However, if they are asked directly, they usually resort to an explanation in terms of conversion of energy[14]. Their intuitive knowledge of the onesidedness of heat (none of the students mentions that heat can be destroyed[15]) collides with the belief of the conservation of a quantity which is imagined to be something like a "fluid". This type of reaction is the same in electricity and in mechanics where charge and momentum often are assumed to be converted out of other forms of energy[14].

The concepts formed obviously are similar to those held in the *caloric theory of heat*. Like the physicists who used this theory, students show an ambivalence as to the conservation of this quantity, and they solve the problem in the same way as was done more than 100 years ago (Section IV): they resort to the use of the concept of energy. However, they do not do so spontaneously.

III. HEAT IN CLASSICAL THERMODYNAMICS: THE DYNAMICAL THEORY OF HEAT

In order to see why believing heat to be an extensive quantity is a misconception in thermodynamics, we have to understand the theory as it is commonly presented. This theory, which was first developed around 1850 by R.Clausius, is based on the so-called *dynamical theory of heat*.

Ever since Calusius, heat has been an *exchange* form of energy. Clausius built his theory on the assumption that heat and work should be universally and uniformly interconvertible in cyclic processes. If W is the work done by a heat engine in one cycle, and Q is the difference between heat absorbed Q^+ and heat emitted Q^- in this cycle, then

$$W = JQ = J\,[Q^+ - Q^-] \quad , \qquad (1)$$

where J is the mechanical equivalent of a unit of heat. This is the expresstion of the dynamical theory of heat.

This does not mean that heat is energy. In particular, heat is not internal energy. *Heat is the name for energy exchanged in thermal processes,* no more, no less. As such, it has to be strictly distinguished from internal energy. Thermodynamics needs this distinction: otherwise it is left impotent when it comes to formulating the First Law. We have to make perfectly clear to students that the first law

$$\Delta U = Q + W \quad , \qquad (2)$$

may not be read as follows: the (internal) energy of a body is the sum of heat and work, suggesting that at any given moment we could say how much heat and how much work the body contains. The quantity Q in (2) has a totally different meaning. The problem is that even physicists do not always grasp this meaning[6], since the words used mask it completely (Section IV). We have to conclude that the majority of students, including those who have had physics and engineering thermodynamics, plus a good number of teachers, are victims of a misconception (Section II): in the dynamical theory of heat, *heat is not contained in bodies!*

There is a simple way by which we can demonstrate that we may not believe heat to be contained in bodies. Take a compressed gas in a cylinder with a piston, with values of temperature and pressure higher than those of the surroundings. How much heat is contained in this gas? Obviously, this question is senseless. If a heat content existed, we should be able to measure changes of this quantity. Different processes can be envisioned which lead to such a change. We can let the gas expand adiabatically, or we can let it cool through heat conduction. Now, since the amount of heat exchanged is different in the two processes, we cannot say by how much the heat content has changed.Therefore, there is no such thing as a "heat content".

IV. THE HISTORICAL DEVELOPMENT

Why is there such a dichotomy between intuitive concepts and the theory of thermodynamics? The historical development can throw some light on this question. On the one hand we will see that there are some reasons and some prejudices which explain why thermodynamics developed the way it did; on the other hand it will become clear that only a small step separated S.Carnot from finishing the theory of thermodynamics on the basis of the caloric theory of heat. In other words, only a small step was needed to found thermodynamics on the basis of intuitive concepts rather than anti-intuitive ones.

The historical development roughly went along the following lines[9,16]. The caloric theory of heat was widely accepted during the period before 1850. In mathematical terms, it simply meant that for the fluids used there exists a heat function (Section V). It was assumed that heat (caloric) was conserved which, again for the cases treated by Carnot and by later thermodynamicists, was acceptable. However, the main problem with the caloric theory of heat can be traced to irreversible processes in which, as Davy´s experiment (melting two blocks of ice by rubbing them) had demonstrated, heat must be generated. Today we know that heat cannot be caloric if we accept that the usual calorimetric measurements determine amounts of heat. In these experiments heat would be generated.

The concept of caloric and a heat function led Carnot to propose the following analogy for the functioning of heat engines: heat falls from a higher to a lower level (temperature), thereby driving the engine just like water drives a water wheel. Carnot proceeded to derive the motive power of heat. However, the result which was based on the caloric theory equired the heat capacities of the ideal gas to be inversely proportional to the ideal gas temperature. If we measure "heat" in the usual calorimetric devices, we get a different answer: the capacities should be constant. Carnot did not decide between this and another solution he proposed, thus forgoing the simplest form of a theory of thermodynamics (Section V).

However, measurements were not accurate enough for deciding if the caloric theory of heat was still tenable. Also, the often cited experiment by Rumford, which is supposed to have demonstrated that heat could not be caloric, did not even prove that caloric was not conserved. Even Joule´s experiments did not show that heat is an energy form: the range of temperatures employed by him was too small for the motive power of heat to be determined experimentally. His experiments simply supported a completely new idea: there is a quantity called energy associated with different types of processes (electrical, mechanical, and thermal) which remains constant during such processes.

Something else was needed for deciding between the two concepts concerning the nature of heat[9]. It was the prejudice of physicists "that heat is not a substance, but consists in a motion of the least parts of the bodies" (Clausius), which suggested that the heat (energy!) capacities of the ideal gas should be constant. Now, nothing in the world of experiment could have suggested such a belief at the time. Still, Clausius sought, and found, a theory which both determined the motive power of heat and allowed for the capacities to be constant. It is irony indeed that Clausius´ solution does not permit us anymore to think of heat as the "motion of the least particles of the bodies", since this would equate heat and internal energy!

We conclude that Carnot could have finished his theory of the motive power of heat on the basis of the caloric theory if he had accepted that heat (caloric) can be generated in irreversible processes (which would have told him that calorimetric devices do not measure caloric[9]). The dynamical theory of heat triumphed not because of nature, and not because of pure reason, but because we want heat to be the irregular motion of the atoms.

Here is the point to say a few words about the usage of terms and expressions in thermal physics. The very word heat, like electricity and motion, suggests an extensive quantity. The term "heat capacity" and "latent heat" make us think of a quantity which is contained in bodies. "Absorption of heat", and "emission of heat" do the same. Why should heat, which has been absorbed, not reside in the body which absorbed it? No wonder that we have such difficulties with thermodynamics if the simplest and most intuitive things are not true anymore. Take mechanics. There we have a different, and neutral, word for the energy exchanged in mechanical processes: it is "work", and not "motion". If we want to learn from other fields of physics, we should call the extensive thermal quantity heat (and not entropy), and "heat" (thermal energy) should be called thermal work. The trouble with thermodynamics is that *the words which make sense in the the caloric theory have been transfered to a context where they simply do not belong*. As Kelvin put it in

1878 ("Heat", Encyclopaedia Brittanica, 9th ed.): "Now that we know heat to be a mode of motion, and not a material substance, the old 'impressive, clear, and wrong' statements regarding latent heat, evolution and absorption of heat by compression, specific heats of bodies and quantities of heat possessed by them, are summarily discarded. But they have not yet been generally enough followed by equally clear and concise statements of what we now know to be the truth. A combination of impressions surviving from the old erroneous notions regarding the nature of heat with imperfectly developed apprehensions of the new theory has somewhat liberally perplexed the modern student of thermodynamics with questions unanswerable by theory or experiment....".

V. THERMODYNAMICS ON THE BASIS OF THE CALORIC THEORY OF HEAT

The way we think about heat, even after much formal schooling, resembles the old caloric theory of heat. Heat is something like a "thermal fluid" which can be stored, and which can flow from body to body. Intuitively, we also know that heat is not conserved (Section II). This suffices for building thermodynamics on an extended version of the caloric theory of heat[9].

We start with a specific theory of calorimetry for a particular type of fluid[16]. This theory served as the basis of just about all the investigations regarding heat from the beginning to Calusius and Kelvin. Only the theory of the conduction of heat was excluded from it. The theory will be outlined briefly. The bodies used in this theory are those which (1) obey a thermal equation of state relating pressure, volume, and temperature:

$$P = P(V,T) > 0 \quad . \tag{3}$$

and (2) for which two further constitutive quantities exist, namely the *latent and specific heats* (Λ_v and B_v):

$$\Lambda_v = \Lambda_v(V,T) > 0 \;, \; K_v = K_v(V,T) > 0 \;. \tag{4}$$

The index v refers to the latent heat with respect to volume, and the specific heat at constant volume. [The first inequality in (3) excludes water in the range of temperatures from 0°C to 4°C from our considerations.] This means that the bodies can be fully described by the values of two variables, namely volume and temperature. The latent and specific heats are related as follows to the *heating* $\dot{S}$ [17]:

$$\dot{S} = \Lambda_v \dot{V} + K_v \dot{T} \quad . \tag{5}$$

The heat (caloric) exchanged in a process P is then given by

$$S = \int_{\mathrm{P}} \dot{S}\, dt \quad . \tag{6}$$

Note that nothing has been said about what heat "really" is. A few consequences of the theory of calorimetry are particularly interesting. First, the bodies subsumed by this theory *can only undergo reversible changes* [9]. Secondly, the latent and specific heats with respect to volume and pressure are related by

$$\Lambda_p = \Lambda_v (\partial P/\partial V)^{-1} \quad , \tag{7}$$

$$K_p = K_v - K_v (\partial P/\partial T)(\partial P/\partial V)^{-1} \quad . \tag{8}$$

Finally, we can derive the Poisson-Laplace Law of adiabatic change ($\dot{S} = 0$) which holds for the *ideal gas with constant ratio of the specific heats:*

$$PV^{\kappa} = \text{const.} \;, \; \kappa = K_p / K_v = \text{const.} > 1 \quad . \tag{9}$$

The inequality follows from Lapace´s explanation of the speed of sound. This speed is always higher than that calculated if the oscillations of the gas were isothermal. The observation that the ratio of the specific heats must be constant will prove to be crucial when we determine the motive power of heat.

We need two assumptions plus the theory of sound (Equ.9) in order to derive the motive power of heat. This development simply finishes what Carnot had left open. The first assumption is that the caloric theory of heat is valid. In the context of our theory of calorimetry his means that there exists a "heat function" $S(V,T)$. As a consequence, the latent and specific heats are related to the heat S by

$$\Lambda_v = \partial S/\partial V \quad ; \quad K_v = \partial S/\partial T \quad . \tag{10}$$

Also, if we consider a fluid body to undergo a (Carnot) cycle, the heat absorbed (S^+) and the heat emitted (S^-) in one cycle must be equal:

$$S^+ = S^- . \tag{11}$$

This means that a heat engine can do work without any consumption of heat. (Compare this with Equ.1, the expression of the dynamical theory of heat.) Heat simply is the driving agency like water in the case of a water wheel.

The second assumption concerns the validity of Carnot´s Axiom[16]. It states that the work done by a heat engine undergoing Carnot cycles only depends on the temperatures of the furnace (T^+) and the refrigerator (T^-), and on the heat absorbed from the furnace (S^+). These two assumptions together with the observation that the ratio of the specific heats must be constant (Equ.9) suffice for determining the relationship between the heat "falling" from the higher to the lower temperature and the work done by it:

$$W = (T^+ - T^-)\,S^+ \quad . \tag{12}$$

The result is analogous to what we know from gravitation. Heat corresponds to the mass of water falling, and the difference of temperatures is compared to the difference of the gravitational potential. The heat capacities of the ideal gas turn out to be inversely proportional to the (ideal gas) temperature T:

$$K_v \sim 1/T \quad , \quad K_p \sim 1/T \quad , \tag{13}$$

which still alows for their ratio to be constant as required by the theory of the speed of sound. Finally, there exists a function E of the body (which we call its internal energy) such that:

$$\dot{E} = T\dot{S} - P\dot{V} \quad . \tag{14}$$

This should now be compared to the theory of thermodynamics based on the dynamical theory of heat. We find that *heat in the caloric theory is the enigmatic entropy* of classical thermodynamics.

VI. CONCLUSION: WHAT IS A MISCONCEPTION?

There are three points we should keep in mind: (1) students and teachers alike need a quantity which they call heat and which they can believe to be contained in bodies; (2) thermodynamics can be built on the basis of the caloric theory of heat (in this form it is structurally analogous to electricity and mechanics); and (3) the classical form of the theory is too limited (it does not prepare students for modern non-equilibrium thermodynamics). These three reasons call for a modern version of introductory thermodynamics.

If we build thermodynamics on the caloric theory of heat (suitably extended by the requirement that heat is created in irreversible processes), an interesting reversal of the roles of intuitive concepts and accepted theory results: it is not the concepts which are wrong, it is the theory which, unnecessarily, makes sound intuitive notions unacceptable. A similar situation exists in mechanics with regard to forces[18]. Fig.2 shows that most students have formed correct concepts if heat is accepted as entropy[12]. Indeed, 17 essays can be taken as a lucid explanation of the concept of entropy, while only two describe the dynamical theory of heat in an acceptable way.

Thermodynamics therefore is a case which forces us to investigate the structure of the theory before we conclude where the misconception lies.

In this case we are allowed to say that the form of the theory is misconceived. The theory disregards valuable information contained in intuitive concepts formed before formal schooling has taken place. For didactic reasons it should be rejected. In an analogous case we would not hesitate to discard such a theory. If we develop electricity along lines we know from thermodynamics, we get a structure which, according to taste, is either unacceptable, or ridiculous, or both[10].

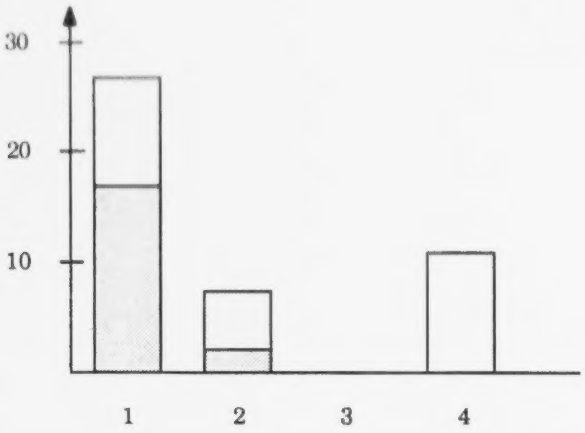

Fig.2: Results of the analysis of 42 essays on heat (ref.12). In the essay, heat can be 1: only entropy (light: only entropy if a reference to energy is dropped); 2: only energy (light: only energy if a reference to storage of heat is dropped); 3: both entropy and energy; 4: neither entropy nor energy, or too vague.

This is not to say that a new structure will present us with a brave new world in which all is well. Recent experience in the teaching of several different subjects suggests[14] that students resort to the wrong use of the concept of energy when they are confronted with the task of recognizing the role of so-called substancelike quantities like electrical charge (electricity), momentum (motion), and entropy (heat). At first sight, it seems that electricity, motion, and heat, can all be created. Since they have an apparent aversion to such a conclusion, students resort to the cure-all: energy. To give an example: charge, which has been used to introduce electrical phenomena, is not created; students rather say that it is converted out of other forms of energy. The real remaining problem thus is the failure to understand the role of energy and its relationship to the substancelike quantities in physical processes[14]. Whether this failure has its roots in experience outside of formal schooling, or whether it is a result of the teaching of physics, I cannot decide at this point. However, since the concept of energy and its conversion is a result of teaching rather than a self-evident notion, I suggest that we should look to how we teach physics if we want to solve the problem.

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ELECTROSTATICS AND ELECTRODYNAMICS - THE MISSING LINK IN STUDENTS' CONCEPTIONS

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Background

The topic of electric d.c. circuits is studied at schools at a variety of levels. At the advanced high school or introductory university level students have usually studied some physics earlier, so that the necessary concepts for analyzing electric circuits should presumably be familiar from their previous studies. At this level, the mathematical tools for treating electric circuits are also available. Indeed, various studies have shown(1,2,3) that students' general understanding does improve with age and instruction, and their mental models concerning current flow become more advanced: primitive models are abandoned in favor of more scientific ones. However, several studies(4,5,6) show that even after extensive instruction, students do not grasp some of the very basic characteristics of an electric circuit. For example, students tend to be "current minded" rather than "voltage minded",(5) confusing cause and effect. Furthermore, the general idea that an electric circuit is an interactive system is not properly understood. Difficulties associated with this feature become apparent when changes are introduced in a circuit at some point and students are asked to predict their effect on currents and voltages at various parts of the circuit.(5) Students, and even their teachers, tend to adopt a "local" approach in their analyses. They have difficulties in considering simultaneous changes in several variables. A related phenomenon emerges from Closset's findings(4) that students tend to adopt "sequential reasoning" across a wide variety of problems. This means that they regard effects in a circuit as occuring in a sequential, directional manner, without feedback. Consequently, a change at some point in a circuit will affect only quantities further downstream, and not anything happening before the point of the original change.

In order to analyze students' reasoning, it is useful to think in terms of the following aspects:
(1) *Quantitative relationships:* these involve casting the physical problem in the form of a set of linear equations (e.g. by using Kirchoff's rules), solving these equations and interpreting the solutions

correctly (i.e. what is the physical meaning of a negative value for a certain current or voltage, etc.)

(2) *Functional relationships:* these involve a qualitative analysis of functional relationships among variables in an electric circuit, namely: an understanding how a change of one component and/or parameter in a system affects the other parameters of the system (e.g. what happens to voltages and currents in a circuit when one resistance is changed?) We emphasize that by this we do not mean an algebraic manipulation and comparison of two consecutive computations, but rather an ability to consider the dynamics of the interplay between the variables in a qualitative manner.

(3) *Processes, Macro-Micro relationships:* these involve the association of phenomena with processes. The behavior of a circuit is described in terms of "macroscopic" variables such as current, resistance and voltage (in its macroscopic manifestation, as measured by a voltmeter). The underlying processes are described in terms of models: motion of charged particles, forces, fields and potentials (in their microscopic sense). Such models relate concepts studied in electrostatics with those which come into play when analyzing electric circuits.

We would like to suggest that all these aspects are necessary ingredients for proper understanding of electric circuits at the level of study under discussion. However, our expectations of student performance in terms of the three aspects defined above are restricted to an appropriate level of complexity. The usual quantitative analysis of an electric circuit (aspect 1) follows well defined algorithms, and can thus be applied even in complex circuits. A qualitative analysis of functional relationships (aspect 2) is more difficult, and will be carried out successfully only for relatively simple circuits [see e.g. the questionnaire in Cohen et. al.(5)]. The third aspect can easily become too complicated for a student to handle, even in simple circuits.(7) For example, what happens to electric fields, drift velocities, etc., when a third resistor is connected in parallel to one of two resistors connected in series in a circuit? Even experts do not carry out such an analysis routinely. However, it would be important for a student to be able to understand the process through which increasing the source voltage will increase the current in a circuit (for example: the electric field inside the conductor increases, hence the force on the charge carriers, their acceleration between collisions, their drift velocities, and hence the current, all increase). Clearly, the first two aspects are sufficient in dealing with conventional problems. Our claim with regard to the third aspect is only that students should be able to occasionally employ microscopic considerations.

Do students attain a satisfactory level of performance in these three aspects of reasoning about electric circuits? The answer to this question depends on the population examined. In Israel, high school students who choose to study physics at the advanced level, usually exhibit a satisfactory level in the first aspect. They are usually able to solve routine problems on electric circuits which are set in the final matriculation ("Bagrut") examinations. The second aspect, of functional relationships, is a source of difficulty for students and even for their teachers(5) not only in Israel but in many other countries as well.(3,4,8)

Very little is known about the third aspect. Available studies concerning students' concepts concentrate mostly on their models for current flow,(1,2,3) and not on the actual microscopic mechanisms and their interpretation in terms of electrostatic entities.

We report here on a preliminary study in which we explored whether students can associate processes occuring in electric circuits with microscopic mechanisms. For example: is the sequence of relations: battery→ potential difference→ electric field→ forces→ motion of charges (current) internalized? These issues were studied through interviews conducted with advanced high school students

after they had completed their studies in electrostatics and electric circuits.

Method

Two criteria guided the design of the interviews. First, the physical situation discussed had to be one which was unfamiliar to the students, so that they would not be able to explain the phenomena by simply reciting what they had heard in class. Second, the process under discussion had to consist of a sequence of events which would be easy to identify. As pointed out by Steinberg,(9) in any simple d.c. circuit, steady state is maintained by a number of cause-effect mechanisms (charge-voltage, voltage-current, current-charge feedback). However, all these are usually hidden mechanisms occuring simultaneously, which may not be easy to separate. We therefore chose to discuss with our interviewees transients occuring in a very simple circuit. The phenomena here (e.g. current in an "open" circuit) are different from those occuring at steady state, since the microscopic processes are easier to separate and identify. On the other hand, high school students do not usually deal with such phenomena in detail, and in order to describe them properly they would have to provide their own explanations.

Eight high school students (ages 17-18) from four different classes were interviewed for 60-90 minutes each. Each student was shown a sequence of circuit diagrams (Fig. 1), and we assured that they could identify the components and understand the circuit. In a preliminary conversation, sign conventions were also agreed on. All students knew that electrons are the charge carriers in an ordinary circuit, and preferred to describe current in terms of electron flow. The galvanometer (G) was specifically described as an ideal instrument which would instantaneously indicate the passage of any current through it. The starting question asked was: "what will G indicate?". The situation of interest to us was (1c). Circuits (1a) and (1b) served as starting points, and (1d) was used for prompts or follow-ups. All students responded correctly to questions concerning the trivial situations (1a-1b).

A varied series of questions, adapted to possible student answers, was designed. For example: Fig. 2 shows a possible flow of the dialogue in cases where students initially said that there was no current in case (1c). Clearly, a variety of branchings developed in the flow of the dialogue in reaction to the different responses students gave. In particular, when a student was not able to come up with any description at the microscopic level, we introduced various prompts into the discussion, e.g. suggesting s/he try to use terms from electrostatics (force, field, potential, etc.).

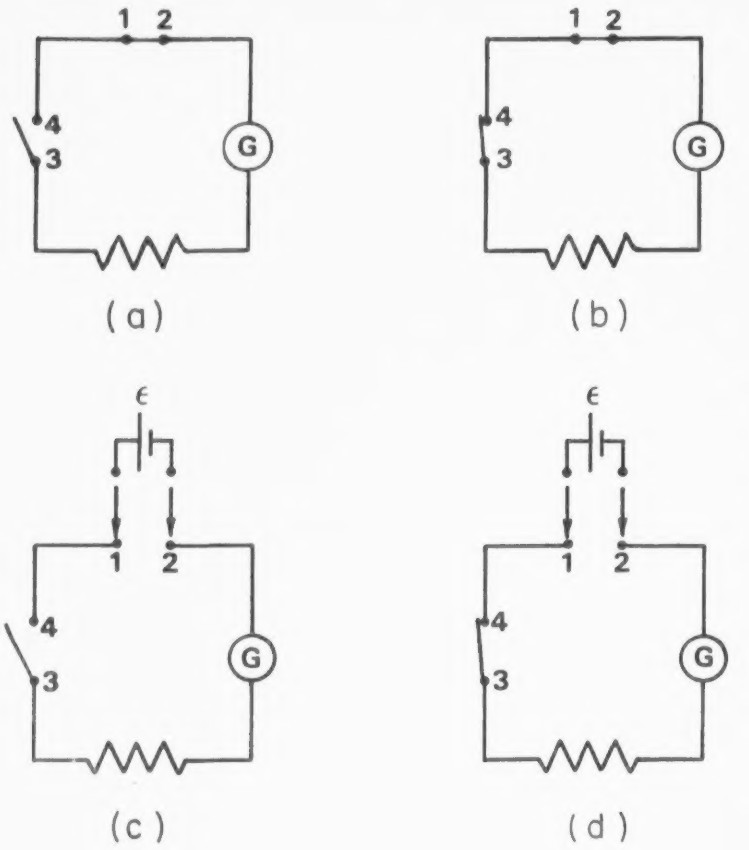
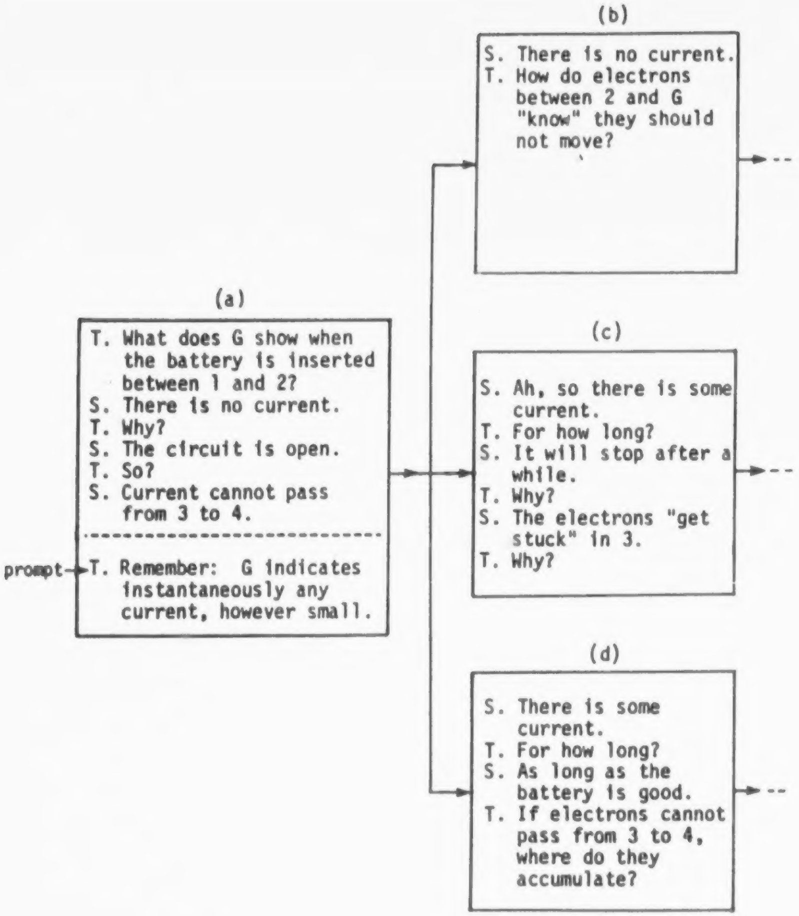


Figure 1

Sequence of circuit diagrams shown to the students. In (c) we start from (a), remove the short between 1 and 2 and insert a battery instead. In (d) we start from (b) and follow the same sequence.



(a)
T. What does G show when the battery is inserted between 1 and 2?
S. There is no current.
T. Why?
S. The circuit is open.
T. So?
S. Current cannot pass from 3 to 4.

T. Remember: G indicates instantaneously any current, however small.

(b)
S. There is no current.
T. How do electrons between 2 and G "know" they should not move?

(c)
S. Ah, so there is some current.
T. For how long?
S. It will stop after a while.
T. Why?
S. The electrons "get stuck" in 3.
T. Why?

(d)
S. There is some current.
T. For how long?
S. As long as the battery is good.
T. If electrons cannot pass from 3 to 4, where do they accumulate?

Figure 2

A sequence of questions asked in conjunction with circuit 1c.

Results and Discussion

Of the eight students interviewed, only one was able to provide satisfactory explanations of the phenomena the questions dealt with, as will be described below. Two more students came close: they first said there would be no current (in situation 1c) but when prompted ("G shows any current") they changed their minds, described the transient and the mechanisms correctly. The other five could not come up with satisfactory answers even after long discussions.

We shall now quote a few excerpts from dialogues between the experimenter (T) and the student (S). We shall then describe the concepts students use and their reasoning patterns, as emerging from all the interviews.

Examples

(I) We first give a short excerpt from the dialogue with the single student who answered all the questions correctly, in order to establish what we regard as a satisfactory understanding at this level. The student was shown circuit (1c), and said that G would initially indicate current, which would decrease and finally stop. Then:

T. Why?

S. At first, there is a potential difference between 2 and 3 and between 4 and 1, so current flows

T. Why does current flow?

S. If there is a potential difference, there is an electric field, and so the electrons are pushed along the conductor.

T. Right. So why will the current stop?

S. If we look at the break at 3, electrons cannot pass to 4, so there is an accumulation of electrons in the wire close to 3. These electrons repel each other and the next ones arriving, so there are forces which oppose the current. I can say that a field opposite to the original electric field develops, so that the potential difference between 2 and 3 becomes less and less. It is like charging a capacitor.

T. And what happens between 4 and 1?

S. It is similar, only in the opposite sense: at first electrons flow into the battery, but at 4 the wire becomes positively charged and the electrons are attracted, until the attraction cancels the attraction by the positive terminal 1.

(II) The student was shown circuit diagram (1c). The initial part of the dialogue proceeded as in Fig. 2(a). Then:

S. Maybe there is some current through G, since the electrons in 2 do not know there is a break in the circuit further down.

T. Is there current between 4 and 1?

S. No, in the battery current flows from 1 to 2 and then the electrons flow from 2 to 3.

T. For how long does this continue?

S. It will stop, because the first electrons stop at the break and there is no room for the next ones.

T. Do you mean that there is no physical space for the coming electrons?

S. No.... the electrons are tiny.

T. So what causes them to stop?

S. When there is a conductor and a voltage, the electrons move. In the break (3-4) there is no conductor, so the electrons stop there. But I do not see why they stop in the wires between 2 and 3 - there is voltage and there is a conductor...

T. What is voltage?

S. Voltage moves electric charge from high to low potential.

T. What do you mean by potential?

S. The work needed to bring the charge to infinity. Now, if the battery creates a potential difference, the electrons move.

T. Why do they move?

S. Because there is a potential difference.

(It was not possible to get beyond this point in the discussion. We therefore introduced additional concepts, in order to trigger a description of the processes involved).

T. Have you learned about the concept of the electric field?

S. Yes, a charge creates a field around it.

T. How is the electric field related to potential difference?

S. Potential difference is the work necessary to move the charge between two points...

(This student was unable to go beyond reciting formal definitions of concepts from electrostatics, and could not describe the processes. This part of the interview was concluded by providing the student with an explanation. Later on, the following dialogue took place).

T. What is the voltage between 3 and 4 after the current has stopped?

S. Zero, since there is no current... Wait - 1 is at the same potential as 4, since there is no current. The same for 2 and 3. So between 3 and 4 I must have the same as between 2 and 1. But this is ϵ.

T. So?

S. I get two different results. Ah... the voltage between 3 and 4 is indeed zero, but the potential difference is ϵ.

The complete interviews with every student were analyzed in detail, in terms of their Macroscopic notions, Microscopic notions, and Micro-Macro relationships. For example, the main notions which emerge from the complete interview with the previous student can be summarized as follows:

Macroscopic notions

- A battery creates voltage.
- Voltage causes current flow if there is a conductor present.
- If there is no current, there is no voltage (but there may be a potential difference...)

Microscopic notions

- Electrons are pushed into the open circuit from the negative terminal only.
- These electrons move through the wire freely as in a pipe, until they stop at a break.
- The student remembers the formal definitions for potential and electric field from electrostatics, but he does not relate the two concepts.

Macro-Micro relationships

This student cannot provide a description of macroscopic phenomena in terms of microscopic concepts from electrostatics. In particular, voltage (macro) is divorced from potential (micro).

(III) The student was shown circuit (1c). The initial part of the dialogue proceeded as in Fig. (2a). Then:

S. There cannot be any current.

T. Why?

S. In a closed circuit, the (-) of the battery pushes the electrons and the (+) pulls them. If 3 and 4 are disconnected, only the (-) pushes but the (+) does not pull. You need both a push and a pull.

T. Why?

S. Because the electrons are held by the nuclei.

T. Which electrons are we talking about?

S. The electrons at terminal 2 (Fig 1c), this is where we have a surplus of electrons.

T. (Trying to prompt) Do these electrons not repel each other?

S. Yes, but since the circuit is open, there is no (+) and they are not attracted.

This student insisted that no current could flow. He was then shown circuit (1d), where he agreed that current would flow. Then:

T. We now open the switch 3-4. What happens?

S. The current stops in the neutral state of the wire, because there is no connection to the (+) of the battery. The electrons which were on the way are held by the (+) of the nuclei in the wire.

At a later part we inquired about the voltage between point 3 and 4 in circuit (1c). Like the student in Example II this student also arrived at a contradiction: voltage equals ϵ on the one hand, but voltage equals zero because "there is no current".

Throughout the interview, this student never used the concepts of potentials or electric fields. All his descriptions were given at a concrete level, involving only charge accumulation and forces on charges. In other words: this student creates his own microscopic descriptions, disregarding what he was taught in electrostatics.

(IV) The student was shown circuit (1c). The initial part of the dialogue proceeded as in Fig. (2a). Then:

S. There cannot be any current.

T. Why?

S. Charges flow from high potential, where there is charge surplus, to low potential, where there is a deficiency. Since 3 is not connected to 1, 3 is not at low potential, there is no potential difference, and no current.

At this point the student was shown circuit (1d), and agreed that current would flow. Then:

T. How did the electrons in circuit (1c) "know" not to move from 2 through G?

S. Ah... maybe they do move at the beginning, but after a while the current will stop.

T. Why will it stop?

S. When the potential at 3 equals the potential at 2, because you transfer electrons, since the battery is like a pump, taking electrons from one place to

> another, then if at one place there will be enough electrons, there will be no need to transfer any more, it will serve no purpose...

Personifications aside ("need", "purpose"), this student had some notion of the role of potentials and charge densities, but he was unable to tie the potential concept to a mechanism. Potentials were not associated to electric fields or forces on charge carriers.

Concepts and reasoning

How do students describe the phenomena? The concepts used by students to explain the operation of electric circuits appear in a number of levels. At the most basic level, "electron densities" are invoked, and current flow is then due to some mysterious "tendency" to equalize densities. Although this description is unsatisfactory from a physicist's point of view, it is enough to satisfy students' needs in many of the usual (closed) circuits they encounter: a battery has surplus of electrons on one terminal, lack of them on the other, hence current flows. A slightly higher level associates the concept of "potential" with electron densities, and then an equally mysterious "tendency" to equalize potentials is invoked. Still no mechanisms are used. "Potential" is a concept for which a formal definition is quoted, but it is not

associated (at this level) with electric fields or forces on the charge carriers. We wish to emphasize that students who spoke in these terms did not associate potentials with fields/forces even when specifically asked to do so.

There are students who feel the need for mechanistic explanations, and speak in terms of forces (repulsion and attraction). Again there are two levels: some will stop at the concrete level, and will not be able to make the connection: forces - fields - potentials. Clearly such students cannot associate the microscopic picture (electrons, forces between them) with the macroscopic parameters of the circuit. Only a minority of the students operate at the higher level where they are able to integrate all the concepts into a coherent picture in which the microscopic concepts from electrostatics tie in with the macroscopic parameters.

In light of these characterizations, it is not surprising that students are thrown off by a situation such as 1c (Fig. 1). When asked to explain why things happen, they search for a picture which will appeal to their intuition. Our intuition, however, is guided by macroscopic, everyday experiences. When asked to explain why the initial current will stop, students speak in terms of "space" available to the electrons in the wire. One student actually spoke of "a train which stops when the first car stops", basing his argument on the claim that charge motion must occur simultaneously in all parts of the wire. Others knew enough about atomic sizes (see Example II), so they would not talk in terms of physical space. At this point, there were students who would simply give up, and admit they had no explanation. Others tried to reconcile the knowledge about small sizes with the intuitive need for a macroscopic type of picture ("space"), and this leads to a variety of models which the students try to make up, using bits and pieces from what they had learned previously. For example, one student spoke of "conduction electrons", stationary positive ions, and electrons hopping from one ion to the next. At a break, the electrons have "nowhere to go". This is actually a more refined picture of the same basic notion of "available space".

Our findings concerning the role of the battery and connecting wires are similar to those of Gott.(10) Two general pictures emerge. In one, the battery is regarded as the only source of electrons which move in the circuit. It injects electrons into the wires, which play no active role. The wires can be either "empty", or consist of atoms (ions + electrons). In the other picture the battery acts as a "pump", causing electrons already present in the wires to circulate around the circuit.

Conclusions and Implications

As noted previously, it is useful to think in terms of three aspects, when dealing with student concepts on electric circuits: quantitative relationships, functional relationships, and macro-micro relationships. Our claim is that all three aspects are necessary for achieving a proper understanding of this topic.

It is entirely possible to operate within the quantitative level through rote learning of algebraic algorithms. This is undesirable. From the practical point of view, it provides no safeguards against nonsensible results. Furthermore, in teaching physics we want to achieve a more meaningful understanding than a mere manipulation of numbers or algebraic terms. In the common tasks of solving electric circuits, students calculate values for one parameter after the other, usually without considering the relationships among them. Operating at the level of functional relationships not only can help one to avoid tedious calculations, but it enables the student to deal with the system as a whole, and consider the interrelationships between its parts (the global approach). In order to fully appreciate the functional relationships, the aspects of cause and effect must be clearly understood. In other words, the concept of voltage must be internalized and applied correctly We have already noted that students are not "voltage minded" Why is that the case, when the concept of potential (and potential differences) is studied thoroughly in electrostatics? One reason may be the predominance of early studies, with their emphasis on various models for current flow Another reason is probably the simple fact that students do not relate the electrostatic potential to the processes occuring in the circuit Put differently, their reasoning may be lacking in the third aspect, as defined above Consideration of macro-micro relationships can lead to a deeper understanding of the phenomena, by providing a meaning to abstract parameters which are used in the analysis In particular, concepts studied in the framework of electrostatics will be integrated into the description of dynamic phenomena, and enable the student to "explain" them

What emerges from the present study is the realization, that this does not seem to be the situation for most students Students are not able to tie concepts from electrostatics into their description of phenomena occuring in electric circuits As already noted, this leads to a number of difficulties First, the concept of voltage remains vague; its formal definitions (quoted correctly) are not utilized operationally Secondly, most students do not create a consistent picture of the mechanisms, and are therefore

unable to explain the phenomena We note in passing, that this situation does not necessarily represent misconceptions, but rather the lack of any clear concepts Consequently, we encountered a variety of contrived explanations Thirdly, we believe that this absence of a micro-macro link impedes students' ability to conceptualize the electric circuit as a system and to appreciate the functional relationships between its parts

Several approaches have been presented in the literature in recent years, suggesting how to start the instruction of electric circuits, so as to emphasize the mechanisms underlying the phenomena For example, Steinberg(9) uses large capacitors to slow down the transients, in order to demonstrate the processes occuring in electric circuits These experiments are used to gradually develop a model for the system Hartel(6) suggests emphasizing the interactive aspects by utilizing the analogy with mechanical systems such as stiff rings or chains Whatever the initial analogies used, it is essential that students realize the limits of their applicability(12) This means that the students cannot be left with such models standing on their own At some point any such model must be abandoned, in favor of a picture based on electrical concepts Clearly, a complete solution which maps charge distributions, electric fields etc , is beyond the level of high school physics. In fact, this is a non trivial task(7,11) even for very simple configurations. However, it is essential to relate charge (current) to forces and fields - those studied in electrostatics, and thus demystify the mechanisms driving the system. This can be done, for example, in a semiqualitative way, using a classical treatment based on Drude's theory. Such an approach can be found in various texts,(13) but it would seem that the necessary "macro-micro" link does not receive enough emphasis, and consequently is not assimilated in students' minds. We believe that by creating such a link, some of the difficulties encountered by students can be resolved.

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USING AN INTERACTIVE VIDEODISC AS A TOOL FOR INVESTIGATING AND FACILITATING STUDENT UNDERSTANDING IN GEOMETRICAL OPTICS *

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Introduction

For the past several years we have been investigating college student understanding in the domain of geometrical optics [Goldberg and McDermott, 1986, 1987]. Our goals have been two-fold: to study how students think about specific optical phenomena (like shadows, mirrors and lenses) prior to formal instruction in their college level science courses; to identify and describe the nature of specific conceptual difficulties exhibited by students after formal instruction. In the latter case, we are particularly interested in the extent to which the students can use ray diagram representations to help make predictions and provide explanations.

The primary source of our data has been individual interviews with students. Each interview involves a sequence of tasks focusing on some apparatus. The student is asked to make predictions and then to explain his or her reasoning, drawing ray diagrams if appropriate. For example, in our work on student understanding of the image formed by a converging lens [Goldberg and McDermott, 1987] the apparatus consisted of a light bulb, a lens and a screen with a clear, inverted image of the bulb on it. See Figure 1. We asked the student to predict how the image on the screen (or the observability of the image) might change if we were to make each of the following changes from the initial set-up: (1) removing the lens; (2) covering the top half of the lens with an opaque card; (3) moving the screen from its original position towards the lens; (4) removing the screen entirely. The responses of the students to

these and other similar questions indicated that many of them had difficulty interpreting and drawing ray diagrams, that they misunderstood the function of the screen, and that they confused the concept of image point and focal point. The lengthy discussions we had with the students during the interviews enabled us to learn a great deal about the nature of their difficulties.

Recently we videotaped the entire sequence of tasks involving the converging lens and produced a videodisc. This is being used, under computer control, to carry out stand-alone interactive interviews with students. Our first goal in doing this is to gather more information about student understanding in geometrical optics. We have found that the computer-videodisc combination allows us great versatility both in the types of tasks we can present to students and in their style of presentation. Our second, and long term goal, has been to use the videodisc technology to provide tutorial instruction. Because our interview tasks seemed very effective in eliciting students' conceptual difficulties, we believe the same tasks would serve as good starting points for helping students overcome their difficulties. Furthermore, the characteristics of the interactive videodisc system promise to make it an effective tool for helping students make connections between real world optical phenomena and ray diagram representations.

In this paper we will describe the computer-videodisc system and how we use it to administer interviews. Then we will discuss certain issues regarding its use as a tool for investigating student understanding. We will conclude with a discussion of its use as a tutorial tool.

Description of the Interactive Videodisc System

To develop the videodisc the author was videotaped demonstrating many of the same tasks that were used during the previous interviews with students [Goldberg and McDermott, 1987]. In addition, close-up pictures were taken of the screen under a myriad of contrived conditions so that the resulting images would match all the students' incorrect as well as their correct predictions. Most of these screen pictures, however, were fakes; that is, they did not represent what would actually be seen on the screen under the conditions of the tasks presented to the students. The intent of taking these "fake" screen pictures was to include them among a visual montage of several possible screen images that students could choose from when visually confirming their predictions. After final editing of the videotape an inexpensive videodisc was produced.[1] After receiving the finished videodisc, the author then used the Authoring System to sequence the interview and to incorporate computer-generated text and graphics on the appropriate video pictures.

Figure 2 shows a sketch of the main components of the computer-videodisc system that was used during the interviews with students. These components are as follows: (1) IBM XT computer; (2) Zenith Model ZVM 135 color monitor that displays both RGB and composite signals--the composite picture comes from the videodisc and the RGB comes from the computer generated text and graphics; (3) Pioneer Model VD-L1000 videodisc player (can be controlled by signals from the computer); (4) VAL Model 1125 Videodisc controller and overlay card; (5) Tecmar Graphics Master video card; (6) QUEST Authoring System from Allen Communications.[2] The Authoring System enables us to control the videodisc player, produce text and graphics overlays and design the sequence of events that will be presented to the student. As of January 1987, this entire system costs approximately $6500.

Description of the Videodisc Converging Lens Research Interview

Most students require between 30 and 45 minutes to complete the converging lens videodisc interview after first turning on the computer and videodisc player. The interview begins with a two-minute narrative by the author telling about the purpose of the research and describing how the computer interview will work. Then the author introduces the apparatus: an unfrosted light bulb that serves as an illuminated object, a converging lens and a translucent screen on which a clear, inverted image of the light bulb filament can be seen. The student is asked to sketch the image on a special form provided for responses to the questions.

The author then proceeds through a series of questions regarding how the image observed on the translucent screen might change if the set-up were changed in some particular way. As each question is posed the author actually demonstrates the change in the set-up and simultaneously the translucent screen is covered (overlayed) with an opaque mask, effectively hiding from view what the student is being queried about. We felt that by actually demonstrating to the student what is being asked some of the ambiguity involved in making predictions about "possible" changes in the set-up would be avoided.

The student is then directed to sketch what he or she predicts would be on the screen and to draw a ray diagram to justify the prediction. Finally, the student is shown a visual montage of "possible" screen pictures (all but one of which are "fakes") and is asked to choose the one that is most like his or her prediction. (As mentioned earlier, these fake pictures represent the spectrum of incorrect predictions that were made by students who were interviewed with the actual apparatus. We would not have been able to include these on the videodisc had we not gathered this information from previous research.) Portions of the computer program are branched so that the next task presented to the student is contingent on the response to the previous question. This process of asking questions, masking out critical portions of the visual scene, and then asking for a visual confirmation of predictions continues through a sequence of between five and nine questions. The exact number of tasks presented to a given student would depend on his or her responses to the first few questions.

After the questions are completed the student is given the option of either seeing demonstrations of the situations he or she was queried about or to quit the interview. (Almost all of the students who have gone through the interview have opted to see demonstrations of most of the tasks. They generally found the questions posed to them to be interesting and were quite curious of the actual outcomes.) Finally, the student turns in the response form which includes his or her screen image prediction sketches, ray diagrams and written explanations.

Some Questions Regarding the Use of the Videodisc Interview

We have tried to design our videodisc interviews so that students could go through them without a facilitator present. The questions are posed directly by the author (on the video monitor). The student writes all responses on a special form and registers his or her prediction with the computer by making a selection from a visual montage of choices.

Thus far we have administered the converging lens videodisc interview to three groups, with a total population of 34 students. All were unpaid volunteers. The first group consisted of 5 students enrolled in a special course required for secondary school science certification. The second group consisted of 13 students enrolled in a one semester activity-based physics course for preservice elementary school teachers. These students had experimented with converging lenses and had studied their behavior qualitatively, but did not draw ray diagrams. The third group consisted of 16 students who were enrolled in an introductory, calculus-based physics course for engineering students and science majors. Their instructor had discussed geometrical optics in class and had drawn ray diagrams. 70% of those students were also enrolled concurrently in a lab course and had already performed experiments with converging lenses. The only major qualitative difference in performance between the three groups on the videodisc tasks was that the third group drew ray diagrams to help justify their predictions, whereas the other two groups only wrote descriptions of their predictions.

Our use of the computer-videodisc system as a vehicle to gather information on student understanding raises a number of concerns or questions. We will describe each concern or question below and then address it, basing our responses on the data gathered during the interviews with the three groups of students mentioned above.

Since we intended for students to work through these interviews by themselves we needed to know the extent to which the questions posed on the video screen were clear and unambiguous to them. The author was present in the room during the times that students in the first two groups engaged in the videodisc interviews. Based on feedback from the first few interviews a small number of editing changes were made in the computer-videodisc program. After those changes, the students seemed to have little difficulty understanding what they were supposed to do. None of the students asked the author to interpret any of the questions being asked.

For the last group of 16 students, the author was present only at the beginning of the interview to make a few procedural comments and to show the student the form on which predictions and diagrams were to be recorded. The author left the room when the student turned on the computer. Although help was available in an adjacent room, none of the students requested any after beginning the interview. All the students in the third group completed the interview completely on their own. We conclude, therefore, that the questions seem to be clear and unambiguous to the students.

Another of our concerns centered on how the student was to communicate his or her thinking to the computer. Eventhough the student was asked to write explanations and draw diagrams on a written form, the computer required input from the student to "know" the proper sequence of tasks to present. In other words, we needed an effective way of letting the computer "know how the student was thinking." Our vehicle for doing this was to present the student with the visual montage of possible choices. This raises two questions. First, does the student actually choose the response closest to his or her written prediction? Second, when confronted with a visual picture of the results of the prediction, does the student change his or her mind? We were concerned that if the student made an incorrect prediction, and then saw what the screen would actually look like if that prediction were confirmed, the student might react by thinking something like the following: "That can't be right. The screen will never look like that. It must be some other answer." Furthermore, if the student is not very confident to begin with, just seeing the alternative choices may encourage the student to change his or her mind before responding to the computer. The effect of this is that the information received by the computer would not be representative of the student's original thinking.

Space was provided on the interview response form for the student to draw a sketch of what he or she thought the screen would look like for each of the tasks. To determine to what extent the students choose the "appropriate" picture from the visual montage, we compared the students' written descriptions of their prediction and their sketches of the screen images with their choices from the visual montages. We found that in less than 2% of the cases did a student apparently choose an inappropriate multiple choice response.

We tried to address our concern about students' changing their mind by telling them, at the beginning of the interview, that even though they might wish to change their mind when shown the visual montage of various choices, they should still write down the choice which corresponded to their original prediction. Then, if they wanted to change their mind, they could indicate their new response at a separate place provided on the form. Of the 184 possible multiple choice responses from the 34 subjects we found that only 8% of the responses showed an indication that the student had changed his or her mind.

Two final questions raised by the use of the videodisc interviews as a research tool have to do with a comparison of the responses of the students who went through the videodisc interviews with the responses of those who were interviewed in our previous study with real apparatus and with the investigator present [Goldberg and McDermott, 1987]. The first question is: How does the quality of their ray diagrams compare? The second is: How do the pattern and frequency of their prediction responses compare?

Space was provided on the interview response form for students to draw ray diagrams to go along with most of their predictions. (Students who had not studied ray diagrams were encouraged to justify their responses with some written comments.) The vast majority of the students in the third group, who had studied ray diagrams in class, made some attempt to answer each question with a diagram. Those diagrams that were incorrect displayed the same types of errors that we found in the diagrams drawn by students interviewed in our previous study with actual apparatus.

With respect to the predictions that the students made, we found the pattern of responses to be very similar in both the interviews using the videodisc and the actual apparatus. For all the tasks that were identical in the two interview formats, the percentage of students from the two groups who gave the same responses matched to within 10%. The accompanying Table shows the results for two of the questions. A complete description of these questions is given in one of the references [Goldberg and McDermott, 1987].

Using the Interactive Videodisc System for a Tutorial Interview

Based on our previous research with students who have studied geometrical optics in their college physics or physical science classes we can make the following two observations: (1) most of the students seemed to have a basic understanding of the behavior of light with respect to rectilinear propagation, reflection and refraction, and could represent this behavior readily by drawing light ray diagrams; (2) many of the students exhibited difficulty using ray diagrams to make predictions about or to explain novel optical phenomena; i.e. examples that they had not explicitly studied in class. This difficulty students have in connecting real world phenomena and the appropriate scientific representation seems to be a general one, found in other areas of physics [e.g. McDermott, Rosenquist and van Zee, 1987].

Recent psychological research provides a clue for an effective instructional strategy to address this kind of difficulty [Tulving and Thompson, 1973]. If knowledge related to external events is to be accessible when those events occur, then it is important for there to be perception of the relevant phenomena when the knowledge is encoded into memory. The implication here is that to enhance the student's ability to connect representations and real world phenomena, it would be most effective for the student to have learned about the representations and phenomena at the same time.

Further support for this contention is provided in the recent work by Brasell in kinematics [Brasell, 1987]. In her study she used the real-time graphing feature of a Microcomputer-based lab on motion to teach students how to draw and interpret position-time and velocity-time graphs representing the actual motion of an object. She found that presenting the graphical representation at the same time as the motion occurred, rather than after a delay of only 20-30 seconds, was a crucial factor in facilitating the students' learning.

The interactive videodisc system provides a vehicle through which students can be presented with diagrammatic representations and pictures of physical phenomena *simultaneously*. With our videodisc system we can overlay ray diagrams on video pictures of the lens set-up. By incorporating this strategy into a tutorial program we hope to be able to enhance the students ability to connect ray diagrams and optical phenomena.

At this time we have just completed a preliminary version of such a tutorial program. The topic is image formation by converging lenses and the program was developed by Sharon Bendall, a member of our research group. The strategy is to first show a demonstration and ask the student to make a prediction, then present as needed appropriate information about optics, and finally present a related follow-up demonstration question. The questions we ask are the same as those we used for our interview tasks. We have found them to be unambiguous and to be useful in bringing out any conceptual difficulties that might exist in the minds of the students.

The information we provide emphasizes the meaning of light rays and the use and significance of light ray diagrams. We found in our previous research that students often misinterpret ray diagrams, and are especially confused by the significance of the special rays. Many students seemed to think that the special rays were necessary to form an image rather than just being convenient to locate the position of the image. Furthermore, they seemed to think of a light ray as if it were an actual beam of light, rather than as a geometrical representation of the direction that the light was travelling.

In our tutorial, we have emphasized this geometrical representation. We overlay ray diagrams right on top of the pictures of the lens set-up. Although we do introduce and use the "special rays," we also draw plenty of other rays representing light travelling in other directions.

Preliminary results with students who have worked through our tutorial indicate that it has promise in helping the students develop a good qualitative understanding of image formation by a converging lens. We are currently designing new tutorials in other areas of geometrical optics and will begin to carefully evaluate their effectiveness in helping students develop the general skill of connecting ray diagram representations with a myriad of examples of optical phenomena.

FOOTNOTES

1. The videodisc was produced by Spectra Image, Inc., 540 N. Hollywood Way, Burbank, CA 91505. The cost was $300.

2. Allen Communication, 140 Lakeside Plaza II, 5225 Wiley Post Way, Salt Lake City, UT 84116. This company can provide most of the components of the entire interactive videodisc system.

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TABLE: Student responses for two tasks presented during interviews using either the actual apparatus or the videodisc. Percentages have been rounded to the nearest 5%. Correct response are indicated with an asterisk(*)

What would you see on the screen if the lens were removed?

Response	ACTUAL APPARATUS (N=22)	VIDEODISC (N=34)
No image(*)	50	55
Erect Image	40	35
Other	10	10

What would you see on the screen if the top half of the lens were covered with cardboard?

Response	ACTUAL APPARATUS (N=23)	VIDEODISC (N=34)
Entire image remains(*)	35	40
Half image vanishes	55	55
Other	10	5

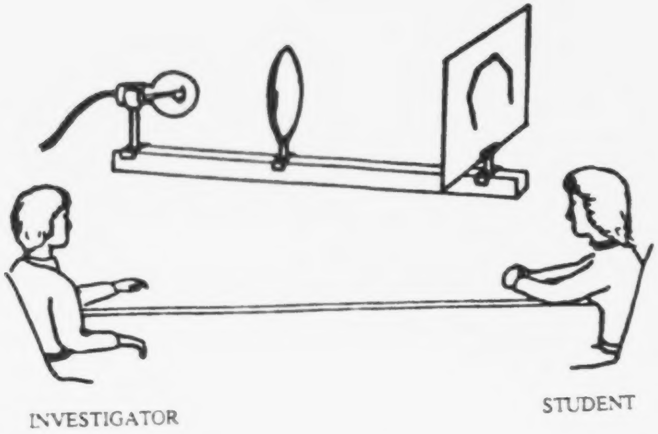
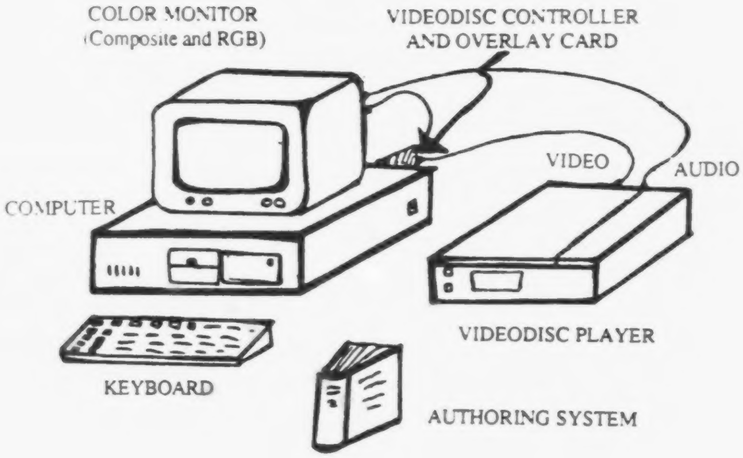


Figure 1



INTERACTIVE VIDEODISC SYSTEM

Figure 2

The Roles of students and teachers in misconceptualization of aspects in "chemical equilibrium"

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School science is being taught after the child has been exposed to the world around him. As a result of this personal involvement, whether visual, audial or manipulative, the child involves himself in a process of generalization of phenomena and abstaction, leading to the formation of concepts concerning the nature of the world around him. The formed concepts are usually limited in their scope as they result from limited experience and are based only on macro and local occurences. These conceptions have been called in the literature "commonsense understanding" (Hills 1983), "childrens' science" (Osborne & Fensham 1982), "everyday life constructs" (Wheeler 1983),"alternate frameworks" (Driver 1981) or "misconceptions".

Our work is part of the large movement in science education that aims to understand and clarify the conceptions of children regarding scientific concepts. The specific work is concerned with the misconceptions associated with the concept of "chemical equilibrium". The label "misconception" is being used, rather than the other phrases, because of the constant comparison of the students' conception with that of the scientific community. This research is not a goal by itself, rather it is considered as neccessary means to improve and deepen the understanding of chemistry.

At school the concept of "chemical equilibrium" is associated only with the study of chemistry. When this concept is introduced in the classroom the student has already a conception of the concept "equilibrium". This conception is a result of experiencing situations such as see-saw, bicycling, balance and , of course, formal studies at school of the concept "physical equilibrium" (Shaeffer 1984). It should be stressed that this conception of "equilibrium" is a limited one in the sense that it represents only the kind of "static equilibrium" i.e. a subordinate concept of the more general one of equilibrium that is abstracted from knowledge concerning "static equilibrium" and "dynamic equilibrium". The teacher who is aware of this prior knowledge (and most of them are not aware of the conception of "equilibrium" and its nature) is now faced with a dilemma of how to introduce the concept of "chemical equilibrium". Should the newly introduced concept be tied to the preconceived one of equilibrium or should it be introduced as a completely new concept , taking care and stressing its unique features (maybe even using another label to exclude the word "equilibrium"). It seems that most of the teachers (the aware minority and the unaware majority) intuitively choose the first option. The teachers whose representation of chemical equilibrium is probably correct or close to that of the scientific community, are using the label "equilibrium" as a short cut for "chemical equilibrium". However the novice learners not having yet different schemas for "chemical equilibrium" and "equilibrium" compartmentalize the information on chemical equilibrium together with the previously conceiced concept of equilibrium, or reject the new information as it is in disagreement with the previous one. This process of subsumption into the prior knowledge associated with "equilibrium" without differentiation of attributes should lead to strong resemblance of the representation of the two concepts. The similarity of the associative framework is supported by the analysis of word associations provided by students to the two key concepts "equilibrium" and "chemical equilibrium".

Table 1. The frequencies of categories of word associations to the key concept "equilibrium" and "chemical equilibrium" in percentages.

N	Colloquial Language	Balance*	General Chemistry Concepts	Chemical Equilibrium
		Equilibrium		
pre(72)	46	12	11	7
post(72)	23	6	13	35
		Chemical Equilibrium		
pre(72)	12	10	46	13
post(72)	1	1	19	56

* These values are included in the Colloquial Language.

Note: Other categories are not presented.

Table 1 gives the frequencies of the categories of free word associations of highschool students who studied an advanced course in chemistry. The results indicate that the conception of the label "equilibrium" at pre-learning is associated primarily with everyday concepts (46%). In contrast, the concept of "chemical equilibrium" is associated with general chemical concepts (46%). The post-learning increase in the word associations categorized as chemical equilibrium concepts (35% for equilibrium and 56% for chemical equilibrium) indicates a diffusion process whereby the concepts of "equilibrium" and "chemical equilibrium" merged to one concept being associated with a very similar framework.

This situation is actually the one that is leading to possible misconceptions. The features of the concept "equilibrium" aquired from physical or everyday experience (macrophenomena) are those of staticity and sidedness (grasping the system as being composed of sides). It represents a system that is composed of two or more forces reaching a situation of balance. However "chemical equilibrium" is a dynamic system featuring reversible reactions . The attribution of dynamism to chemical equilibrium cannot be abstracted directly from macrophenomena, rather it is part of the model concerning the structure of matter. The possible transfer of features and attributes from equilibrium to chemical equilibrium will thus lead to a misconceived concept of "chemical equilibrium".

It is of interest that the first introduction of the concept of "equilibrium" to chemical reactions had the notion of balance of chemical forces exactly as that used in physics. Guldberg and Waage who introduced the idea of dependence on "active mass" concerning reversible reactions still viewed the equilibrium state as resulting from the equality of forces exerted by the opposing reactions. Although they talked about reversibility, it is not clear that they had a dynamic conception of the equilibrium state (Lindauer 1962). Nowadays chemical equilibrium is being treated by two different approaches. The thermodynamic treatment considers chemical potentials (that are analogous to potentials in physics) and can lead to a representation, essentially similar to that in physics, i.e. the balance of "chemical forces". Although the thermodynamic treatment represents a formal mathematical approach, dealing with macroscopic observable quantities and being detached from any micro-model, it implicitly assumes the existence of a microscopic mechanism for reversibility, i.e. dynamism. The second approach is the kinetic approach which is the dominantly used in school. In this approach the equilibrium state is defined by the equality of the rates of the forward and reverse reaction. This model is relatively understandable and it provides a useful model for quick intuitive problem solving.

As already mentioned the features of rest, staticity and sidedness, if transfered to chemical equilibrium cause a basic misconception. The scientific representation of chemical equilibrium should consider all the components (both reactants and products) as one dynamic reversible system. A system approach is very important in the analysis of chemical equilibrium. The system approach is strengthening the conception of dynamism and reversibility, whereas the conception of sidedness is reminiscent of a static resting view.

The preconception of the concept of equilibrium is not the only source leading to possible misconceptions. There is also a buildup of knowledge from school formal studies in chemistry and other sciences that seem to cause dissonance with features of chemical equilibrium. These are numerous but in this discussion only those that pertain to the above mentioned misconceptions are discussed. One of these is the stochiometric relationship of the reacting species in a chemical reaction. The students view the stochiometric relationship as representing the relative amounts of the species actually present in the system. Another piece of knowledge is the nature of a "constant" whose value is constant for a whole category, whereas in chemical equilibrium it is only constant for one specified system at a given temperature.

The mentioned difficulties, in addition to other possible misconceptualizations, have been studied and mapped by different methods. The methods used were: word associations

tests (Gussarsky & Gorodetsky), free sorting of concepts (Gussarsky & Gorodetsky 1987), and specifically designed misconception tests (Gorodetsky & Gussarsky 1986, Wheeler 1978, Johnston, MacDonald & Webb 1977). These methods tried to evaluate the degree of influence of learning at school on the resolution of the conflicts among the various conceptions. A neccessary condition for this resolution to occur is the awareness of the teachers to the possible misconceptions. In this presentation results for students' problem solving concerning certain aspects of chemical equilibrium, taught by 4 teachers, are presented. The teachers were graded qualitatively on the basis of free sorting of 18 central concepts and an interview. The criteria for grading were the degree of awareness and conscious realization as to the role of the qualities of dynamism, reversibility and system thinking in the conception of chemical equilibrium. This reflected to what extent teachers actually provided the minimal conditions for the aquisition of the scientific conception. Teacher #1 had the closest conception to that of the scientific community. It is of interest to note that teacher #4 actually confessed that it did not occur to her that such problems exist and that she was not trying in any way to evaluate the students' conception regarding these aspests. This was considered as nonsensitivity to possible misconceptualization of the concept.

Another important means for conceptual change is providing examples. From the interview with the teachers we could learn about the examples or other means that were used to transmit the realization of a non static microworld. Teachers #1,#2 and #3 used common everyday examples such as a playing basketball team, that is kept at 5 although players are exchanged, in addition to chemical ones. All teachers used physical chemical phenomena, such as liquid - gas equilibrium, to illustrate the reversibility and dynamic aspects of the equilibrium state. In addition, chemical experiments such as the equilibration of

$$Fe^{+2}(aq) + Ag^{+1}(aq) \rightleftharpoons Fe^{+3}(aq) + Ag^{0}(s)$$

were performed. The reversibility of the reaction is illustrated by starting from different starting materials producing a mixture that contains all the possible components. This example has its shortcomings in that it illustrates the reversibility of the reaction in general but it does not necessitate reversibility and dynamism at the equilibrium state. A common analogy used by students is that of a pendulum that oscilates from side to side (i.e. is "reversible") till it comes to rest.

As said before, the microscopic behavior cannot be deduced from the macrophenomena and it is part of the model concerning the structure of matter. Thus there is an intrinsic difficulty in integrating this feature into the conception formed from everyday experience. It is hard to intertwine the two conceptual vines, that of public knowledge and that of personal knowledge (Pines & West 1986), and their conflict is leading to the introduction of misconceptions as to the features of chemical systems, in general, and the nature of the equilibrium state in particular.

Table 2 gives the grades achieved by students of an advanced course in high school chemistry in a questionnaire on specific aspects of chemical equilibrium (Gorodetsky & Gussarsky 1986). It shows that the post learning achievements on dynamism and reversibility are quite low. This is particularly evident in the analysis of one of the problems in the questionnaire. The problem regards a saturated solution of NaCl to which radioactive solid NaCl is added. After a while the phases are separated and the students are asked to judge as to where radioactivity will be found. Up to 24% of the students thought that radioactivity will be found only in the solid. In another problem regarding dynamism at the equilibrium state, up to 13% thought that, at equilibrium, the molecules are at rest. Although the numbers are not very high, they are surprising, because the students had studied the subjects of the molecular nature of matter, gas laws and behavior of solutions.

Table 2. The grades achieved by advanced chemistry students in a misconception test regarding certain aspects of chemical equilibrium.

Teacher	Dynamism & Reversibility	Sidedness	LeChatellier's Principle	Constancy of K
#1	57(34)**	70(60) *	71(34) **	52(15)**
#2	53(31)**	65(37)**	91(37) **	82(12)**
#3	56(28)**	58(42)	81(28) **	38(21)
#4	39(22)	45(45)	52(35)	70(10)**

Note: The numbers in brackets are prelearning grades.

* $p \leq 0.05$, ** $p \leq 0.01$

The feature of sidedness reflects the inability of treating the system as a whole - as a unity, but rather splicing it into constituents such as "reactants" and "products" or "left" and "right" sides. This conception, that probably originates from balancing a see-saw or balance is strengthened by the very nature of the taught subject of chemical equilibrium. There is constant shifting between system analysis and a detailed analysis of the constituents, piece by piece, as in calculations of constituent concentrations, or the application of the LeChatellier's principle. The moderate grades on sidedness (Table 2) indicate that this representation of "two sides" is quite abundant. This mental image probably gives rise to the comparatively good performance on some problems that require the application of the LeChatellier's principle. It is of interest that the performance on this aspect is differential; there is a marked difference in the performance on problems that are associated with changes on one side versus problems that require changes on both sides. A general problem of this kind would be: $A + B \rightleftharpoons C$, after the system reaches equilbrium some of B is removed and the system is let to reach equilibrium again. Usually two possible questions are asked; i. What will happen to the concentration of A (changes on the same side) ? ii. What will happen to the concentration of C (changes on the other side)? Table 3 gives some results of the grades on problems of this kind. The surprising point is the pre-learning performance rather than the post performance.

Table 3. The grades achieved by advanced classes in chemistry on problems regarding changes on different sides of the chemical equation.

Teacher	Changes on same side	Changes on other side
#1	74(35)	87(70)
#2	65(12)	71(35)
#3	63(38)	88(71)
#4	55(14)	73(38)

Note: The numbers in brackets are pre-learning grades.

There was no reason for the different grades achieved on the two kinds of problems on the pretest other than the mental image of balancing the other side when a disturbing element is introduced in an equilibrated system.

The conflict between the detailed analysis of the two sides of the reaction and system thinking is reflected via the confusion between stochiometric relations and the equilibrium constant. The students find it confusing to realize that the stochiometric relation, in itself can not determine the amounts of substances actually present at chemical equilibrium but that knowledge of the equilibrium constant is essential. Thus, when referred to the system

$$H_2\,(g) + I_2\,(g) \rightleftharpoons 2HI\,(g)$$

and asked what will be the molar concentration of HI at equilibrium, if started from 1 mole/liter of each H_2 (g) and I_2 (g) a probable answer will be 2 moles/liter. This answer is based on the stochiometric relationship without any consideration of the equilibrium constant of the reaction. It is of interest that this feature (Table 4) is dominant with about 40% of the studied population regardless of the nature of the teacher.

Table 4. Percentages of students that choose the different answers on the mentioned problem.

Teacher	2 Moles/liter of HI	More Information needed
#1	34(54)	55(14)
#2	53(59)	47(18)
#3	25(71)	75(13)
#4	46(60)	55(20)

Note: The numbers in brackets are pre-learning grades.

A similar compartmentalization of sides is probably inherent in dealing with the equilibrium constant itself. Although apriori it seems to be very simple to remember, that at a given temperature, K is constant, the performance on this aspect is not too high (Table 2). Some students do believe that a change in concentration of either reactants or products will result in a change in K. This again may creep from the mental attention to the ratio of reactants and products concentration, rather than grasping the system as a unity with a constant "equilibrium constant".

As already mentioned, there is a dilemma as to how to introduce the concept of chemical equilibrium - as an extention of the already existing conception of equilibrium or as a new concept with its attributes. It seems from the research reports that there is no clear differentiation between the concepts of equilibrium and chemical equilibrium, and inspite learning, attributes of one concept are assigned to the other. This persistance probably rests on psychological factors, such as the nature of procedural knowledge or set effect, on class evaluation that is insensitive to detect either preconceptions or misconceptions, and on the lack of need for change by the student. The literature on conceptual change (Hashweh 1986, Posner 1983) agrees that conceptual change is unlikly to occur unless the individual views his conception with some dissatisfaction. Whether the origin of this is anomalies as perceived by the students or some other conceptual conflict, it is a crucial point for conceptual accomodation. The question is how can the conflict and resolution be achieved most efficiently. If the new concept of chemical equilibrium is being introduced with reference to "equilibrium", there is always the possibility for selective assimilation as well as rejection processes to take place, avoiding the stage of conflict. The time table should be first to establish the two complementary conceptions saparately and letting the student himself or by outside assistance face the possibly conflicting aspects. It is the authors' belief that chemical equilibrium should be taught on its own merrits, stressing the systems it is concerned with and its attributes. This calls first for the awareness of the teachers to the problems involved. At a later stage it is also their responsibility to call students' attention to the possible conflicting aspects of "equilibrium" and "chemical equilibrium" and with mutual help reach a resolution, i.e. abstraction to a more formal concept of equilibrium. The conflicting situation not only serves as an initiator to conceptual change, it also provides good conditions for deep and seriuos information processing.

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Physics Novices' Judgments of Solution Similarity: When Are They Based on Principles? *

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This study is concerned with the types of information used by problem solvers in deciding whether or not two problems should be solved similarly. The consensus of numerous studies (Chi, Feltovich & Glaser, 1981; Niegemann & Paar, 1986; Schoenfeld & Herrmann, 1982) of expert and novice problem solvers is that experts sort problems primarily on the basis of the problem's "deep structure" (e.g. principles, concepts or heuristics that could be applied to solve the problem), while novices sort problems mainly according to "surface features" (e.g. problem jargon and descriptor terms). However, the extent to which experts might rely on surface features, and the extent to which novices might rely on deep structure in their decision making processes have not been examined in any detail. The following two questions will be addressed here: 1) Are novices capable of using deep structure to categorize problems according to similarity of solution, and if so, under what conditions? and 2) Do experts consistently categorize problems according to similarity of solution solely on the basis of deep structure, and if not, what other criteria do they use and when do they use it?

The study of the mental representation of word problems is important for both cognitive and pedagogical reasons, because the representation of a problem plays an important role in generating a problem solution. Although there is some debate concerning the exact mechanism by which a problem is represented (McDermott & Larkin, 1978; Chi, et al., 1981), there is general agreement that the mental representation of a problem is built as the problem is read, the formation of which implies some categorization of the problem. For experts at least, the categorization process suggests possible solution strategies (Hayes & Simon, 1976; Hinsley, Hayes & Simon, 1978; Newell & Simon, 1972; Simon & Simon, 1978). Therefore, the appropriateness of the categorization can directly influence the ability to generate a successful solution to the problem.

In accord with these ideas, research in the domain of physics indicates that ability to sort problems according to deep structure increases with level of expertise: Ph.D. physicists are the most capable of consistently categorizing problems according to deep structure, followed by advanced graduate students and college seniors, with college freshman relying mainly on surface features for categorization (Chi, et al., 1981). This suggests that if novices' categorization skills could be improved, improvement in their problem solving skills might follow. The insights gained from the present and other studies could prove instrumental in designing pedagogical approaches aimed at improving novices' categorization skills.

A common paradigm that has been used to study expert and novice differences in the perception of solution similarity among word problems is the card sorting task: subjects are given a stack of index cards, each containing a written word problem, and are asked to sort the cards into piles of problems that would be solved similary. The subjects are not expected to actually solve the problems before sorting them. Although this technique has led to a greater understanding of how novices and experts classify problems, there are certain inherent limitations which make it less than optimal for the investigation of more detailed questions. The task demands that subjects find a categorization scheme that deals with all problems simultaneously; this usually leads to considerable variation in the number and composition of the possible problem piles making data analysis cumbersome. More importantly, it is difficult, if not impossible, to extract detailed information concerning the competing influences of surface features and deep structures on the categorization schemes used by subjects of varying levels of expertise.

In order to gain more detailed information concerning the competing influences of deep structure and surface features on problem categorization, we designed a similarity judgment task not unlike one that has been used in studies of object categorization (Rosch and Mervis, 1975; Mervis, 1980; Mervis and Crisafi, 1982). In our task, the subject is presented with three problems, a model problem and two comparison problems, and asked to decide which of the two comparison problems would be solved similarly to the model problem. The relationships of the comparison problems to the model problem could be varied, so as to assess the relative contributions of surface features and deep structure in the decision making process.

Any given comparison problem could match the model in surface features (S), deep structure (D), both surface features and deep structure (SD), or neither surface features nor deep structure (N). To form each 3-problem item, comparison problems were paired together with the following constraint: one and only one of the two comparison problems could match the model problem in deep structure, thereby guaranteeing that there would always be a single correct answer. This meant there could be four types of comparison problem sets: 1) S-D, 2) S-SD, 3) N-D, and 4) N-SD.

If it is true that experts always base their categorization decisions on deep structure, then for each 3-problem item experts should select the comparison problem that matches the model problem in deep structure. For novices, the pattern of results should be quite different, assuming that novices base their categorization decisions solely on surface features. When only one of the two comparison problems matches the model problem in surface features, as for pairs S-D and N-SD, the novice should choose the comparison problem matching the model problem in surface features 100% of the time. This strategy would predict that the novice should be always wrong when given an S-D comparison problem pair, since the surface feature problem is a compelling distractor, and should be always right when given an N-SD pair, because surface features co-occurs with deep structure. However, if both of the comparison problems share surface features with the model problem (i.e. an S-SD pair) or if neither shares surface features with the model problem (i.e. an N-D pair), then performance should be random, or 50% correct. In neither case should there be a preference for one comparison problem over the other if the subject only considers surface features, since both alternatives should appear equally good or equally poor.

Method

Subjects

Novices. Forty-nine undergraduate students at the University of Massachusetts who had completed the first semester physics course for majors or for engineers, and received a grade of B or better, participated in this study. Seven of these subjects were eliminated, five due to attrition, one because of an extremely low level of performance, and one because of ceiling-level performance, leaving a total of 42 subjects. The subjects were participating in a study with ten hour-long sessions, for which they were paid fifty dollars. The task reported here was performed in the second session.

Experts. Eight Ph.D. physicists and two advanced physics graduate students who had passed the qualifying exam and were nearing completion of the Ph.D. participated as experts in this study. Two of the experts were eliminated because of low levels of overall performance (56% and 69% correct versus a mean of 91% for the remaining eight) and because their patterns of performance differed substantially from those of the remaining eight experts.

Task Items

Each task item was composed of three elementary mechanics problems that were similar in type and level of difficulty to problems in Resnick and Halliday (1977), a commonly used introductory text in college physics. The problems were each three to five lines long and contained only text (no pictures or diagrams). For each item, one of the three problems was identified as the model problem, while the other two were the comparison problems. The subjects were to indicate which of the two comparison problems they believed "would be solved most similarly" to the model problem.

Types of Comparison Problems. A comparison problem could share different numbers of and types of characteristics with its model problem. Four types of comparison problems were possible. The comparison problem could match the model problem in: 1) surface features, meaning that the objects and descriptor terms that occur in the problems are similar, 2) deep structure, or the physical principle that could be applied to solve the problem, 3) both surface features and deep structure, or 4) neither surface features nor deep structure. These four types of comparison problems were termed S, D, SD, and N, respectively. The following is a sample model problem and the four comparison problems that were constructed to accompany the model:

Model Problem

A 2.5 kg ball of radius 4 cm is traveling at 7 m/s on a rough horizontal surface, but not spinning. Some distance later, the ball is rolling without slipping at 5m/s. How much work was done by friction?

S Alternative

A 3 kg soccer ball of radius 15 cm is initially sliding at 10 m/s without spinning. The ball travels on a rough horizontal surface and eventually rolls without slipping. Find the ball's final velocity.

D Alternative

A small rock of mass 10 g falling vertically hits a very thick layer of snow and penetrates 2 meters before coming to rest. If the rock's speed was 25 m/s just prior to hitting the snow, find the average force exerted on the rock by the snow.

SD Alternative

A 0.5 kg billiard ball of radius 2 cm rolls without slipping down an inclined plane. If the billiard ball is initially at rest, what is its speed after it has moved through a vertical distance of .5 m?

N Alternative

A 2 kg projectile is fired with an initial velocity of 1500 m/sec at an angle of 30 degrees above the horizontal and height 100 m above the ground. Find the time needed for the projectile to reach the ground.

In constructing allowable pairs of comparison problems to accompany a model problem, two comparison problems of different types were paired together, such that one of the two comparison problems matched the model problem in deep structure, while the other did not. This constraint led to four types of comparison problem pairs: 1) S-D, 2) S-SD, 3) N-D, and 4) N-SD. Thus, a response was considered correct if the subject chose the comparison problem that matched the model problem in deep structure.

Model Problems. In addition to varying the types of comparison sets, the principles used for solution and the objects in the model problems were also varied. There were

initially eight model problems, later narrowed to five. Each model problem used in the study appeared four times, once with each of the four types of comparison sets. The deep structure and surface features of the eight model problems were as follows:

Model #	Deep Structure	Surface Structure
1	Forces (Statics)	Spring, Friction
2	Energy	Spring
3	Linear Momentum	Two Blocks, Spring
4	Work-Energy	Rolling Ball, Friction
5	Angular Momentum	Rolling Ball, Friction*
6	Angular Momentum	Spinning Stick, Collision*
7	Linear Momentum	Collision*
8	Forces (Dynamics)	Friction, Motion

* Not used in the final analyses

Thirty-two items were developed, each composed of one model problem and two of its comparison problems. Each of the eight model problems appeared four times, once with each of the four types of comparison problem sets.

Preliminary analysis of the expert data indicated that the eight model problems were not of equal difficulty, $F(7,49) = 6.04$, $p<.0001$. Mean performance for the eight model problems ranged from 100% to 59% correct. Since it was important to establish a baseline of problems in which experts consistently agreed on the answer, the three model problems in which the lowest mean performance was observed were elimnated. These were model problem 5 (72% correct), model problem 6 (69% correct), and model problem 7 (59% correct). After eliminating the 12 items that used model problems 5, 6, and 7, there was no longer a model problem main effect in the expert data, $F(4,28) = 1.65$, $p=.1885$. Eliminating these items from the novice analyses did not significantly alter any results.

Procedure

The experiment was run on IBM compatible PC's, to which a three-key response unit was attached. The subject was told to read carefully the model problem and the two comparison problems which would appear below it. Then they were to decide whether comparison problem A or comparison problem B would be solved most like the model problem, and press either the button labeled A or the button labeled B on the response unit to indicate their decision. The items were presented in random order, with no limit imposed on time to respond. After every 5 items, the subject was given the opportunity to take a brief rest. Most subjects completed the task within 45 minutes.

RESULTS

The performances of the 42 novices and the 8 experts were compared in a 2 (Groups) by 4 (Comparison Types) by 5 (Model Problems) analysis of variance. Overall, the experts chose the alternative that matched the model in deep structure (91% of the time) significantly more often than did the novices (60% of the time), $F(1,49) = 83.03$, $p<.0001$. Thus, in general, experts were better able to determine whether two problems would be solved similarly.

Comparison Types

Performance on the four Comparison Types indicated that they were not of equal difficulty, $F(3,147) = 31.31$, $p<.0001$. Averaged over all 50 subjects, the means for each Comparison Type (in order of difficulty) were: SD: 33% correct, SSD: 62% correct, ND: 72% correct, and N-SD: 92% correct. However, since there was also a Group x Type interaction, as predicted, $F(3,147) = 12.31$, $p<.0001$, it is more meaningful to discuss the differences between Comparison Types within each group separately.

Experts. If experts base their decisions about the similarity of solution of two problems on deep structure only, then Comparison Type should have no influence on experts' performance, and the predicted means for each Comparison Type are 100% correct. However, there was a significant main effect of Comparison Type, $F(3,21) = 3.77$, $p=.0261$, indicating that surface features do exert some influence on experts' decision making processes (see the means for the four Comparison Types in Table 1). Note that the relative ordering of these means is the same as that for the novices (see below), suggesting that experts tend to make errors in the same kinds of conditions as novices.

However, this tendency for experts to make more errors in certain classes of items is only suggestive, since there were no significant differences between any of the four Comparison Types when they were compared pairwise. In fact, the 99% confidence intervals for three of the four Comparison Types do include the predicted 100% correct: SD: $70\%<M<95\%$, S-SD: $76\%<M<100\%$, N-D: $88\%<M<100\%$, and N-SD: $92\%<M<100\%$. Hence, for this set of model problems, experts appeared to base their decisions mainly on deep structure.

Table 1: Predicted and Observed Performance of Experts and Novices

Comparison Type	Experts Predicted	Experts Observed	Novices Predicted	Novices Observed
S-D	100%	82%*	0%	24%*
S-SD	100%	90%	50%	56%
N-D	100%	95%	50%	68%*
N-SD	100%	98%	100%	91%*

* Differs from the predicted value (p<.01)

Novices. Assuming that novices base their decisions on surface features alone, then Comparison Type should have a major influence on performance, as our initial predictions suggest. In fact, there were substantial differences in performance among the four Comparison Types, F(3,123) = 114.86, p<.0001 (See the means for the four Comparison Types in Table 1). All pairwise comparisons were significant at p<.0282 (the highest p value was p=.0047, here controlled for 6 tests).

Two observations can be made about these data regarding the role played by surface features in novices' decision making processes. On the one hand, the presence of a comparison problem that matches the model problem in surface features only adds to the difficulty of making a decision about whether two problems would be solved similarly, regardless of whether or not the second comparison problem matches the model problem in surface features in addition to matching it in deep structure: the combined mean of the two Comparison Types with surface feature distractors, S-D and S-SD, was 40% correct versus 80% correct for N-D and N-SD combined. On the other hand, if a comparison problem matches the model problem in both surface features and deep structure, then the decision is facilitated: the combined mean of the two Comparison Types in which the surface feature match co-occurs with the deep structure match, S-SD and N-SD, is 74% correct versus 46% correct for S-D and N-D. Clearly, surface features not only play a major role in novices' decision making processes, but can both help and hinder the process of deciding whether two problems would be solved similarly. However, the observation that surface features play such a large role does not necessarily imply that deep structure plays no role in novices' decisions. This will be the next issue to be discussed.

Recall that in the introduction to this paper, we made a set of specific predictions for the performances of novices in each of the four Comparison Types, assuming that they considered only surface features in their decision making process. Briefly, these predictions were: 1) S-D: always choose S, resulting in 0% correct, 2) S-SD: S and S-D are equally good choices, resulting in 50% correct, 3) N-D: N and D are equally poor choices, resulting in 50% correct, and 4) N-SD: always choose SD, resulting in 100% correct. However, the confidence intervals for each Comparison Type suggest that the assumption that novices consider only surface features in deciding on solution similarity does not give a complete account of the data; deep structure does seem to play a role in novices' decision making processes.

For S-D items, the 99% confidence interval was 17%<M<31% correct, versus the predicted 0% correct. The fact that the lower limit of the confidence interval is so far above 0% correct suggests that for at least some model problems, novices were able to utilize deep structure in making their decision. In fact, for S-D items, the mean percent correct for the five model problems ranged from 7% for model problem #2 (Spring/ Energy) to 50% for model problem #1 (Spring-Friction/Force). Thus, although novices displayed a definite preference toward selecting the surface feature distractor in S-D items, by no means did all subjects ignore deep structure in their decisions, even in the S-D condition where they should have been most prone to do so.

Performance on the S-SD items was no different from the predicted value of 50% correct; the 99% confidence interval was $48\% < M < 64\%$. There was considerable variablility in the mean percent correct scores for the five model problems, which ranged from 24% for model problem #4 (Rolling Ball-Friction/Energy) to 78% correct for model problem #8 (Friction-Motion/Forces). In general, adding matching surface features to the deep structure alternative did increase the probability that subjects would choose it over the surface-feature-only distractor, but subjects were not significantly more likely to choose the deep structure alternative over the surface features distractor.

In contrast, performance in the N-D items, where all surface feature similarity was eliminated from both comparison problems, indicates that novices chose the deep structure alternative at a rate significantly above the predicted level of 50%; the 99% confidence interval was $60\% < M < 76\%$ correct. Mean performances on the five model problems were 64% correct and higher, except for model problem #2 (Spring/Energy), on which performance was 21% correct. These findings suggest that when an item did not permit a decision based on matching the model problem to one of the comparison problems on surface features, subjects were able to determine which alternative would be solved similarly to the model problem more often than not. In order to do so, subjects had to be able to make some use of the problems' deep structure.

As expected, when surface features and deep structure co-occurred in a comparison problem and the other comparison problem matched the model problem on neither surface features nor deep structure, performance was quite high.

However, the confidence interval in these N-SD items did not include 100% as predicted; the 99% confidence interval was $86\% < M < 96\%$ correct. For this Comparison Type, it is actually not clear why subjects would ever have chosen the N alternative, since it did not share any characteristics with the model problem. It is possible that some unintentional similarity between the model problem and the false alternative was noted or that the subjects did some guessing.

Experts versus Novices. The mean performance of the experts was significantly higher, at $p < .0001$, than that of the novices for each of the four Comparison Types, except for the N-SD type where performance was quite high for both groups (see Table 1). As noted earlier, although the experts' performance is higher than the novices' in all four Comparison Types, the relative ordering of the means across the four Comparison Types is the same for both experts and novices.

Model Problems

The analysis of the combined expert and novice data indicates that the five Model Problems were not of equal difficulty, $F(4,196)=4.65$, $p=.0013$. Recall that for experts, there was no significant difference among the five Model Problems. However, for novices, the Spring-Friction/Forces items (76% correct) were easiest, followed by Friction-Motion/Forces (63% correct), Rolling Ball-Friction/Energy (60% correct), Two Blocks-Spring/Linear Momentum (59% correct), and Spring/Energy (41% correct). In addition, there was a significant interaction of Comparison Type and Model Problem, $F(12,588)=4.34$, $p < .0001$, indicating that the ordering of means for the five Model Problems was not the same for each of the four Comparison Types. These results suggest that the difficulty of making a decision based on deep structure may be related to the context of the problem. Subjects probably perform better with problem situations that they understand better.

DISCUSSION

This study focused on investigating the extent to which experts and novices use deep structure and surface features to categorize physics problems. We hypothesized that experts would categorize problems solely on the basis deep structure and that novices would categorize solely on the basis of surface features. According to these hypotheses, quite different patterns of performance were predicted for experts and novices. However, neither experts nor novices behaved completely in the manner experted.

Although the five model problems selected for analysis were those on which experts categorized predominantly on the basis of deep structure, there is some evidence that experts were distracted by surface features. Experts' lowest performance occurred on the S-D items, where surface features and deep structure were in direct competition, indicating that even experts can experience difficulty ignoring surface features. This conclusion is supported by the fact that we found it necessary to drop two experts because their patterns of performance were unlike those of the other eight experts, and were, in fact, more similar to the performance patterns predicted for novices. Since there is no obvious explanation for why these two experts sorted in the manner they did, their performance serves as a warning to those researchers inclined to make global statements on the basis of small numbers of experts: not all "experts" behave like the "typical" expert.

On the other hand, the results from the novice data suggest that novices do not focus exclusively on surface features when categorizing problems. In the condition where novices should have been most distracted by surface features, namely the S-D items, the deep structure comparison problem was not infrequently chosen. Further, when no matching surface features were present in the comparison problems, namely the N-D items, novices were able to choose the comparison problem that matched the model problem on deep structure over the no-match comparison problem with a frequency that was significantly higher than the predicted 50%. The significance of these results is enhanced by the fact that they reflect consistent trends in performance across subjects. Thus, although our findings basically support the conclusion that surface features serve as the most compelling attributes used by novices in categorizing physics problems, our findings also suggest that novices do not entirely ignore deep structure. Under the right conditions, it might be possible to develop further novices' tendency to consider deep structure through instruction.

Categorization on the basis of surface features has generally been viewed as an impediment to successful problem solving by the cognitive research community, yet many mathematics and science textbooks do not develop an approach that would lead students to recognize the arbitrariness of surface feature cuing. Our results suggest that surface features in word problems could be used to hone students' problem solving prowess. The assumption made by most traditional approaches to teaching problem solving is that it is easiest for students to recognize that two problems would be solved similarly if they share surface features. Our research indicates that this is a valid assumption, suggesting that it makes sense to give students problems to solve that match worked-out examples in both deep structure and surface feature attributes while they are learning a new topic. However, if one stops here, students will likely get the impression that surface features are more important than they really are in deciding on a solution strategy for a problem. It is therefore important to vary the problem context so as to eliminate surface feature similarity as the student learns to apply the problem solving technique to new types of situations. Finally, problems can be given that match the earliest examples in surface features, leading students to believe that they should be solved similarly, but which actually require a different solution strategy. In other words, the students' concept of what problem cues can be used in selecting particular problem solving strategies should be challenged in order to promote flexible problem solving. The practice of grouping problems with similar surface features together, as is commonly done in many popular mathematics textbooks under headings such as "age problems" and "per cent problems," is in direct conflict with this goal since it is likely to mislead students into thinking that all such problems share a common solution strategy.

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Declarative and Procedural Knowledge and Isomorphism of Speed Problems

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The studies on the solution of speed problems were conducted in either informal or formal settings. In the first kind of studies, subjects are presented with a concrete situation and they are required to make judgements regarding the relative speeds, durations and distances of the moving objects. The solutions and judgements can be either qualitative or semi-quantitative, which are based on the perceptual physical relationship between distance, speed, and duration, since the numerical values of the variables are not given (e.g., Piaget, 1970; Siegler and Richards, 1979). In the second type of studies the subjects are required to solve speed problems which are presented verbally and include numerical values for the variables. Here the solutions can be quantitative and involve mathematical symbols and procedures (e.g., Simon and Simon, 1978; Gorodetsky et al., 1986). The study of Gorodetsky et al. (1986) makes it clear that the most serious deficiency in students knowledge structures were the great variety of misconceptions in the time concept and its use and in their inability to represent correctly time and distance relations which require to use relationship between starting times, starting points, or between directions of the moving objects. Possible interpretations to the latter result are: (a) the time concept was not developed to the degree required to solve speed problems of non-trivial nature, and therefore was missing from the students' declarative knowledge, or (b) the rule expressing the relationship among the three concepts, either was not acquired, had inappropriate procedural attachments or components, or could not be used under the specific conditions in the presented speed problems.

The interesting issue regarding the acquisition of the three-concept relationship among speed, duration, and distance was raised by the conflicting findings from the two types of studies, i.e., in informal and formal settings. It was claimed by researchers in the informal studies that the three concepts were fully integrated by the age of 12. However, the study by Gorodetsky et al. (1986) which belong to the formal type, demonstrated that by the age of 17 most students (over 60%) were unable to use the three-concept relationship in the solution of speed problems. A plausible explanation is that the difference between these conclusions is due to the different contexts in which the judgements were made. It can be hypothesized that even if the three-concept relationship was acquired, it can be used differently in (a) concrete, familiar everyday situations which neither require nor enable the use of this relationship in the precise manner (e.g., by representing it symbolically) and (b) relatively unfamiliar situations in which the numerical values of the variables are given and the three concepts must be symbolized, their relationship expressed mathematically, and is manipulated algebraically.

This analysis points that a sharp contrast exists between the findings from the research on algebraic solution of speed problems and the developmental claims that the three concepts and their physical relationship are mastered by the age of 12 or 13.

This discrepancy and the other findings from Gorodetsky et al. initiated the analysis described in the next section. This analysis explores the semantic structure of speed problems and their solutions in the formal settings, enables us to understand the complex structure of speed problems, and helps to identity bugs in the representation of speed problem elements. Based on this anlaysis, we define an isomorphism between speed problems, which faciliates the transition from a difficult problem to an easier isomorphic one.

DECLARATIVE AND PROCEDURAL KNOWLEDGE OF SPEED PROBLEMS

In Harel and Hoz (in preparation), the following categorization of speed problem elements were made. A problem element is a verbal statement describing a knowledge state. A problem element can be either a quantity element or a relation element. A quantity element reflects an aspect of at least one quantity (e.g., "the speed of a car is 80 km/h," or "what is the time required for car 1 to get to A?"). A relation element pertains to a relation that exists between quantity elements or directions of motions (e.g., "the speed of car 1 is larger than that of car 2").

Problem elements can either be mentioned in the problem statement or inferred from other elements, thus being in the problem space (Anderson, 1985). A mentioned element can be either specified in the problem statement, i.e., include a datum, or unspecified in the problem statements (e.g., the time required for the car to traverse the given distance does not appear in the problem statement). The set of mentioned unspecified elements contains a subset of missing value elements, both quantities and relations, whose numerical values are required (for example, "find the distance between A and B").

Problems in various domains may include different problem elements. Thus, speed problems are characterized as incorporating the following specific problem elements: (1) Quantities. Duration (DU): The total period for time in which the car was in motion; Stationary (ST): The total period of time the car was stationary; Time (T): The period of time that passed from the moment the car started to the moment the car arrived (i.e., duration + stationary); Chronological Time; Speed; and Distance; and (2) Relations between the former quantities, or between the directions of motion (i.e., same or different).

The relations in speed problems on two moving objects can be classified according to the quantities included in the relation: Du-Du: Relation between duration quantities. T-T: Relation between time quantities. St-St: Relation between stationary quantities. Dis-Dis: Relation between distance quantities. S-S: Relation between speed quantities. These relations can be classified into three categories: *basic relations*, *direct relations*, and *indirect relations*. The involved quantities may belong either to one car or to two cars.

Basic relations are the simplest and temporal relations that indicate whether the following five quantities are same or different: Starting times (STs), terminal times (TTs), starting points (SPs), terminal points (TPs), and directions of the two cars (Dirs). Basic relations are presented either explicitly or implicitly in every speed problem involving two moving objects, and serve to infer (according to logic rules) the relations among problems quantities that are needed to solve the problem. For example, if the two cars have the same starting and terminal chronological times, then they travel for the same period of time.

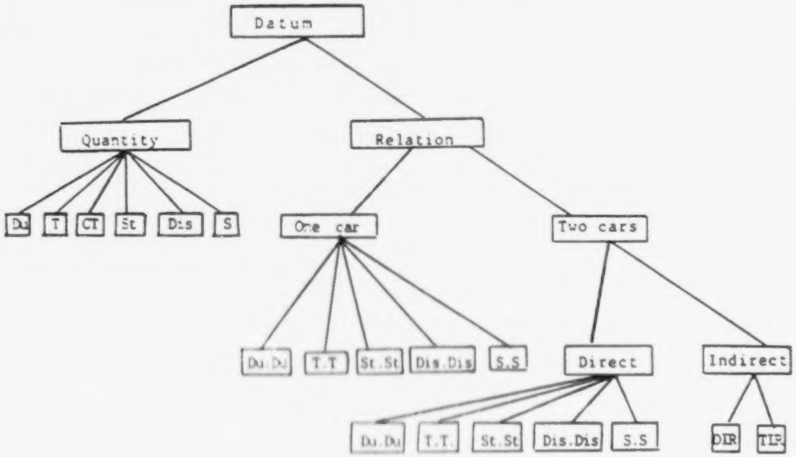
The remaining specified relations that pertain to quantities of both cars are either direct or indirect. A *direct relation* is one which can be transformed into an equation (or inequality) without using any basic relations. An *indirect relation* is one which cannot be transformed into an equation unless certain basic relations have been used.

There are two groups of indirect relations: The *Distance-Indirect-Relations* (DIR), and the *Time-Indirect-Relations* (TIR). DIR relations are those which involve the time-determining basic relations between STs (starting times) and TTs (terminal times). Note that speed relations are basic relations since constant speed does not have starting value, terminal value, or direction.

The derivation of both DIR and TIR relations is achieved through the use of declarative and procedural knowledge which was described up to here. The solution of speed problems necessitates that DIR and TIR relations be inferred from basic relations, represented individually and used for the representation of the whole problem, and translated into equations.

The declarative and procedural knowledge required for the solution of speed-problems involve the following processes: (1) the establishment of correspondence between problem elements and the problem quantities, and (2) the formation of their representations and the whole problem representation. For this analysis, we assume that the process by which the numbers appearing in the problem statement are linked with the appropriate mentioned problem elements is essential for inferring other problem elements (either quantities or relations). Similar arguments were proposed by Kamii (1970). Such links are formed in order to coherently and completely represent certain problem elements, as well as the whole problem situation. The possible kinds and their subkinds of elements to which a datum in a speed problem can referred to are depicted in Figure 1.

Figure 1. References of a datum in a speed problem.



The formation of representations of a problem as a whole is based on the first process, i.e., the correspondence between these elements. While the first process is related to the understanding of the role and nature of the mentioned elements, the second process is related to inferences which the problem solver makes from these elements. An outcome of this process might be making symbolic representations for the inferred missing value elements. To clarify this point consider the following situation: "Car 1 started at 4:00 and car 2 started at 6:00. Car (1) arrived 2 hours after car 2. Car (2) arrived at 11:00." In order to represent this situation the problem solver has first to understand the situation and identify each datum in the mentioned elements:

1. 4:00 represents chronological-starting-time quantity of car (1).
2. 6:00 represents chronological-starting-time quantity of car (2).
3. 11:00 represents chronological-terminal-time quantity of car (2).
4. 2 represents a relation between the terminal times of cars (1) and (2); the terminal time of car (1) is (2) hours greater than that of car (2).

The problem solver can now turn to infer the indirect relations from these specified elements.

5. Elements 2 and 3 imply that the duration of car (2) is 5 hours.
6. Elements 1 and 2 imply that car (1) traveled two hours before car (2) started.
7. Element 4 implies that car (1) traveled 2 hours after car (2) had arrived.
8. Elements 3 and 7 imply that car (1) arrived at 1:00.
9. Elements 1 and 8 imply that the duration for car (1) is 7 hours.

The kinds of inferences in more complex situations are depicted by the possible symbolic representations of problem elements in Figure 2 and Table 1.

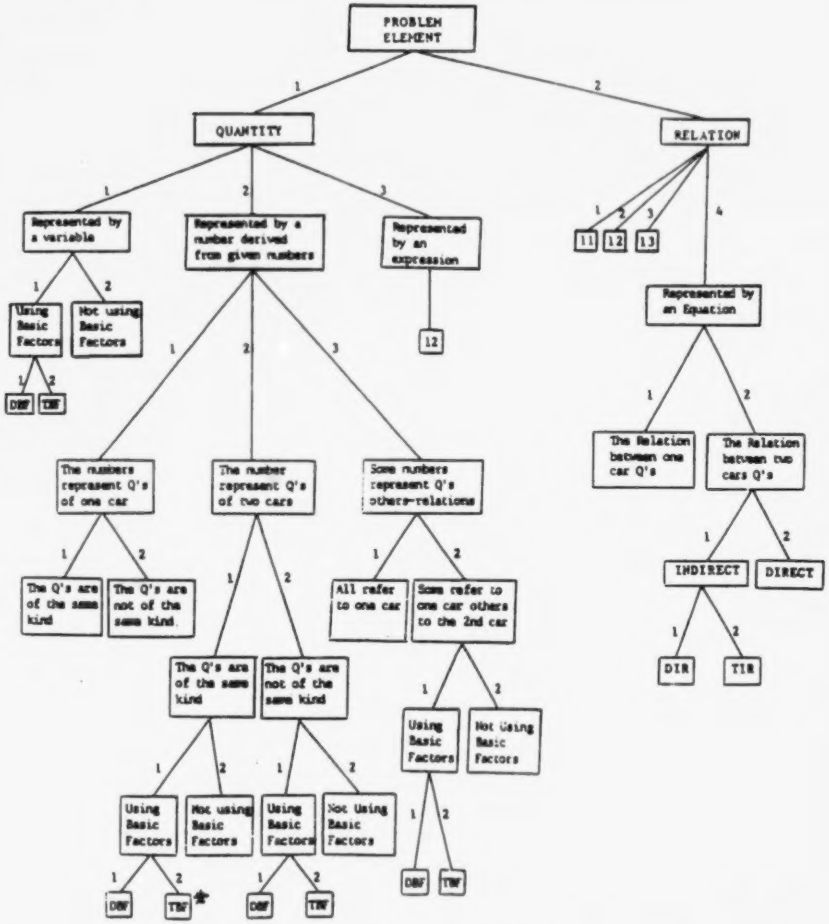
There are four different ways to represent problem elements:
(1) by a variable, (2) by a given number or one derived from given numbers, or (3) by an algebraic expression that includes numbers or variables, but not only numbers, or (4) by an equation. The latter possibility holds for relation elements but not for quantity elements. (See Figure 2 and Table 1.)

Figure 2 shows possible ways of representing problem elements symbolically. We labeled the links between the nodes to allow the identification of the nodes. For example, the node, "Representation by a number derived from given numbers", is identified as 12 since the links between its consecutive nodes are labeled by 1 and 2; or the node "TBF" marked by * is coded by 122112. Examples for some of these nodes and the symbolization process involved are given in Table 1.

The representation by a variable can be achieved by either using basic relations, namely, Distance-Basic-Relations (DBRs) or Time-Basic-Relations (TBRs), or not using basic relations.

When the representation is done by numbers that are given or were derived from numbers given in the problem statement, or by an expression, there are three cases: (a) The numbers or expressions represent quantity elements of one car, (b) the numbers or expressions represent quantity elements of two cars, or (c) some numbers or expressions represent quantity elements and others represent relation elements. In the cases (a) and (b), the quantities can be either of the same kind (for example, all of them are time quantities) or of different kinds (for example, one quantity is a speed and the other is a distance quantity). In case (b), the involved relations are either DBRs, TBRs or indirect relations. In case (c) it can be that all the quantities and relations refer to one car, or some refer to one car and others to the second car. In the latter case they can be either DBRs, TBRs or indirect relations.

Figure 2: References of a problem element in a speed problem.


PROBLEM ELEMENT
1
2
QUANTITY
RELATION
1
2
3
Represented by a variable
Represented by a number derived from given numbers
Represented by an expression
1
2
3
4
11
12
13
1
2
Using Basic Factors
Not using Basic Factors
12
Represented by an Equation
1
2
DBF
TBF
1
2
3
1
2
The numbers represent Q's of one car
The number represent Q's of two cars
Some numbers represent Q's others-relations
The Relation between one car Q's
The Relation between two cars Q's
1
2
1
2
1
2
1
2
The Q's are of the same kind
The Q's are not of the same kind.
All refer to one car
Some refer to one car others to the 2nd car
INDIRECT
DIRECT
1
2
DIR
TIR
The Q's are of the same kind
The Q's are not of the same kind
1
2
Using Basic Factors
Not Using Basic Factors
1
2
1
2
Using Basic Factors
Not using Basic Factors
Using Basic Factors
Not Using Basic Factors
1
2
1
2
1
2
DBF
TBF
DBF
TBF
*
DBF
TBF

Note: Examples for problem elements in some of the nodes are given in Table 1.

Table 1

NODE		PROBLEM ELEMENT	SYMBOLIZATION PROCESS	NOTES
1112	a	Two cars left at the same time points A and B and met at C.	The duration of one car is x hour the duration of the second car is also x hours.	In order to represent the two durations by the same variable (x), one needs to use the given basic factors. In this case TBF's.
112	a	As in 1112	The duration of one car is x hours, the duration of the second car is y hours.	Basic factors are not involved in this symbolization process.
	b	Two cars left the same point, and traveled in different directions, They stopped at two points 200 km apart.	One car traveled x km, the other car traveled y km.	As in 112a
1211		The car left at 8:00 and arrived at 12:00.	The duration of the car is 12 - 8 (= 4) hours	The duration quantity was derived from two given quantities of the same kind, and both refer to one car.
1212		The car traveled 5 hours at speed of 40 km/h.	The distance that has been traveled by the car is 40 x 5 (= 200).	The distance quantity has been derived from two given quantitites of different kinds, but both refer to one car.
122111		Two cars started one toward the other. One car started from point A and traveled 70 km. The other car started from point B and traveled 80 km.	The distance between A and B is 70 + 80 (= 150) km.	The distance quantity has been derived from two given quantities of the same kind and refer to the two cars. DBF's are involved in the process of deriving the quantity 150.

NODE		PROBLEM ELEMENT	SYMBOLIZATION PROCESS	NOTES
122112		Two cars started at 8:00 and met at 12:00	The duration of each car is 12 - 8 (= 4).	As in 122111. Instead of distance, here we have duration. Instead of DBF's, TBF's are involved here, since the words "started" and "met" must be translated to relations between the starting times and the terminal times respectively.
12212		One car started at 8:00 and arrived at 12:00. The second car started at 9:00 and arrived at 11:00.	The two cars traveled 4 hours.	TBF's are not involved in this symbolization process.
122211		Two cars left the same point and traveled in opposite directions. After 2 hours the distance between them was 100 km. One car traveled at 20 km/h.	One car traveled 100 - 20 x 2 (= 60).	The distance quantity of one car has been derived from different kinds of quantities which refer to the two cars. The derivation involves DBF's.
1231		One part of the distance between A and B has been traveled by a car at speed of 50 km/h in 2 hours. The second part which is 10 km greater than the first part, has been traveled in 3 hours.	The speed of the car at the second part of the distance is $\frac{50 \cdot 2 + 10}{3}$	The speed quantity has been derived from numbers representing quantities and relation.
12322		Two cars traveled 150 km in 2 hours One car without stationary the second car with stationary. The speed of the first car is 5 km/h more than the speed of the second car.	The second car stationed $\frac{150 - 2 \cdot (\frac{150}{2} - 5)}{\frac{150}{2} - 5}$	The speed quantity has been derived from numbers representing quantities and relation.
1312		The car traveled the distance between A and B at speed of 75 km/h.	The duration of the car is $\frac{x}{75}$.	The speed quantity has been represented by an expression, when the items in the expression represent quantities of different kinds.

NODE	PROBLEM ELEMENT	SYMBOLIZATION PROCESS	NOTES
132111	As in 112_b	One car traveled x km, the other car traveled 200 - x km.	The distance quantity of the other car has been represented by an expression. This expression has been constructed by ugin DBF's.
132212	Two cars started one toward the other from two points. One traveled at 75 km/h, the other traveled at 80 km/h.	The distance between the departure points is $75t \cdot t + 80 \cdot t$ when t is the duration of the two cars.	TBF's has been used when t has been taken as a variable denoting the duration of the two cars. Moreover the process of symbolizing the distance involves DBF's.
133212	Two cars left at the same time at point A to go to point B, 120 km apart. The speed of one car is greater than the speed of the other car. Therefore she arrived a ½ hour earlier to B.	The speed of one car is $\frac{120}{t}$, and the speed of the second car is $\frac{120}{t+\frac{1}{2}}$ where t is the duration of the first car.	The process of symbolizing the speed at the second car involves TBF's.
2112	One car is faster than the other car.	The difference between the speeds of the two cars is x.	No basic factor has been used.
21111	Two cars started from A and traveled at some direction. One car stopped at C, the second car stopped further at point B.	The distance between C and B is x km. The distance that has been traveled by the second car is x km greater than the distance that has been traveled by the first car.	DBF/s are involved in the process of symbolizing the relation between the two distances.
2212	The car accelerated by 10 km/h and traveled at this speed 5 hours.	If the car did not accelerate she would have traveled 50 km less.	The number 50 represents relation; the difference between the distance with the accelerated speed and the distance with the original speed.

PROBLEM ISOMORPHS

Speed problems on two moving objects can be solved by the construction of the following system of four sets of equations. Each set expresses relations that were either formulated in the problem statements, inferred from the formulated ones, or underlie the situation (physical principles).

D set -- the equations representing the distance relations.

T set -- the equations representing the time relations.

S set -- the equations represents the speed relations.

These sets of equations represent the problem situation of a speed problem; their solution requires an auxiliary set of equations:

A set -- the equations representing the physical relations among the variables speed, Duration, and distance (D=SxT) for every path traveled by a moving object.

It is possible now to define isomorphic speed problems. Given a speed problem, if it is solvable, then it can be represented by a system of D, T, S, and A sets of equations. Two speed problems are isomorphic, if they can be represented by the same systems D, T, S, and A of equations.

This definition is not trivial as it may look, even if it seems inevitable. To see this notice that a relation of one type of equation, such as $x = y + a$, cannot be transformed algebraically into a relation of another type, such $x + y = a$. The condition in the problem statement determines the type of relations (either $x = y + a$ or $x + y = a$) that is expressed in equations of D, T, or S set. The definition is therefore clear and unambiguous, since the semantic types of relation do not enable two problems to become isomorphic when they are not, no matter what algebraic wizardy is used.

The value of this isomorphism between speed problems can be immediately seen if we consider the difficulties experienced by students in the algebraic type of studies. In Gorodetsky et al.'s study (1986) most 9th to 11th graders were unable to solve the following problem: "Two cars go from A to B and return to A without delay. One car goes to B at the speed of 60 km/hour and returns at 30 km/h. A second car goes both ways at 50 km/h. Which car returned to A first?" According to our approach, this problem is isomorphic to many other problems, one of which is the following. "Two cars go from A to C, passing through B, the midway between A and C. One car goes from A to B at the speed of 60 km/h and from B to C at 30 km/h and the second car goes from A to C at 50 km/h. Which car arrived at C first?"

Students suffered no difficulties in solving the latter problem (by computing the times needed for each car to arrive at C, and then comparing them), but most of them were unable to solve the former problem. Their solution strategies demonstrate that the main difficulty lied in the inability to deal with the TBRs (Time-Basic Relations) and DBRs (Distance Basic Relations). Most students were confused by the relationships among the two cars' TTs (terminal times) and STs (starting times), which they perceived as being different due to speed differences. Other students even confused DBRs with TBRs. Had these students realized the isomorphism of the given problem to an easier one, they were much more successful in their solutions.

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AGGREGATE VERSUS INDIVIDUAL ELEMENT INTERPRETATIONS TO FACILITATE PART-WHOLE REPRESENTATIONS

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Background

The interpretation of quantity as an aggregate or as individual elements has an impact on the solution of problems which tap numerical knowledge. Markman (1979) demonstrated this when she identified two types of natural concepts, collections (e.g., "forest," "army") and classes (e.g., "tree," "soldier"), and showed that children are less successful on problems described in class terms than on problems that include collection terms. For example, when children were presented with equivalent displays, those asked "what's more, my soldiers, your soldiers, or are they both the same?", were less successful than those asked "what's more, my army, your army, or are they both the same?"

Markman's explanation for this finding was that a collection label imposes a higher organization on perceptually discrete objects; this helps children think of the objects as a whole or aggregate. According to Markman, problems which tap knowledge of cardinality require attention to a whole, or an aggregate; this makes a collection structure more advantageous. This explanation appears satisfactory for the problems Markman used in her investigations, since they required qualitative answers. However, this explanation does not seem to be sufficient for problems requiring quantitative answers. For example, when a problem includes the cardinality of two sets and asks for the difference, a solution involving one-to-one correspondence requires attention to the individual elements. Support for this can be found in Hudson (1980) who showed that for young children, Compare problems such as "how many more birds than worms are there?" are much more difficult than problems formulated to focus attention on the individual elements. Hudson achieved this by presenting the problems in the form: "Suppose the birds all race over and each one tries to get a worm. How many birds won't get a worm?. Notice that even though both problems utilize class terms, the latter places additional emphasizes on the individual elements.

Attempts to explain Hudson's and Markman's findings have been made by several investigators. Greeno and Johnson (1985) hypothesized that the important mathematical knowledge for solving arithmetic word problems includes the principles of set theory: cardinality, subsets, complementary sets, unions, disjoint sets, and set differences. This hypothesis was further elaborated by Kintsch and Greeno (1985) in their model of solving arithmetic word problems. The model is based on two primary components, a linguistic component that constructs representations of text and a mathematical component that constructs representations of quantified sets and set relations. For example, the MAKE SET schema transforms "Tom has 5 marbles," into a representation including a set of marbles, the magnitude 5, and an owner named Tom. Other mathematical schemata designate subsets and perform arithmetic operations. According to this analysis, Markman's findings were interpreted as indicating that collection terms facilitate representations that include reference to sets, which helps children arrive at correct conclusions. However, this analysis does not indicate how the representations of class terms differ or how they hinder a successful solution as compared to collection terms.

We hypothesize that the representations of a quantity as an aggregate or individual elements include reference to sets but these representations differ in important ways that influence arithmetic word problem solving. In our analysis we develop characterizations of these representations in the context of the Part-Whole schema and suggest empirical studies to substantiate this hypothesis.

Representations of Magnitude

The Part-Whole schema is considered an important part of arithmetic knowledge in that it specifies relationships among triples of numbers (e.g., 7, 5, and 2) (Resnick, 1983). It has been particularly useful in accounting for performance in arithmetic word problem solving (Kintsch and Greeno, 1985; Resnick, 1979). One of the uses of the Part-Whole schema has been to demonstrate how different problem structures influence solution strategies (e.g., Riley, Greeno, and Heller, 1983). For example, Compare problems provide the problem solver with two cardinalities and require that the solver find the difference. Combine problems also present two cardinalities but request the sum. The task faced by the problem solver is to construct a representation of the text that enables him to utilize his knowledge of the Part-Whole relation. Most current models of arithmetic word problem solving assume one basic Part-Whole schema. We hypothesize two instantiations of the basic Part-Whole schema that are based on the conceptual difference between representations of quantity as aggregates or individual elements.

Size Part-Whole and Counter Part-Whole

A quantity of n objects can be represented in two ways: (a) as one set of n objects, an aggregate. For example, the quantity "3 soldiers" can be interpreted as "1 set of 3 soldiers." (b) as n sets of 1 object, individual elements. For example, the quantity "3 soldiers" can be interpreted as "3 sets of 1 soldier." Schematically, we denote these two representations by (1)[n] for the aggregate interpretation and (n)[1] for the individual elements interpretation. Generally, the x and y in (x)[y] will be called, respectively, the counter and size. The term counter is used because it enumerates the sets; the term size is used because it indicates the measure of a set.

To see the application of these instantiations of the Part-Whole schema, let a be the whole with parts a_1 and a_2. The relationship $a = a_1 + a_2$ can be represented, depending upon the interpretation of each of these quantities as individual elements or as an aggregate, in 2 x 2 x 2, or eight different ways. Among these eight representations only two involve the same interpretations as aggregates or individual elements of the three quantities a, a_1, and a_2: These are the following:

1. If both parts and the whole are interpreted as individual elements, then the Part-Whole relationship will be represented according to the Counter Part-Whole schema, i.e., $(a)[1] = (a_1)[1] + (a_2)[1]$;
2. If both parts and the whole are interpreted as aggregates, then the Part-Whole relationship will be represented according to the Size Part-Whole schema, i.e., $(1)[a] = (1)[a_1] + (1)[a_2]$.

If all three components of the Part-Whole relationship do not have the same interpretation, then it is necessary for the problem solver to change some interpretations so that a complete correspondence is achieved. For example, if a and a_1 are interpreted as individual elements (i.e., as (a)[1] and $(a_2)[1]$, respectively) but a_2 as an aggregate (i.e., as $(1)[a_2]$), then the triple a, a_1, and a_2 cannot be specified by either one of the Part-Whole schema instantiations. Therefore, to satisfy the constraint of equivalent interpretation among a, a_1, and a_2, a mental process of interpreting a and a_1 as aggregates or else a_2 as individual elements is required. To illustrate, notice that the sentence "3 sets of 1 child and 1 set of 4 children" is more difficult to conceptualize than the sentence "3 sets of 1 child and 4 sets of 1 child" or "1 set of 3 children and 1 set of 4 children."

Proposed Experiments

The proposed study for examining the validity of this analysis will be based on replicating and extending some of Markman's experiments. Preschool children will be presented with displays of objects and asked a series of questions. The first question will require the child to make a qualitative judgment whereas the other questions will require quantification. There will be nine basic problem types constructed from Compare, Combine, and Equalize problems involving exclusively aggregate terms, individual element terms, or a combination of both. An example of problems involving exclusively aggregate terms is:

This is my army. (Display of 8 soldiers)
This is your army. (Display of 5 soldiers)

qualitative question : What's more, my army, your army or are they both the same?

Compare question: How much bigger is the size of my army than yours?

Combine question: How big would the size of army be if we put our armies together?

Equalize question: What could I do to have an army the same size as yours?

An example of problems involving individual elements terms is:

These are my soldiers. (Display of 8 soldiers)
These are your soldiers. (Display of 5 soldiers)

Qualitative question : What's more, my soldiers, your soldiers, or are they both the same?

Compare question: How many more soldiers do I have?

Combine question: How many soldiers do we have altogether?

Equalize question: What could I do to have the same number of soldiers as you?

An example of problems involving a combination of aggregate and individual elements terms is:

This is my army. (Display of 8 soldiers)
These are your soldiers. (Display of 5 soldiers)

Qualitative question : What's more, my army, your army or are they both the same?

Compare question: How much bigger is the size of my army than yours?

Combine question: How big would the size of army be if we put our armies together?

Equalize question: What could I do to have an army the same size as yours?

A second study will involve the presentation of arithmetic word problems similar to those used by Riley, Greeno, and Heller (1979). The new major manipulation will be the inclusion of problems phrased in both aggregate and individual element terms. The primary data for these two studies will include solution time, reasoning strategies, and answer correctness. In general, we expect to find differences on all three measures. We expect that the results on qualitative problems will be similar to those of Markman. Namely, problems involving aggregate terms will show better performance than those involving individual element terms or combinations of both. On the other hand, we expect that quantitative problems involving individual elements terms will be solved faster and more accurately than problems involving aggregate terms. Quantitative problems involving a combination of aggregate and individual element terms are expected to take the longest time to solve and will also involve the greatest proportion of errors. Even though we can not make explicit predictions, we expect that these different types of problems will also lead to marked differences in problem solving strategies.

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Strategies for Self-Regulated Learning from Computerized Practice in Arithmetic

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In a rapidly changing world, a most important task of educational research is to investigate how children adjust themselves to novel learning environments. This paper describes how good students self-regulate their learning of new arithmetic concepts and algorithms in a learning environment induced by a widely used computer-managed system, named here CBPA (Computer-Based Practice in Arithmetic). The fact that students actively learn in that system is noteworthy because the system is designed to provide drill as a complement to class instruction rather than to teach new material.

To satisfy the primary objectives of individualized learning, the CBPA system enables each student to advance in the levels of practice at his or her own rate. Because each student advances at different rates in several different strands (arithmetic topics), a large proportion of the above-average achieving students receive practice in advance of the concurrent class material. Thus, many students, mostly the better learners, receive CAI exercises whose solution requires knowledge of concepts and algorithms not yet available from class instruction. Observations in a previous study (Hativa, 1988a) reveal that high-achieving students use a wide range of strategies for dealing with the problems encountered in the CAI work. This study has been designed to further identify and to categorize these strategies.

Method

For investigating students' solution processes, this study used the naturalistic method of observations coupled with individual interviews. Subjects were the 20% highest-achieving students (as measured by the respective computer class reports) in each of six classes. The classes comprised of 2nd through 4th grades in a Tel Aviv suburban school for medium to high SES population. Each of the subjects was observed by one of two observers during at least five CAI ten-minute sessions, once in every 6-8 consecutive sessions, during a four-month period. The observer recorded on paper every exercise that the student received, all the steps of the student's solution and the computer response. Immediately after the computer session, the observer interviewed the student and asked him or her to describe and explain the method for solution for each type of exercise and the source of the student's knowledge of how to solve this type of an exercise.

It is human nature that a person tries to make the best of a situation. Our observations and interviews reveal that "the name of the game" for high achieving students is the fastest advancement in the hierarchical levels of the CAI system, in spite of the fact that "success" on a problem does not necessarily represent mathematical understanding. However, conclusions that children fail to develop mathematical understanding because of the teaching environment are premature. This is powerfully illustrated by the fact that many high achievers manage to use the particular CAI environment in ways that first lead to their technically identifying algorithms for solution without understanding and eventually lead to their remarkable learning with understanding of mathematical algorithms and concepts. These points are illustrated and discussed below.

Strategies That Motivated Students Use for "Technical" Advancement in the Hierarchical Levels of Practice

High achieving students manage to provide correct answers to exercises that they do not fully understand. This is achieved by applying strategies that are, in fact, resourceful problem solving methods.

Of the general (as opposed to domain specific) problem-solving strategies identified in research literature, the ones relevant to this work are: problem decomposition; means-end analysis; planning; analogy; checking the solution; comparing solutions with worked examples (Gick, 1986); and embedding the problem into a larger one (Clement, 1984).

Our observed students used many of these listed strategies and a few additional ones, which are described below.

1. Using analogy

"Learning by analogy is the mapping of knowledge from one domain over to the target domain, where it is applied to solve problems." (VanLehn, 1986, p. 152). In the CBPA system, students use frequently solving-by-analogy strategies, particularly in the domain of decimals and negative numbers. Students accomplish this by searching in memory for similar operations, algorithms, patterns, or rules with whole numbers; performing the required operations with whole numbers; and then adjusting the answers to the particular domains by adding the decimal point or the negative sign respectively.

1.1 Analogy by keeping a pattern

(a) keeping the pattern of arithmetic operations to new notations

Example: Most students solved correctly horizontal exercises such as: "0.2+0.3=?" and "(-1)+(-4)=?", or exercises such as "0.04+0.39=?" and "0.921-0.589=?" that were written vertically, the first time that the students received them. This was achieved by analogy to addition and subtraction with whole numbers.

(b) keeping the pattern of a sequence to new notation

Example: all of the high achievers observed solved correctly exercises of the type: -45, -40, ? , -30, -25 or 1.5, 1.6, 1.7, ?, 1.9. They explained that they followed the pattern with whole numbers and then added the "minus sign" or the "period" in order "to make the added number look like all other numbers" in that sequence.

1.2 Analogy by generalization to new situations

(a) generalizing notations to new situations

Example: A fourth-grade student learned from final computer answers to solve correctly exercises of the type:

$$\frac{53}{100} = 0.53$$

A week later he received a new type of exercise consisting of a two-digit number to be divided by 1000. His first answer was:

$$\frac{53}{1000} = 00.53$$

His explanation was that he compared this problem to the previous type that included two zeros in the "100" and one zero in the answer. Thus, he assumed that each additional zero in the denominator required an additional zero before the "period". This explanation reveals the strategy of search for a similar problem and the generalization of the decimal notation to the new situation. His erroneous answer reveals his lack of understanding of the decimal point concept.

(b) generalizing operations to new situations

When students receive new types of exercises that they do not know how to solve but that involve the use of arithmetic operations, students activate knowledge of arithmetic operations with whole numbers.

Example: "$\frac{2}{3} + \frac{5}{6} = \frac{?}{?}$" Many students tried: $\frac{7}{9}$ $\left(\frac{2+5}{3+6}\right)$ that is, the students added numerators and denominators separately.

(c) generalizing rules to new situations

Example: A fourth grader received a new type of exercise which required rounding 3.5 to a whole number. She immediately typed a "4". Later she explained that she had already practiced rounding of 35 to tens (ans. 40) and 350 to hundreds (ans. 400) and in the case of 3.5 she applied the same rule.

2. Synthesizing previously known rules

When students receive a new, unfamiliar type of an exercise, they activate a search for relevant knowledge in memory. In using this process, students may identify several different rules that the students then synthesize, in order to solve the new problem.

Example: A fourth grader who, through the CAI system, practiced negative numbers without fully understanding this concept, received a new type of exercise which required completing the missing number in " 4 + ? =0 ". She immediately typed as an answer a "(-4)". She explained that she used her experience with a previous sequence of exercises which required the adding of a positive whole number to a negative whole number (e.g., "5+(-3)=?"). She remembered that in that sequence, whenever she added two numbers which were "positive and negative of the same number" (e.g., "3+(-3)=?"), the answer was "0". Thus, for a successful solution, she synthesized her knowledge from that previous experience with her knowledge of the logical rule for solving equations that she had already practiced with positive numbers: "given that a+b=c and a+?=c then b is the missing part".

3. Using means-end analysis

Means-end analysis is the strategy of reducing the difference between the current state and the goal of the problem by applying appropriate problem-solving operators (Gick, 1986). In order to use this strategy, students should be provided with the answer or the goal of the problem. Only a small proportion of exercises in the CBPA curriculum provide a final answer or an explicit goal. This occurs primarily in exercises that provide multiple-choice options for answers.

Example: A bright second grader received the following type of exercise which required choosing the right option for an answer:

If 40 + a = 63 then:
1. a = 63 + 40
2. a = 63 - 40
3. a = 40 - 63

These exercises were designed to practice the rules of equality--that is, that one can subtract the same number (40) from both sides of an equation and get an equivalent expression. The observed student who had not studied these rules explained his correct solution as follows: "The computer asks me what number I should add to 40 in order to get 63. This is very easy--I know that I have to add 23. Then I compute the result of each of the three given answers and see which of these gives me 23. 63+40 is too large but 63-40 gives me the right answer". This student compared the given and the expected results and worked to discover a method to reduce the differences between them.

4. Examining special cases

This strategy is primarily used in exercises designed to examine knowledge of rules of arithmetic (e.g., operations with units of computations--"0" and "1"; and the commutative, associative, and distributive laws). A typical question would be: "Is the equality: aXb:c = a:bXc true for every a,b,c?" or: "If a-z=a then (choose the correct answer) 1. z=0; 2. z=1; 3. z=a". Many of the observed students who had not yet studied these rules in their class arithmetic, substituted the English letters in the equation with numbers (e.g., in the first example, a with 1, b with 2 and c with 3); computed mentally (1X2:3=2/3 and 1:2X3=3/2); and made conclusions on the basis of the results of computations.

5. Embedding the problem into a larger one

A third grader received, for the first time, exercises of the type: "identify which one of four two-digit numbers is divisible by 4". The first two groups of exercises were: "62, 81, 72, 69" and "59, 70, 78, 96". For both groups he immediately typed the correct solutions (72 and 96 respectively). He explained later that these numbers were multiplications of 8 and therefore they should also be multiplications of 4. This student embedded the problem of division by 4, a multiplication table that he remembered by heart up to 40, into the problem of division by 8, a multiplication table that he remembered by heart up to 80. It was easy for him to compute mentally very quickly that 96 is also a multiplication of 8 (80+8+8).

6. Using heuristics to shortcut the search for solutions

A few good students, when faced with an impasse, used heuristics to eliminate computational steps that seemed inappropriate, in order to shortcut the search for the correct solution. These heuristics were based either solely on knowledge of arithmetic operations or on familiarity with the way the particular CAI curriculum worked. The students knew what level of difficulty of mental computations they could expect from the CAI curriculum, considering the fact that making computations on paper while on-line was forbidden. A typical rationale provided by students using heuristics was "it looks logical to try this operation" or "the computer cannot ask me to do such a difficult computation".

Example: A fourth grader, after practicing a sequence of exercises with multiplication of exponents ("$5^3 X 5^4 = 5^7$"), received for the first time division of exponents: " $5^7 : 5^4 = 5^?$ ". She typed correctly a "3" on the first trial (the exponent, 3, is computed by: 7-4). She explained that she did not know why this answer was correct but she assumed that because addition (3+4) worked well in the multiplication exercises, it could not also be good for division. Next she considered the division of 7 by 4 but rejected this option because "this division was not good" (that is, it did not yield a whole number that she believed was expected by the computer as an answer). Thus, after the mental elimination of addition and division she started with subtraction.

7. Using probabilistic considerations

In contrast with medium-to-low achievers who were observed working with the same CAI system, the observed high achievers only seldom tried to guess when they knew neither the answer nor the algorithm for solution. However, when the better students were faced with the need to choose the correct answer out of a few given options and they did not know the underlying method, they often tried educated guesses.

Example: A third grader received exercises that required identifying out of four three-digit numbers the one divisible by 9 (e.g., "277, 530, 450, 624)". This pupil did not know the rules of divisibility by 9. He did not even know that such rules existed. Thus, he tried to divide mentally each of the four given numbers by 9. Because of the time limitation, he missed frequently the correct answer. Thus, he developed an heuristic procedure for reducing the amount of numbers to be divided by 9. This procedure enabled him to arrive frequently at the correct answer within the short time framework. Step 1 was--looking for a number that ends with a "0" because, according to his explanation, "these numbers are easy to divide". Thus, in the example above, he first tried 530:9, immediately knew that 53 was not divisible by 9, and then he tried 450:9 which worked. In step 2 he used a probabalistic strategy, as follows: if there were two out of the four numbers that started with the same digit, he started by typing that digit. If the computer told him that this digit was incorrect, these two numbers were eliminated at once and only two additional numbers remained to be examined. If that digit was correct, he needed to divide by 9 only the two numbers that started with that digit. To illustrate: given the exercise "423, 610, 498, 536", he examined first the 610 (and eliminated it before typing because "only 63 is divisible by 9") and then he typed a "4" (first digit of both 423 and 498). The computer accepted the 4 and thus he knew that the correct answer was one of these two numbers. He promptly divided 423 by 9 mentally and produced the correct answer within the time limit.

8. Inducing from worked examples, using trial-and-error

When all the previously listed strategies do not work, good students resort to getting help from the computer. They then either press any key three times (for three trials) or they press a particular "advance" key. Both actions result in a display of the correct answer with all intermediate steps. This answer remains on the screen for only a short time (from 6 to 9 seconds). The students concentrate on the displayed answer, trying to identify the algorithm used. Thus, by using the solution displayed by the computer as a worked example, students induce frequently from that example (either from the first one or, using trial-and-error, from several similar examples) the correct algorithm for solution.

Support for this description is provided by students' verbal explanations of their methods for solutions. In their interviews, when asked about how they find the correct solution to an unfamiliar exercise, students' most typical explanations are: "this is the way the computer wants it"; "I tried and it came out right"; "when I solved it in the way I thought I should, I got an error and therefore I had to answer the other way around"; and "the computer showed me how to do this".

Example: A fourth grader who had not yet studied negative numbers received exercises of the type: "Replace the "?" by >, < or = in "-4 ? -19". She explained that when she first received this type of exercises she typed "<" because 4 is less than 19 but "the computer said that she was wrong" and that the correct answer was ">". Since then she knew that every time that she received the same type of exercise, she had to "reverse the logical order"

.

Through this trial-and-error strategy, it happens that students induce from final answers algorithms that are "local", that is, algorithms that work for that particular type of an exercise but that do not work for the general case.

Example: A bright second grader received a sequence of exercises that required converting mixed numbers into fractions, the integral part of the mixed numbers being the same in all of them--1. For example:

$1\frac{4}{5} = \frac{?}{5}$ $\qquad$ $1\frac{2}{3} = \frac{?}{3}$

The student failed to solve the first two exercises in that sequence but he then solved all the following ones correctly. When the session was over, he explained that for solving 1 4/5 he added 4+5 and wrote 9 for the "?". He did the same with 1 2/3-- replacing the "?" with 5 computed as 2+3 (that is, he did not refer at all to the integral part of the mixed number). This example shows that the boy solved this sequence of exercises correctly without really understanding the process involved.

9. Acquiring human help

All the highly motivated students that we observed were not ready to accept failure in identifying answers to problems. If they did fail during the computer session, they tried to get the teacher's help, and sometimes to get help from their neighbours (classmates). If they did not succeed in receiving the required algorithm for solution, they saw to it that they remembered the problematic exercises and they looked aggressively for help off-line. They used several sources for off-line help: In-school they asked the teacher during arithmetic lessons or during breaks between the lessons. They also discussed their problems with their classmates, usually during the breaks between lessons. Outside school they asked family members and friends for help.

How do good students acquire their knowledge?

We have seen thus far a variety of strategies that high-achieving students use to advance in the hierarchical levels of CAI practice, very frequently with no understanding of the underlying concepts involved. However, after a while, in their interviews, they demonstrate good understanding of these concepts. How do these students learn new concepts from final answers to exercises without receiving any organized, orderly, structured instruction? This question should be furthermore investigated. I may only assume that some part of the understanding of the new concepts is developed through the practice itself, and the rest is acquired through obtaining human help. The better students are curious human beings and usually like to understand what they are doing. Thus, when they ask for external help--the teacher, parents, and siblings, these students ask not only for the techniques but also for explanations--for the why's.

Discussion

The designers of the CAI system involved here created a special learning environment, different from the classroom environment, for enhancing individualization of students' work (this goal is stated in the introduction to teachers' handbook in the CBPA curriculum). At face value, students' work with the CBPA system seems to be very dull. The screen display is B&W with almost no graphics and no animation. Thus, it does not include essential features believed to provide motivation for students' learning that are found in the newer microcomputer software D&P programs. However, as we see in this article, the highly individualized curriculum leads to situations and consequences not foreseen by the system's designers. We see here a flourish of resourceful problem-solving strategies by high achievers for mastering skills technically. These skills are learned from material that is designed to provide practice of already known material rather than to teach new material. However, this study suggests that good students learn not only the techniques for solving the exercises, but eventually they acquire a good understanding of the concepts involved. In contrast, evidence from observations of medium-to-low achieving students (Hativa, 1988a, 1988b) suggests that many of these latter students, when getting material that either has not yet been taught in class or that they have already forgotten, do not acquire full understanding following their mastering of the techniques. However, I believe that even mastering skills without understanding bears positive prospects for facilitating their future learning. It is possible that when the class teacher eventually presents this material in class, it will be easier for these students to understand it because they have already mastered the underlying technique for solution. This point needs further investigation.

This article describes the multiple strategies that high-achieving students use to self-regulate their learning in this CAI-induced learning environment. Self-regulated learning is defined as a student's active acquisition and transformation of instructional material (Mandinach, 1984). A similar phenomenon was observed in other contexts as well (Siegler, 1986). Siegler contends that there are good reasons for students to know and to use a variety of strategies. Strategies differ in their accuracy, in the amounts of time they require, in their memory demands, and in the range of problems to which they apply. Strategy choices involve trade-offs among these properties so that pupils can cope with cognitive and situational constraints. The broader the range of strategies that children know, and the more effective their procedures for choosing which strategy to use in a given situation, the better they can adapt to the demands of changing circumstances."

In order for students to be able to effectively use a wide range of strategies in practicing with the CBPA system material new to them, students need to have most of the following aptitudes (Hativa, 1988a): quick thinking (the final solutions stay only a short time on the screen); very good induction and deduction abilities; very good memory capacity; high motivation; competitive spirit; persistence; and aggressiveness in looking for answers. Further research should take place to investigate how to teach medium to low achieving students to use strategies and what another type of support these students should receive for effective learning in a CAI environment which is different from class environment.

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Use of Core Propositions in Solving Current Electricity Problems

by

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INTRODUCTION

At the university of Minnesota we are in the process of developing science instruction for elementary and middle school teachers. One unit of instruction is on current electricity. The instruction was based on the conceptual change model proposed by Posner, Strike, Hewson, and Gertzog (1982). In addition, we considered the problem solving literature that suggests students' intellectual behavior while solving problems is adaptive - that is, students apply their knowledge in various ways depending on the specific features of the problem that confronts them (Newell and Simon, 1972).

These two considerations and the fact that previous studies of individuals' knowledge of current electricity was done with different populations necessitated that the answers to the following questions be obtained before the instruction could be designed:

(1) What as the teachers' prior knowledge of current electricity? Is this knowledge different from the knowledge of other populations of students?

(2) How did the teachers use their knowledge to solve unfamiliar problems. Did they consistently use the same set of propositions to solve unfamiliar problems, or did they use different combinations of propositions to solve different problems?

This paper reports the results of this preliminary (preinstructional) analysis.

METHOD

The Subjects

The subjects were five middle school science teachers (grades 6 - 8) and eleven elementary school teachers (grades 2 - 5) from a small rural town in Minnesota. The teachers were enrolled in an integrated physics and science teaching methods course. Only five of the teachers had a physics course in their college backgrounds. They had 4 to 25 years of teaching experience (mean = 12 years)

The Current Electricity Pretest

The written pretest consisted of 13 questions with 6 questions consisting of 2 or more parts. Most of the questions referred to a circuit diagram consisting of a battery, wires, and identical bulbs. These questions asked the teachers to (1) predict what happens to the brightness of a bulb if a change is made to the circuit (a bulb is shorted or a second bulb is added in series or parallel), (2) compare the brightness of two bulbs in the same circuit, (3) compare the brightness of bulbs in different circuits, or (4) compare the amount of current at different points in a series or parallel circuit (see Table 2 for a sample test question).

The teachers were asked to explain their reasoning for each question.

The test was divided into two parts administered at two successive class sessions. After a teacher had completed each part, a researcher read through his/her responses and circled key words or phrases. The teacher was then asked to "tell as best you can what the word/phrase means to you as you used it in this question." In general, words like current, conductor, resistance, energy, and power as well as phrases such as "the path of least resistance" and "energy is equally dispersed" were circled. The teachers wrote their explanations on the back of the test pages.

Analysis

To determine the teachers' core propositions, the responses of all of the teachers to all of the pretest problems were examined using the following questions as a guide:

What is current?
What does a battery do?
What happens when current leaves the battery?
What happens when current encounters a bulb?
What happens when current encounters wire?
What happens when current encounters a junction?

These questions were not asked directly on the pretest. The core propositions in these categories were inferred from the reasoning the teachers used to solve the problems.

From this first examination, a list of propositions for each category was generated for the entire group of teachers in this study. For example, the propositions about what happens when current encounters a wire included:

Copper wires are excellent conductors of electricity.

Wires disperse (spread out) the current.

Wires simply conduct the current to the bulbs.

Wires use up or weaken the current (wires have resistance).

The longer the wire, the more current is used up.

The smaller the diameter of the wire, the more current is used up.

Wires use up much less current than bulbs.

This list represents all of the propositions about the effect of wire on current that this group of teachers used to solve one or more of the pretest problems. An individual teacher could have one or more of these propositions.

For each teacher, the major statements or clauses in their pretest responses were numbered (e.g., Q5a.6 means the 6th statement in response to question 5a). Then each pretest was examined problem by problem to determine the teacher's core propositions in each category. The researcher recorded all statement numbers which indicated that the given proposition was used by the teacher to answer the problems. At the same time, idiosyncratic propositions for each teacher were also recorded.

After this procedure had been completed, the accuracy of the results was checked by having a second researcher complete the procedure for 4 cases. Any differences of opinion were resolved by returning to the original statements. There were no disagreements on the core propositions for any subject -- differences arose only in the number of statements which supported the presence of a few of the core propositions.

Finally, a matrix was formed of core propositions by subjects. From this matrix it was possible to determine the core propositions common to all teachers, as well as patterns of different sets of core propositions held by different groups of teachers. Two cases were dropped from the sample: One teacher did not answer enough questions on the pretest to determine her model, and one teacher was also an electrician, so he already had essentially the correct current electricity model. The results of the analysis for the remaining 14 cases are reported below.

RESULTS

The Common Core

The core propositions common to all of the teachers were related to their beliefs about the nature of current and the function of the battery. At first sight, these teachers seemed to have different views of the nature of current. Regardless of their initial definitions of current, however, all of the teachers treated current as energy when they were asked to predict or compare the brightness of bulbs. This result is in agreement with other studies of secondary and college students (for example, Osborne, 1981; Riley, Bee, and Mokwa, 1981; Von Rhoneck, 1983)

It is interesting to note that when asked specifically to define current, most teachers revealed imprecise and inconsistent notions about the relationship between current, electricity, charges, and energy, as illustrated in Table 1. In fact, four teachers had assimilated learned textbook information into the more primitive "clashing currents" model commonly held by children

Table 1

Examples of Current Definitions

Case 1: Current is the flow of electricity from a positive area to a negative area. Positive is one type of electrical current flowing away from the energy source; negative is one type of electrical current flowing toward the energy source. Electricity is a source of energy which is caused by the charging of positive and negative particles.

Case 6: Current is the intensity of the flow of charges (positive and negative attractions). The closed switch engages the charges within the battery so the wire can conduct energy to the bulb (carry positive and negative forces from the action of the particles of molecules).

Case 7: Electric current is a current powered by electricity. The battery provides the electric current. Electricity is the power provided by the battery.

Case 8: Electrons flow in a closed circuit from the negative pole to the positive pole of the battery. Electrons flow from high density to low density areas. Current is the energy with which the electrons travel.

Case 12: Current is the flow of electricity through the closed circuit. Electricity is the source of power used to create some form of light or heat. The components of electricity are electrons and neutrons.

(Osborne, 1981; Riley et. al., 1981). For the one-bulb circuit these teachers stated that positive charges flow from the positive end of the battery and negative charges flow from the negative end of the battery. When the currents meet at the bulb filament, they neutralize each other, producing heat and light.

The teachers had two common beliefs about the function of the battery:

(1) The battery is the source of the current (i.e., the circuit is initially empty of the stuff that flows through it).

(2) The battery releases the same, fixed amount of current to any circuit.

The misconception that the current flows from the battery through "empty" conductors (Tiberghien, 1983) is not surprising since it is reinforced in most elementary and middle school textbooks. These textbooks usually define current as the flow of charges (or electrons), the battery as the source of these charges, and wires as good conductors of the charges. Textbook presentations of current electricity seldom include the fact that the charges are already present in the conductors or the analogy of the battery to a pump that circulates the charges around a closed system of conductors.

The notion that a battery always releases a fixed amount of current to a circuit is also common with secondary and college students (Closset, 1983; Cohen, Eylon, and Ganiel, 1983; Shipstone, 1984). Teachers frequently buy batteries which are labeled "1.5 Volts", "9 Volts" etc.. Since voltage and current are not distinct concepts, it is not surprising that teachers believe the battery releases the fixed amount of current they think is specified on the label. Another possibility is that the fixed current notion may be related to a deep, intuitive misconception about the lack of feedback mechanisms in passive, inanimate systems. Since batteries cannot "know" what is hanging on them, they can only release the same amount of current every time a circuit is hooked up. A similar misconception exists in mechanics about the reaction or normal force. Since tables, chairs, walls, etc. cannot know how hard they are being pushed, they can only push back the same amount every time.

Only one teacher had a different model of the function of the battery. She had apparently learned that for two bulbs in parallel, the potential difference across each bulb is the same, so each bulb receives the same current. She generalized this learned rule to the following:

> As long as the battery (volts) is the same, the same amount of energy is released to every bulb in the circuit.

She applied this rule to all the questions on the pretest; regardless of the circuit, all bulbs have the same brightness as a single bulb. This example is consistent with the study by Eylon and Helfman (1985), who found that physics students overgeneralize example problems in texts to apply to new problems.

The Sequential and Static Models of Current Flow

The teachers had two different notions about what happens when the fixed current leaves the battery and encounters a bulb, one static and one dynamic. The majority of the teachers (ten) had a dynamic model of current flow as defined by the four propositions below:

(1) The fixed current flows out of the battery and does not decrease or diminish until it reaches a circuit element that "uses up" some of the current.

(2) Bulbs use up current.

(3) The brightness of a bulb depends on the amount of current flowing to the bulb.

(4) When there is more than one bulb on a circuit path, each bulb uses up some of the fixed current, so all bulbs receive less current.

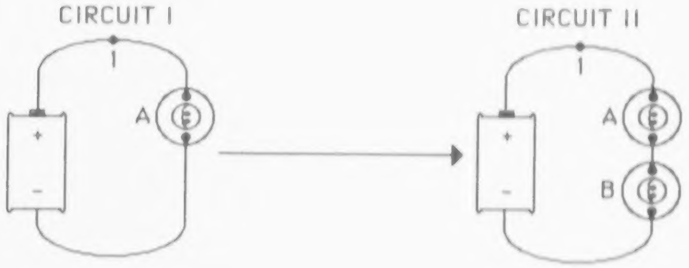
The first three propositions are commonly called the "sequential" model (Closset, 1983; Shipstone, 1984). The last proposition was included in the teachers' common set of core propositions because it was consistently used to answer several questions on the pretest.

Notice that the first and last propositions are contradictory. The first proposition implies the lack of a feedback mechanism. The last proposition implies that there is some mechanism whereby current "knows" that other bulbs are in the circuit before the current reaches the first bulb. This feedback mechanism was not defined by any of the teachers.

The contradiction in the two core propositions generated inconsistent patterns of explanations as illustrated in Table 2. All of the teachers who had a sequential model of current predicted that the current was not affected by the addition of the second bulb (Question 4c on Table 2). However, they also predicted that the first bulb would dim when the second bulb was added in series (Question 4a). They failed to notice that the propositions which they had used to explain one situation were contradicted by the propositions used to explain

Table 2

Inconsistent Dynamic Model Explanations for Series Circuit Questions



4a. What happens to the brightness of bulb A when a second identical bulb is added to the circuit as shown? Explain your reasoning.

Bulb A dims because the second bulb uses up some of the (fixed) current.

4b. In circuit II, how does the brightness of bulb A compare to the brightness of bulb B? Explain your reasoning.

Bulb B is dimmer than bulb A because bulb A uses up some of the current, so less current flows to bulb B.

4c. Compare the amount of current in the wire at point 1 before and after the second bulb is added (i.e., In which circuit is there the most current flowing through the battery, or is the current the same?) Explain your reasoning.

The current is the same in both circuits. No current is being used by the bulbs at point 1, so the number of bulbs does not influence the current at this point.

another related situation. This lack of consistency has also been found in studies of students core beliefs in mechanics (for example, Champagne, Gunstone, and Klopfer, 1985).

In the sequential model of current, there is an implicit dependence as a function of time - the current is not modified until it reaches a component that consumes current. In contrast, the static model of current ignores the initial current flow and focuses instead on the end result, as defined by the two propositions below:

(2) The fixed current (energy) is equally dispersed through the wires and to all the bulbs in a circuit.

(2) The brightness of any bulb in the circuit depends on the amount of energy the bulb receives.

Only one teacher consistently applied the static model to answer all the questions on the pretest. The static model leads to the prediction that the current is the same at all point in a circuit, and all bulbs in a circuit (series or parallel) are the same brightness because they each receive the same amount of current.

One of the other cases has been described previously. The remaining two cases will be described in a later section.

Series Circuits

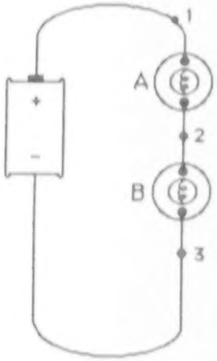
Despite the fact that ten teachers in the study had a common set of core propositions, there was a large variability in their predictions of the brightness of each bulb in a series circuit (see Table 3). This variability stemmed from two sources. First, the teachers had different core propositions about the direction of current flow associated with their sequential model For example, if current flows from both ends of the battery (n = 1), then the two bulbs are the same brightness because they each receive the same amount of current (Sequential 3 on Table 3). If current flows in one direction (n = 9), however, then the second bulb encountered is dimmer than the first bulb because some of the current is used up by the first bulb. Since there are two directions current can flow, there are two different predictions: if current flows from the positive terminal of the battery, bulb A is brighter than bulb B; if current flows from the negative terminal, bulb B is brighter than bulb A (Sequential 2 and 3 on Table 3).

Teachers also had different core propositions about the effect of wires on current: wires simply conduct the current to the bulbs (n = 6), or wires use up current (n = 4). For example, the teachers who believe that wires use up current tended to generate the rule, "the farther the bulb is from the battery terminal, the dimmer the bulb" (see Sequential 2 on Table 3)

Finally, variations in the sequential-model responses to the series circuit problems resulted from adjustments teachers made to account for the learned fact that two bulbs in series are the same brightness. Five of the ten teachers exhibited this adaptive behavior. There were two kinds of adjustments made: change one of the core sequential-model propositions, or switch to the static model explanation. Two of the teachers changed their core proposition about what causes the brightness of a bulb from "brightness depends on the amount of current flowing to the bulb" to

Table 3

Examples of Series Circuit Explanations



12a. How does the brightness of bulb A compare with the brightness of bulb B? Explain your reasoning.

12b. Compare the amount of current at points 1, 2 and 3. Explain your reasoning.

Sequential 1: (Current flows from + to -)
Bulb B is dimmer than bulb A because bulb A uses up some of the current, so less current flows to bulb B. The current decreases from point 1 to point 3 (1 > 2 > 3).

Sequential 2: (Current flows from - to + and wires use up current)
Bulb A is dimmer than bulb B because it is farther away from the battery, so some current is used up by the wires and bulb B. The current decreases from point 3 to point 1 (3 > 2 > 1).

Sequential 3: (Current flows from both poles of the battery)
Bulbs A and B are the same brightness because they receive the same current from the + and - ends of the battery. The current at points 1 and 2 are equal; the current at point 3 is less (1 = 2 > 3).

Adjusted Sequential 1: The total current decreases from points 1 to 3 because the bulbs are using up current (1 > 2 > 3). But the bulbs are the same brightness because identical bulbs use up the same amount of current.

Adjusted Sequential 2: Bulbs A and B are the same brightness because the (fixed) current is equally dispersed to both bulbs. But the current decreases because the bulbs use up the current (1 > 2 > 3).

"brightness depends on the amount of current used up by the bulb" (Adjusted Sequential 1 on Table 3).

Three of the teachers switched to the static model to explain the equal brightness of the bulbs on the first series circuit question, which did not ask for a comparison of the current at different points in the circuit (Question 4b shown in Table 2). These teachers did, however, notice the contradiction between their static-model explanation and their sequential-model core beliefs on the second question (Question 12 shown in Table 3), where they were also asked to compare the current before, between, and after the bulbs. For this question, one teacher crossed out his static model explanation and predicted that bulb A is brighter than bulb B. Another teacher simply said, "I know they are the same brightness - why?". The third teacher said that "in this series circuit, bulb A will be slightly brighter than bulb B." They all predicted that the current would decrease from point 1 to point 3 as the current was used up by the bulbs.

Another teacher never resolved the difference between her knowledge of the facts and her belief in the sequential model of current flow. She gave two explanations for every series circuit question, one static and one sequential, as illustrated by her responses to Question 12:

> "On the one hand I believe the current is the same at all three points because the circuit is complete (current is equally dispersed) and because I think the bulbs are the same brightness like Christmas tree lights. Nevertheless, I think that the current would be stronger at point 1 before any is utilized and weaker at point 3 after current has been used."

Parallel Circuits

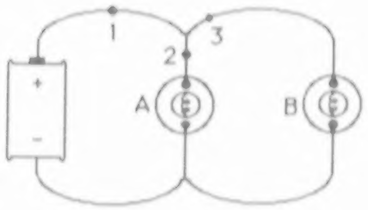
Additional variability in the teachers' use of the sequential-model core was evident when their responses to parallel circuit problems were considered. The wide variability in their predictions about the brightness of the bulbs arose from the different core propositions they had about (1) the direction of current flow, (2) the effect of wires on current, and (3) what happens to current at a junction.

Eight different sequential junction models are illustrated in Table 4. Two of the teachers did not recognize a junction in any of the parallel circuit problems on the pretest: one treated parallel circuits like series circuits (Model 3 on Table 4), and the other focused only on the distance from the negative pole of the battery (Model 2). Only three teachers consistently used one of the junction models (Models 6, 7 and 8 respectively) in all the parallel circuit problems.

Five teachers used two or more junction models for the different problems. These teachers seemed to be influenced by their spatial perception of the different parallel circuits. For example, one teacher predicted that two bulbs in parallel are the same brightness because the current divides equally at the junction (Model 7 on Table 4). For three bulbs in parallel, however, he predicted that the bulbs are successively dimmer as the distance from the battery increases because more current flows down the first path than the second path, and more current flows down the second path than the third path (Model 6). Apparently, the added "distance" in the three-bulb parallel circuit prompted a different model of what happens at a

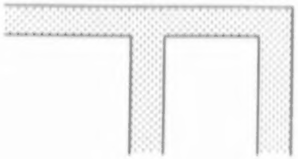
Table 4

Models of Junctions in Parallel Circuits



5c. Compare the amount of current in the wires at points 1, 2, and 3. Explain your reasoning.

Junction Not Recognized



1. The current is the same at all three points (1 = 2 = 3) because the current has not yet reached the bulbs (or is not hindered by any bulbs as it returns to the battery).

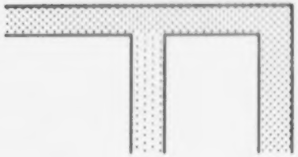
2. The current is largest at point 2 because it is closest to the negative pole. The current at point 3 is greater than at point 1 because of the distance to the negative pole. Point 1 is carrying the least amount because of the bulbs A and B using up the electricity (2 > 3 >1).

3. The current at points 1 and 2 are equal because it has not yet reached the bulb. The current at point 3 is least because the current has been used in passing through bulbs A and B (1 = 2 > 3).

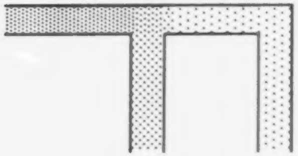
4. The current at points 1 and 2 are equal because it has not yet reached bulb A. The current at point 3 is least because the current flows around the circuit with bulb A before it flows through the circuit with bulb B (1 = 2 > 3)

Table 4 (continued)

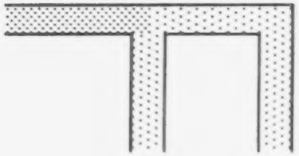
Current at Junction Divides



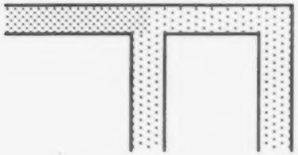
5. The current at points 1 and 3 are about equal. Slightly less current flows down the path to point 2 (1 = 3 > 2)



6. The current at point 1 is largest because it has not yet split. The current at point 2 is larger than the current at point 3 because more current flows through the closer path (1 = 2 + 3; 2 > 3).



7. Points 2 and 3 would each have half of the current at point 1 because it would divide evenly when the split occurs (1 = 2 + 3; 2 = 3).



8. The current at point 1 is the sum of the currents at points 2 and 3. The current at points 2 and 3 is the same because current follows the path of least resistance. Since the paths have the same resistance, the current through each path is the same (1 = 2 + 3; 2 = 3).

junction. This variability in their use of junction models also suggests that the teachers may have been constructing propositions on a problem by problem basis.

The teachers exhibited similar inconsistencies in their use of their core propositions about the effect of wires on current flow in parallel circuits. Different problem tasks seemed to cue the use of different propositions. For example, one teacher ignored the effect of wires on current in the two-bulb parallel circuit. For the three-bulb circuit, however, he predicted that each bulb would be successively dimmer because "more current is used in traveling the extra distance to the bulbs."

Apparently, many teachers did not know under what conditions to apply either their junction or their wire propositions. For example, the teachers who believed that wires use up current did not know how long a wire has to be before there is a noticeable effect on the current. Consequently, they often gave inconsistent answers to similar problems on the pretest. This type of error has been noted in many problem solving contexts (see, for example, Reif, 1985).

Other Cases

Three cases remain to be considered. As reported earlier, one teacher used an overgeneralized rule that as long as the battery (volts) is the same, the same amount of energy is released to all bulbs in the circuit. The remaining two teachers are described below.

One teacher seemed to have an incomplete dynamic model of current flow to bulbs. He believed that a fixed amount of current flows from both ends of the battery. However, he appeared to use the following "empirical" rule to answer the questions on the pretest:

> The further away the bulb is from the battery, the dimmer the bulb

This rule was not tied to any stated belief that either the bulbs or the wires use up current.

Another teacher seemed to use random fragments of learned knowledge, the static model, and the dynamic model to answer the questions. He did not appear to have any consistent model of current flow. For example, he said that two bulbs in series are the same brightness because "there is the same amount of DC current entering the bulbs and leaving the bulbs.". However, he said that the current at a point near the battery is the same before and after the second bulb is added in series because "current does not diminish greatly until it passes through more resistance (copper wire and filament)."

CONCLUSIONS

As expected, most of the elementary and middle school teachers in this study had a sequential model of current flow :

> The battery is the source of current. It releases a fixed amount of current (energy) that circulates around the circuit. This fixed current is not modified until it reaches a circuit component that consumes the current. The current is then successively consumed by each component of the circuit. Bulbs use up or consume current. The brightness of a bulb depends on the amount of current flowing to the bulb.

In addition, these teachers included a proposition not normally reported as part of the sequential model:

> When there is more than one bulb on a circuit path, each bulb consumes some of the fixed current, so all bulbs receive less current.

The inclusion of two contradictory core propositions led to inconsistencies in their answers to several questions. The teachers apparently failed to notice these contradictions.

Despite their common core sequential model of current flow, there was a very wide range of answers to the questions on the pretest. Knowing that teachers have a common set of core propositions is not sufficient to predict performance on a given problem or set of problems. The variability in their predictions arose from (1) the different core propositions teachers had about the direction of current flow, the effect of wires on current, and what happens to current at a junction, and (2) the adjustments they made to their core propositions to account for previously learned facts that contradict the predictions of the sequential model.

Moreover, the teachers did not consistently apply their core propositions across similar problems. The perceptual cues in the problem task tended to sway their judgement about which propositions to apply. In addition, they do not appear to know the applicability conditions for their propositions.

These results suggested two decisions about the instruction that followed. First, two fluid flow analogies were introduced to the teachers (rather than just the "correct" analogy). The teachers were guided through a series of experiments to decide which model consistently accounted for all of the observed facts. That is, the epistemological commitment to internal consistency was specifically built in to the instructional sequence. The activities also included predictions and discussions about what happens when there is a junction of wires or a short circuit.

Second, the instruction included a sequence of guided experiments for the construction of a set of qualitative current electricity rules. These rules were designed to enable the teachers to make consistent predictions about what happens to the total current in the circuit and the brightness of each bulb when (1) a bulb is added to or removed from a circuit path, (2) a bulb is shorted, (3) a path of one or more bulbs is added parallel to the battery, and (4) a path of one or more bulbs is added that is not parallel to the battery. The qualitative rules had applicability conditions built into them.

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INFORMAL GEOMETRY IS THE TRUE GEOMETRY

by

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In the schools today formal geometry (with its postulates, definitions, theorems and proofs) is usually considered to be the apex or goal of learning geometry. Informal geometric topics and activities which do not fit into the formal structures are often given second class status and relegated to the domain of mere motivation or help for those who are not smart enough to learn the "real thing" -- formal geometry. I am a mathematician and as a mathematician I wish to argue that this so-called informal geometry is closer to true mathematics than is formal geometry. I do not believe that formal structures are the apex or goal of learning mathematics. Rather, I beleive, the goal is understanding -- a seeing and construction of meaning. Formal structures are powerful tools in mathematics but they are not the goal. I don't blame teachers for giving formal geometry too much emphasis; mostly I blame my fellow mathematicians because we have done much to perpetuate the rumor that formal systems are an adequate description of the goal of mathematics.

As an example consider the notion of 'straight line'. I claim that this notion is not now and never could be entirely encompassed by a formal structure. I am talking here both about the notions of 'straight line' as used in everyday lanquage and the notions as used by mathematicians. In fact, these various notions are closely interrelated through the felt idea of straightness that underlines them all. Ask any child who hasn't had formal geometry or any research geometer and they will tell you that "straight" means "not turning" or "without bends". (Of course the research geometer is likely to mumble something containing the formal notions of "affine connection" or "covarient derivative", but if pressed for what that means he will admit that it is a formalization of "not turning".) Now "not turning" clearly has a different meaning from "shortest distance". So both the child and the research geometer have a natural question: Is a "non-turning" path always the "shortest" path? And, if so, why? They then look for examples of "non-turning paths". (The child can do this by imagining and/or observing non-turning crawling bugs on spheres and around corners of rooms.) They can then convince themselves that the great circles are the straight lines on the sphere. (This is not something to assume, it is something to check; and it has meaning in the sense that a crawling bug on the sphere whose universe is the surface of the sphere will experience the great circles as straight.) It is then clear that going three-quarters of the way around the sphere on a great circle is a straight path but not the shortest path. (Going one-quarter of the way around in the opposite direction is shorter.) Thus a straight path is not always the shortest. (This can also be seen in situations where it is sometimes a shorter distance to go around a steep mountain rather than to go straight over the top.) But on the sphere it is true that every shortest path is straight. So the question becomes: Is the shortest path always straight? The research geometers have proved that this is true on any smooth (no creases or corners) surface which is complete (no edges or holes) and the basic ideas of their proof can easily be conveyed to high school students. But then the child might think about a bug crawling on a desk with a rectangular block on it and notice that there are two points on either side of the block such that the obvious straight path joining these points is not the shortest path and the shortest path is not straight. These explorations, whether by the child or the research geometer, are a good examples of doing mathematics (or in this case, doing geometry) and they are not encompassed by any formal system. The mathematician will use formal systems to help in the explorations but the driving force and motivation and ultimate meaning comes from outside the system. It comes from a desire which the mathematician shares with the inquisitive child -- the desire to explore the human ideas of "straightness" and "shortest distance".

So, should we teach formal structures? Definitely, yes. But not in geometry. The power of formal structures does not come through clearly in geometry -- it would be better to look at the formal structure of a group with its various examples in geometric symmetry groups and number theory. The emphasis on formal structures in school geometry obscures the meaning of geometry and does not in the context in which it is used add any power.

THE ACQUISITION OF CONCEPTS AND MISCONCEPTIONS IN BASIC GEOMETRY - OR WHEN "A LITTLE LEARNING IS DANGEROUS THING"

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I. INTRODUCTION

In an extensive survey of prerequisite knowledge among students entering Israeli Junior high schools, it was found that achievement, even of a minimal level, in geometry is dramatically lower than achievement in arithmetic, in spite of the fact that the syllabus specifies one hour per week for intuitive geometry throughout the 6 years of primary school.

In order to understand the reasons for this situation, we began to investigate the acquisition of basic geometry concepts in two schools in which the teaching was well organized and the teachers were reliable. The population in one school was disadvantaged or culturally deprived, as officially defined in terms of socio-economic and cultural criteria. The second school had a culturally non-deprived population. In both schools, all the students in grades 5, 6, 7 and 8 participated in the research.

If students are to learn these geometry concepts, then it is important for elementary school teachers to be comfortable

with them, so we also investigated 142 preservice elementary teachers (PRE) and 25 in-service elementary teachers (ST) in Israel.

The content and tasks were sampled from the primary school geometry syllabus and included identification, drawing and reasoning connected with basic examples, their attributes and some relationships between them.

This is a "bottom up" kind of research, in the sense of Resnick (1983); i.e., "theoretical analyses that worked from task performances to the kind of knowledge children must have in order to engage in the performances ... theory of understanding on the basis of detailed analyses of procedures used in performing tasks". By this we "are forced to recognize relatively small changes in cognitive structures".

The explanation of children's behaviour leans on theories or partial theories when appropriate, and it leaves room for the development of a "microstage theory". (This in contrast to research based on a single theory, or "top down" type of research.) In spite of the fact that the research was "bottom-up" in its execution, we will present things in a different order. First we describe briefly some theoretical frameworks which will be used in the interpretation of the research findings and then we shall describe some types of student (and teacher) misconception patterns of behavior concerning basic concepts in geometry. We shall understand the term misconceptions to include concept images (conceptions) which are both partial and/or contain wrong elements.

These misconception patterns of behaviour fall into the following categories

a. Misconceptions which endure - they have the same pattern of overall incidence from one grade level to the next, and for students, pre-service and in-service teachers.

b. Misconceptions which decrease with concept acquisition, as one would expect.

c. Misconceptions which increase with concept acquisition.

II. SOME THEORETICAL FRAMEWORKS

a. The van-Hiele model (Wirszup 1976, Hoffer 1983, Crowley 1987).

This is the most comprehensive theory concerning geometry learning and has been discussed extensively in the last decade. It has thus served as the focus of interest for a substantial part of geometrical research, collectively called van Hiele based research. The part of the model relevant to basic geometry learning is the first three levels and we use them in some of the explanations of our findings.

The first level: Visualization. "Geometric concepts are viewed as total entities", and their examples (geometric figures) "are recognized by their shape as a whole, that is, by their physical appearance, not by their parts or properties".

The second level: Analysis

Figures are recognized as having parts and are recognized by their parts. The students discover and analyse the figure's properties.

The third level: Ordering or informal deduction. "At this level students can establish the interrelationships of properties both within figures and among figures. Thus they can deduce properties of a figure and recognize classes of figures. Class inclusion is understood. Definitions are meaningful. Informal arguments can be followed and given." (Crowley p.3).

In student behaviour, according to van-Hiele, first level visual elements play a major role. One can see the concept examples, can draw them, can even "touch" them:- In the Agam project of visual education (Eylon et al, 1985), young children trace the perimeter of the figure with their fingers, "create" the figure with their bodies, walk along the perimeter of the figure, etc.

Children grasp the whole appearance of the figure and perhaps even the figure's main attributes via visual codes. At this point we need perceptual theories which try to explain how we register and remember these codes.

b. Perceptual Aspects

Bryant (1974) tries to explain the perceptual processes in which "children perceive and interpret their surroundings". The theory is based on experimental evidence and was influenced by other theories (Helmholtz, Piaget, and the Gestalt theory). The main issues of the theory are:

- Children perceive their surroundings via relative codes.

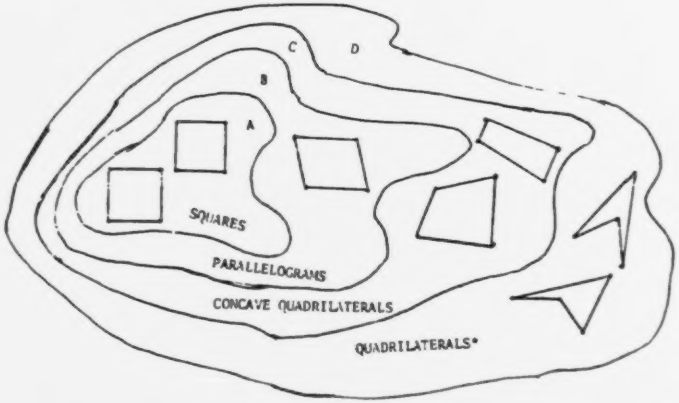
When they become older they can remember absolute properties of objects around them, because they have developed some strategies for coping with them. (The transition from van Hiele first level to the second?)

- "The basic weakness of relative codes is that they are only adequate for direct comparisons between objects presented simultaneously".
- A relative solution to the relative code weakness is "to connect separately presented objects through their common relations with the same framework" - The frame of reference effect.
- The external frame of reference is used to connect "objects" on the same continuum and this is done by deductive inferences.

Bryant emphasises the positive developmental role that external frames of reference have, whereas others (e.g. Witkin et al, 1977) emphasize their negative aspect as sources of error. In an investigation of the influence of "the surrounding organized field" on the person's peception of an item within it, they claim that an individual who, in perception, cannot keep an item separate from the surrounding field (is relatively field dependent), is likely to have difficulty with problems where the solution depends on items which are used in a different organization of the given field.

c. Mathematical structure frameworks

Some misconceptions in basic geometry can be understood better if we relate them to the mathematical structure in which the concepts are embedded. In other words, the mathematical relationships between concepts, their examples and non-examples, and their relevant (critical) attributes play a major role in concept acquisition. One structure which has a considerable effect in basic geometry learning may be called *the opposing directions inclusion relationship*, between concept examples on the one hand and the properties (attributes) of the examples on the other. For example, consider the following sets, which are related by inclusion: $A \subset B \subset C \subset D$.



* We do not include quadrilaterals with intersecting sides.

The above inclusion relationship is formed by the concept examples - the set of squares contains only some of the elements of the set of parallelograms; etc. But if we look at the critical attributes of each of the above sets (- a critical attribute is an attribute which an example of a concept must have in order to be an example of that concept), we get an inclusion relationship in the opposite direction. Thus the "smallest" set of figures (Set A - the squares) has the "largest" set of critical attributes, which contains the set of critical attributes of B (parallelograms), etc.

According to the van-Hiele theory the inclusion relationship is understood when the student is at the third level. We may conclude that the understanding of the two directions relationship is the "top" of level 3.

This typical mathematical structure causes many difficulties as we shall see later.

III. MISCONCEPTION PATTERNS OF BEHAVIOUR

a. Misconceptions which endure - or - prototype examples.

Every "concept" has a set of critical attributes and a set of examples. In the set of concept examples there are the "super" examples: - the prototypes (Rosch and Mervis, 1975); that is the popular examples. In other words, all the concept-examples are mathematically equal, because they conform to the concept definition and contain all its critical attributes, but they are different one from another psychologically. (All examples are (mathematically) equal, but some are more equal than others - with apologies to George Orwell.)

In our investigations we found that the patterns of behaviour with respect to concept examples in identification and construction tasks are very similar across student age groups, for teachers, for boys and girls, etc.

The following are three examples:

The right-angled triangle identification

(This example was reported in Hershkowitz et al., 1987).

Figure 1 shows a comparison between students in different grade levels of elementary school, preservice teacher (PRE) and inservice teachers (ST), in the identification of right-triangles in three different orientations. (In the task they had to identify all the right triangles in a given collection of some 10 triangles.)

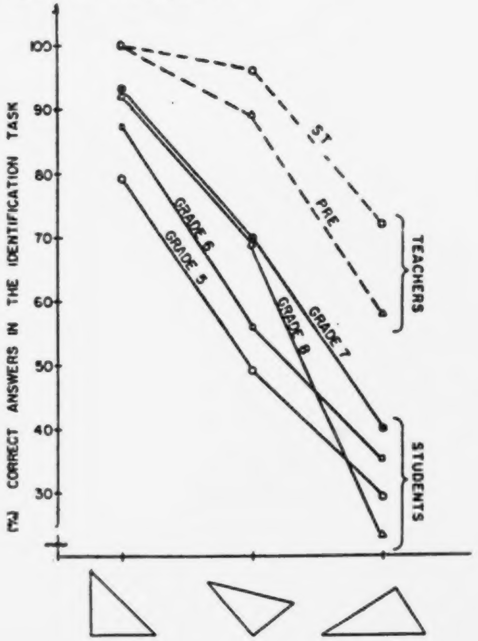
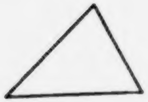


Figure 1

This example is typical of the negative effect of a frame of reference. Because the page-sides are vertical - horizontal (and our surroundings are also vertical-horizontal to a large extent), students, pre-service teachers and in-service teachers have difficulty in the identification of those triangles whose perpendicular sides are not in the vertical-horizontal (prototype) position. This difficulty decreases with age and experience, but the pattern remains very similar.

- The isosceles triangle identification

Mark all the isosceles triangles among the following: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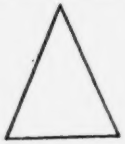
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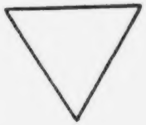
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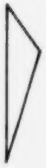
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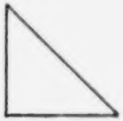
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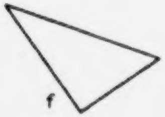
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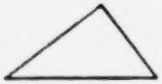
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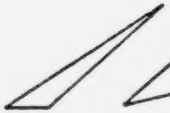
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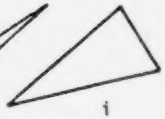

i

There are four isosceles triangles: b, c, e and i. Figure 2 shows the percentages of those who succeeded in correctly identifying these triangles through the grade levels, for two different divisions of the population: boys and girls, and disadvantaged and non-disadvantaged students.

The prototype is the isosceles triangle (b) "standing on its base". As in the previous example, we have again the frame of reference effect. But we can explain the results in a different way: The triangle c is equilateral and the triangle in example e is right-angled.

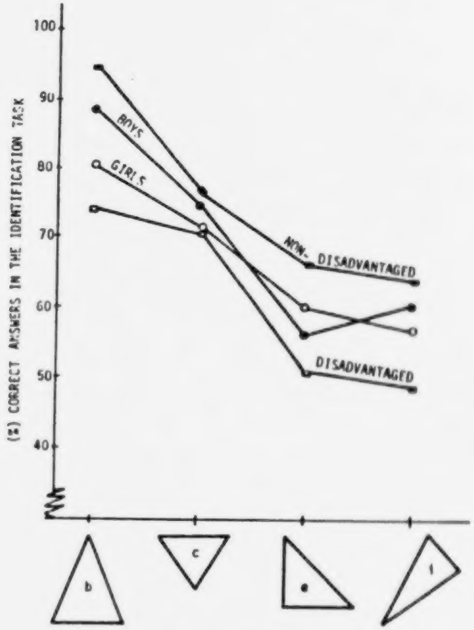


Figure 2

It may be that the additional non-critical attribute in each of these examples distracts from the critical attribute. Or, in other words, the fact that the triangle is an "element" in two different familiar sets of triangles is confusing.

The "bitrian" and "biquad" examples

(The bitrian example was reported in Hershkowitz et al, 1987.)

The purpose of these items was to investigate the role of a verbal definition in the formation of the relevant concept.

We "invented" the following definitions.

- A bitrian is a geometric shape consisting of two triangles having a common vertex. (One point serves as a vertex of both triangles.)

- A biquad is a geometric shape consisting of two quadrilaterals having a common side.

One half of each of our groups (students and teachers) was asked to identify bitrians and biquads among other shapes, the other half was asked to construct two bitrians and two biquads.

The frequencies of the bitrian shapes constructed by students and teachers (PRE and ST) are shown in Figure 3.

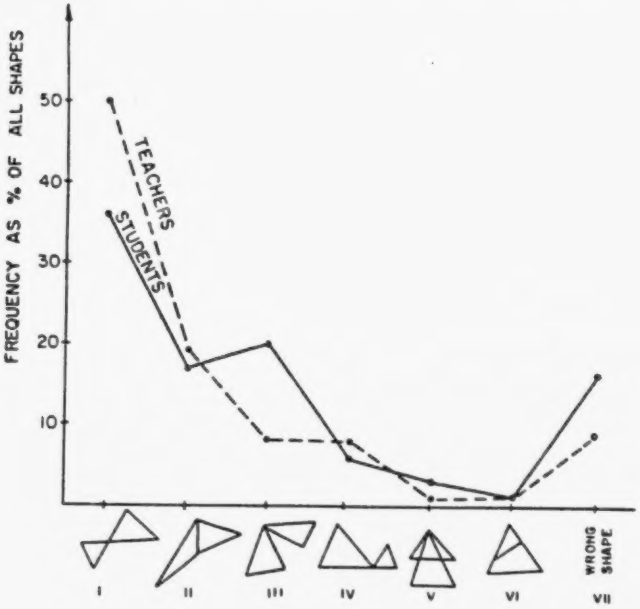
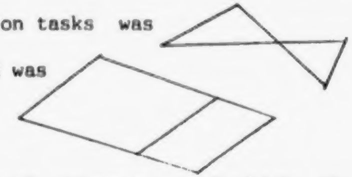


Figure 3

The pattern for teachers and students is very similar. More or less the same pattern was found for other analyses of these tasks using independent variables such as grade level, sex, socio economic status and students and teachers (teachers were better in the identification tasks but the pattern remained the same).

The most popular bitrian figure, both in the identification and construction tasks was and the most popular biquad was



Prototypes of concepts with which the students (or teachers) are already familiar (such as right-triangles, isosceles triangles, quadrilaterals) were explained, as we saw above, by perceptual effects, or by non-critical dominant attributes. It is likely that in addition, teaching contributes to the establishment of the prototypes. Thus the textbook and the teacher present mostly the prototypes. Here in the bitrian and biquad examples, where the "concept" is presented verbally (without even one example), we have the same prototype phenomenon. (It is worth noting that whereas the patterns for the two concept examples is so permanent with age, the patterns for non-examples are different. Thus performance improves with age (grade level) - the number of those who identified non-examples correctly increased from one grade level to the next, and the number of those who construct wrong figures decreased.)

How did individuals form their concept image from the concept verbal definition?

Figure 4 shows the responses to the biquad construction task of 5 grade 6 students (student E drew another 4 examples, all correct; the other students drew only the one example shown for each.)

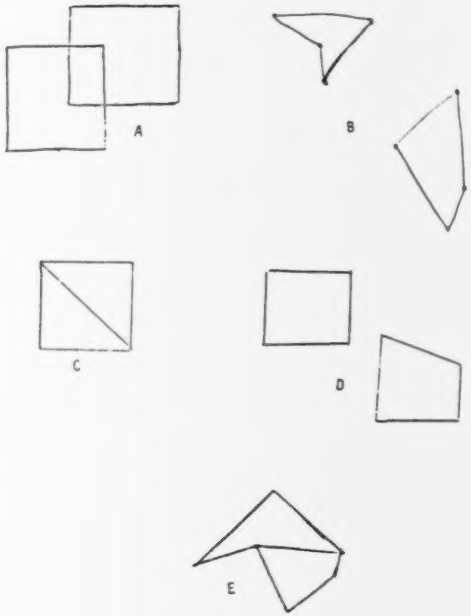


Figure 4

From these responses we can obtain some information on the transition from the verbal definition to the concept image formation: - The definition includes (i) 2 quadrilaterals, (ii) which have one side, (iii) in common.

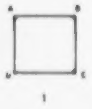
Students B and D pay attention to attribute (i) only, student A draws two quadrilaterals which have something in common but not a side, student C draws the common side but the figures are triangles. Only student E considered the 3 critical attributes of a biquad. We can observe some stages in the formation of concept examples from the concept definition, but the question why some of the examples become more popular (the prototype phenomenon) is still left to speculations.

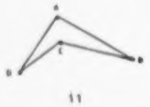
b. Misconceptions which decrease with concept acquisition - or - expected patterns

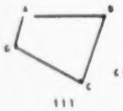
The quadrilateral example

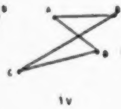
(This example was reported in Hershkowitz and Vinner 1983).

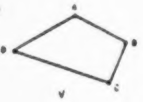
Among the following shapes indicate those which are quadrilaterals











For each shape that is not a quadrilateral explain why.

The concept examined here is quadrilateral. The concept examples and critical attributes will be slightly different for different concept definitions. Thus, "a quadrilateral is a closed four-sided figure" includes figure iv as an example, whereas "closed four-sided figure whose sides do not intersect", does not, etc..)

The percentage of students in the different grades who indicated an exampl as a quadrilateral is given in Figure 5.

There is clearly a great improvement in identifying shape v (convex quad.) and shape ii (concave quad.) with age as can be expected, from grade 5 to grade 7.

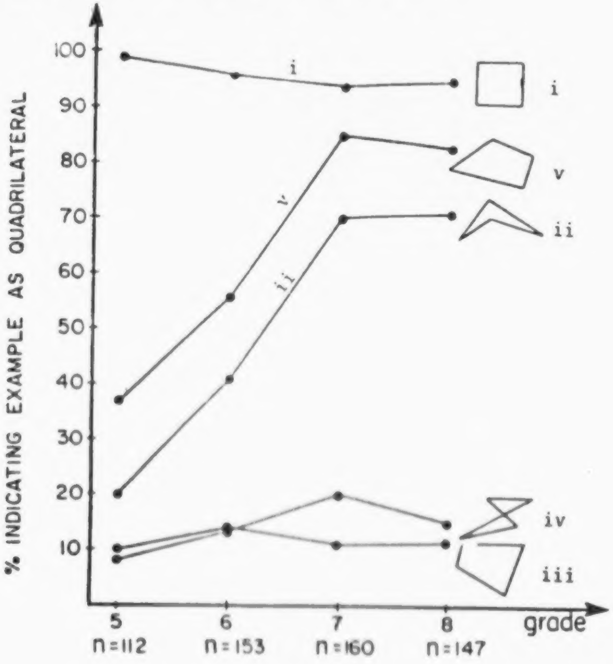


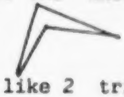
Figure 5

On the other hand, in all grades, most of the students identify a square as a quadrilateral. "Constant behavior" is also found in the identification of the "non-closed" shape iii and of shape iv, which has intersecting sides. For most of the students, in the different ages, these two shapes are non-examples. We can conclude that here also we have the prototype phenomenon. In addition, we found by using Guttman scale analyses, a valid hierarchical structure in the attainment of the concept examples. This implies that:

(i) when students accept an example as a quadrilateral example they accept also the "easier" examples (i.e. the acceptance of figure ii implies the acceptance of figures v and i, etc.);

(ii) the distribution of students at different stages (accepting ii, v and i or accepting v and i or only i) on the scale moves upwards from grade 5 to 8. That is, there is a decreasing number of students who have a concept image consisting of the square only and an increasing number who have three figures in their concept image of quadrilateral. This is again an expected learning behaviour.

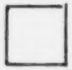
Development with age (grade level) was found also in students' reasoning. Among the different reasons which students gave when they decided that a given shape is not a quadrilateral, we can detect certain categories.

i) Reasons based on the appearance of the whole figure (first van Hiele level). For example "  does not look like quadrilateral: it looks like 2 triangles". (This type of reason can lead to correct or incorrect responses.)

(ii) Non-acceptance of "2 labels" for one figure. For example " is not a quadrilateral because it is a square" (This type of reason usually leads to an incorrect response.)

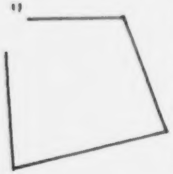
iii) Reasons based on non-critical attributes, usually attributes of a prototype example. For example, - "All the figures, except the square, are not quadrilaterals because they may have equal sides but do not have equal angles".

"and, on the other hand,

"-  is not a quadrilateral because a quadrilateral has 4 unequal sides".

(The two types of reasoning, using the special attributes of the prototype lead to incorrect responses.)

iv) Reasons based on critical attributes. For example,

"  is not quadrilateral because it is not closed, therefore it is not a polygon, and every quadrilateral is a polygon." (This type of reasoning usually leads to correct responses.)

The last two categories can be considered to reflect the second van Hiele level, because students use attributes (concept attributes in iv, concept example attributes in iii, in their reasoning, where the reasoning based on non-critical attributes leads to incorrect responses and the reasoning based on critical attributes leads to a correct response. In addition we can say that some reasons in category iv even reflect the third van Hiele level, because they show some understanding of the inclusion relationship (as, for instance, in the above example - "... and every quadrilateral is polygon".)

In figure 6 we illustrate the change with age (grade level) of the percentage frequency of the first and last two categories of reason for figure iii (the "open figure"). (The non-acceptance of the "2 labels" category did not occur in this example).

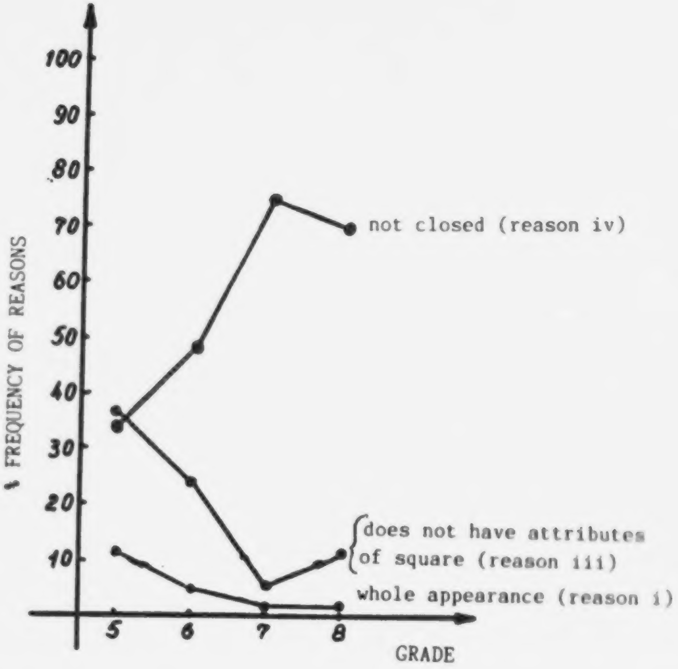


Figure 6

The frequency of reasons based on the figure's whole appearance (van Hiele first level) is very small and decreases across the years, whereas the number of reasons based on the concept or the concept examples attributes (van Hiele second level) is quite high across the years. These van-Hiele second level reasons are of two types which change differently through the grade levels. The number of correct reasons, based on the main concept-attributes (the critical attributes), increases dramatically, whereas the number of incorrect reasons, based on the prototype, decreases. The prototype examples (the squares) are the "smallest" subset of figures within the set of concept examples. Thus we can say that, with the years, more students start "to behave" correctly according to the structure of the "opposing directions of the inclusion relationship" (the third van Hiele level).

c. Misconceptions which increase with concept acquisition

Not always does student behaviour on geometrical tasks follow an expected pattern. One example is exhibited in the responses to the following item (Figure 7, the percentage frequency of correct drawing is shown against each figure).

Draw the altitude to side a in each of the following triangles

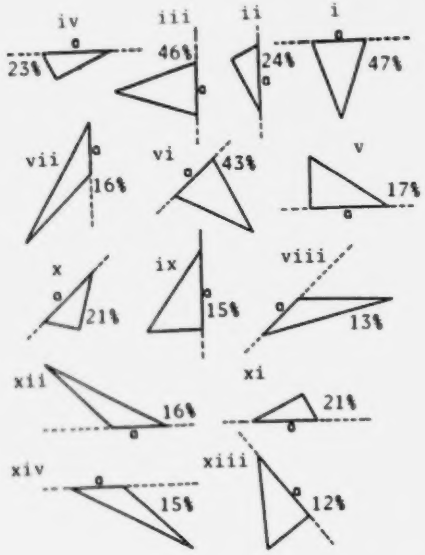


Figure 7

In this item we can distinguish types of triangle which we shall describe as follows: isosceles; i, iii, vi; unequal sides; ii, iv, x, xi;

obtuse-angled; vii, viii, xii, xiv; right angled: v, ix, xiii. Each appears in 3-4 different orientations. The frequencies of success indicate that the orientation (or frame of reference) effect is not effective here. This suggests that the process of concept formation in this case is different from that in the right-angled triangle identification example.

In addition we found here also a hierarchical structure. Those who succeeded in drawing the altitude in the obtuse or right-angled cases, succeeded also for the unequal sides and isosceles triangles.

The incorrect responses provide an interesting analysis. Some examples of such responses are shown in Figure 8.

We can analyse student responses according to the critical attributes of the altitude to side a: (i) The perpendicular (ii) from the vertex opposite (iii) side a or its extension. Students B and D considered only two critical attributes, and A not even one. But, student B systematically drew a median to a and student D the perpendicular bisector. Student A was also systematic in drawing some segment inside the triangle from one of the vertices on a. Student C drew the altitude to a in the two first triangles, but in an obtuse-angled triangle, where the altitude is outside the triangle, and in the right-angled triangle, where the altitude is one of the sides, he drew the altitude to a side other than a, presumably in order to obtain an altitude inside the triangle.

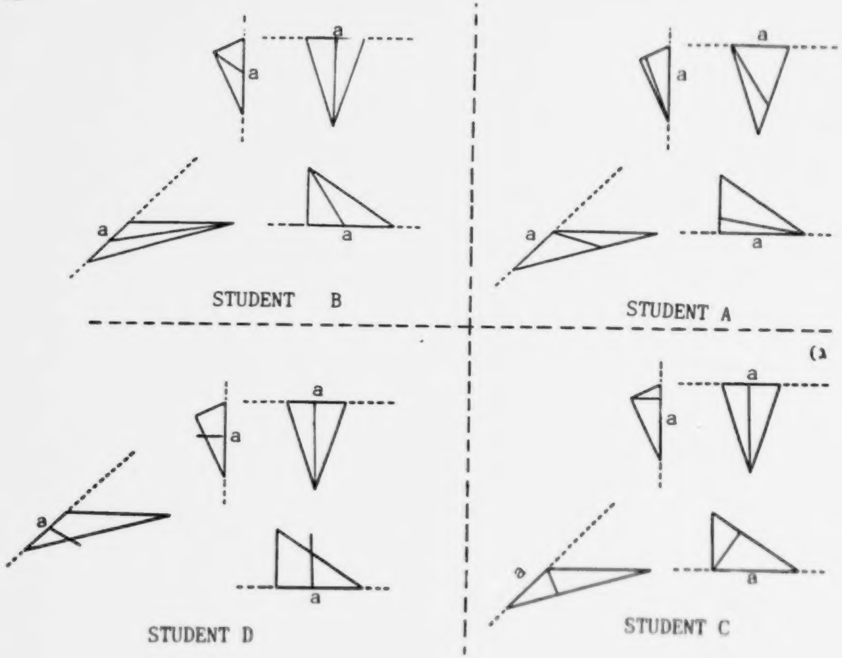


Figure 8

A majority of the students have an altitude concept image which contains segments inside the triangle only. For an isosceles triangle this usually leads to a correct response, because the altitude is the median and the perpendicular bisector as well. In the unequal sides triangles, the correct response also does not contradict this image. But in the other two types of triangle this concept image can only lead to an incorrect response. Again we have here a prototype example of the concept which is the source for misconceptions.

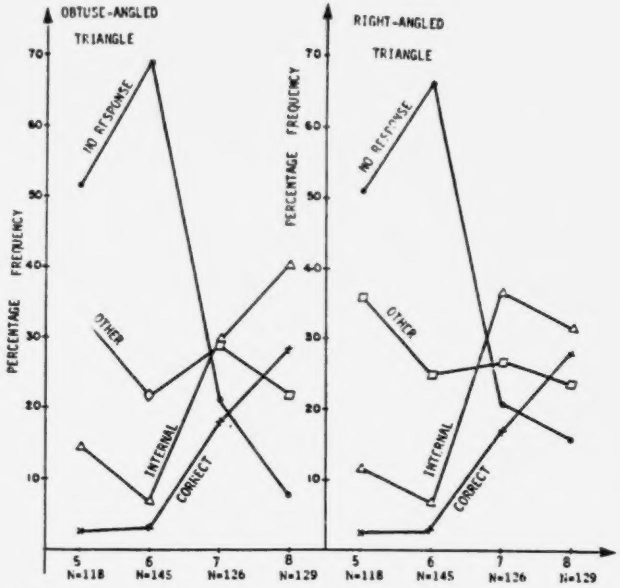


Figure 9

Figure 9 describes quantitatively student responses for the obtuse and right-angled triangles across the class grades. The number of students who did not respond to this task was very high in grades 5 and 6 (we will return later to the increase from grade 5 to 6), decreases dramatically in grade 7 and even further in grade 8. This would seem to reflect the teaching. The subject is taught at the end of grade 6 and then repeated as part of the teaching of area calculations in grades 7 and 8. The interpretation of this "no response" curve is that students start to think that they know the concept. But what have they learned?

The number of students that have formed the concept correctly increases with grade as one would expect, but even in grade 8 it is less than 30% of the population. In parallel, there is another group whose number increases - those who formed the main misconception that an altitude is a segment inside the triangle. The number of "other errors" is relatively constant through grades and also quite large. Before formal learning, most of the population admit that they are not familiar with the concept and they do not produce any response. Of those who do respond, most produce mainly wrong and unsystematic answers. The formal learning, at least at this stage, produces 2 groups - those who had formed the correct concept and those who had formed the dominant misconception, while the number of those who produce "other" misconcepts is stable.

It is worth noting that teachers had similar difficulties; in a group of 20 in-service teachers only 8 drew the correct altitude in all 4 types of triangle. (Activities with teachers in order to improve this situation are described in Hershkowitz et al, 1987). We analysed the population according to sex, and the pattern repeated itself. But some interesting slight differences appeared in an analysis of culturally deprived versus non-deprived students, (Figure 10).

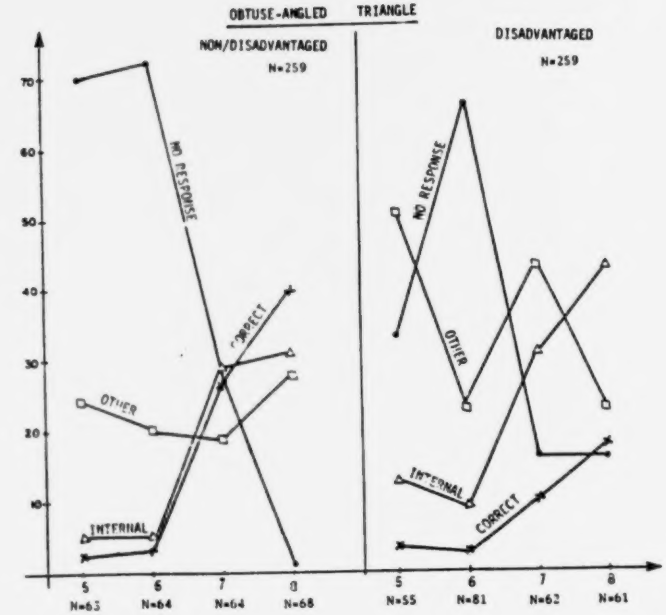


Figure 10

More non-disadvantaged students formed the correct concept, whereas more disadvantaged formed the main misconcept. The category of "other errors" is much higher in the disadvantaged population. In addition, the "no response" category is very different in the 2 populations: more disadvantaged students tend "to gamble"; they do not admit that they don't know.

IV. DISCUSSION AND SUMMARY

If one analyses geometrical task performance according to the geometrical concept examples, we will always find examples which are more popular than others - the prototype phenomenon. (This was true in all the above examples. In the quadrilateral task the square example is the prototype and in the altitude task the altitude inside the triangle is the prototype). The creation of prototypes in the various tasks is caused in different ways:

(i) Perceptual reasons. I.e., the negative effect of the page sides (the external frame of reference) on the identification of right-angled triangles.

(ii) Teaching methods. I.e., the same partial success in the identification of right-angled triangles, given in different orientations can be explained by the unbalanced selection of examples given in textbooks and by teachers. Usually the vertical-horizontal example is given. Similarly, teachers and textbooks tend to present the isosceles triangle "standing on its base". The altitude example is perhaps the worst of this sort. Many teachers are themselves not familiar with an "outside" altitude or an altitude which coincides with one of the sides, which leads directly to the prototype examples.

(iii) Prototypes formed by the mathematical structure

There are many geometrical concept prototypes which are the elements of the "smallest" subset in the set of concept examples. For example, the subset of squares in the set of quadrilaterals. These prototypes are the concept examples with most attributes and it seems that this may make them the prototype examples, (This fits very well Rosch and Mervis' (1975) theory on prototype formation in everyday concepts).

It seems reasonable to assume, however, that there are also other sources in the formation of these prototypes. Thus the prototypes created in the biquad and bitrian examples can be only partially explained by using some combination of (i) and (iii).

In addition, we saw, that the prototype examples have an effect on the concept formation.

At the first van Hiele level, students judge all other examples (and non-examples) by comparing their whole appearance with the prototype whole appearance. This leads, of course, to only partially correct performance. Thus students identify only some of the concept examples (e.g., the right-angled triangle task) and this behaviour is constant across the years, and drow wrong elements in some of the examples (e.g., the altitude in obtuse or right-angled triangles).

At the second van Hiele level, when students make attribute-based judgement, the prototype effect leads them astray because they lean on prototype attributes which are non-critical to the general concept. Only those students who base themselves on the concept's critical attributes, seem to understand the inclusion relationships between geometrical concepts. In other words, only these latter students can move up to the third van Hiele level (e.g. the quadrilateral task).

Acknowledgment

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The fruitful discussions I had with each of them contributed substantially to the consolidation of ideas in this paper.

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AN INVESTIGATION OF SOURCES OF MISCONCEPTIONS IN PHYSICS

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Introduction:

Research efforts on misconceptions in science have implicated a number of factors as sources of misconceptions (Johnstone & Mughol, 1976; Helm, 1980; Ivowi, 1984). These include teaching, normal language usage, everyday experience of the material world, textbooks and examination papers. Helm (1980) suggested that the mis-match between students' intellectual development and the demands made upon them by the content of their courses could also be responsible for the observed misconceptions. Using Bruner's (1960) argument that a child is ready to learn when the teacher is ready to teach, and the crucial factor of teachers effectiveness in class in the formation of concepts by students, Ivowi (1984) stressed the mis-match between societal demands of students and the provision made by the same society for the learning experience of students in schools as a factor affecting the effective formation of concepts in students during the teaching and learning of science.

Recently, Head (1986) proposed five possible origins of misconceptions in students. These are everyday experience and observation, confusion about analogies, use of metaphors, peer culture, and innate origin of some ideas. In

terms of the age and ability of students, he saw a genuine pedagogic problem because during the developmental progression in thinking of a child from pre-operational to formal operational thinking, ideas that are consistent or contradictory to conventional science must be formed. This places emphasis on teaching in an attempt to properly guide a child through formation of concepts. Since misconceptions are bound to exist at one stage or the other of a student's intellectual development, their sources need to be identified and pedagogic strategies deviced to minimise their occurrence in science teaching and learning.

So far, no empirical evidence exists for the sources of misconceptions cited in the literature. The purpose of this study therefore is to seek direct empirical evidence for the sources of students' misconceptions in Physics.

Method:

The instrument for this study consists of problem descriptions and tasks performed by students. The problems cover motion, conservation principles and fields while the tasks relate to stating and explaining answers to the problems and indicating the sources of information used in answering the questions in the tasks. All the odd-numbered tasks solicited answers and their explanations; while the even-numbered tasks required the students to indicate, from a suggested list, their sources of knowledge. The problems and the odd-numbered tasks were basically the same as those used in Ivowi (1985 and 1986a) except that in the present study, no options were provided for the students. They were allowed to respond freely to the task so that any answers and explanations that tallied with already identified misconceptions from previous studies could be followed to identify their sources from the information given in the even-numbered tasks. Since the stems of the problems were the same as those used in previous studies and the odd-numbered tasks sought exactly the same information of answer and explanation, the validity and reliability of previous tests are assumed for this study.

The subjects for this study were upper form 6 physics students of ages 17-19 years. This restriction became necessary because previous studies (Ivowi, 1985 and 1986a) had shown that the upper form 6 students performed significantly better than the form 5 students and that the test was more suitable for the upper form 6 students than for the form 5 students. These students were drawn from twelve secondary schools within Lagos that offer the higher school certificate courses leading to the 'A' level General Certificate of Education (GCE) examinations. Altogether, 77 out of 159 students responded to the instrument used for this study.

The administration of the instrument on the students was done with the help of the physics teachers in the schools investigated. The test was taken in April 1987, a few months before the GCE examinations in June 1987. Since most schools delayed instruction in fields, especially magnetic and electric fields, the time of the administration of the test appeared best as the students were expected to have covered the course content and would be revising for their examinations.

In analysing the results, only instances of identified misconceptions were of interest and in such cases the sources

of such misconceptions were identified. The frequency of occurrence of each of the possible sources was counted against each misconception and the χ^2-statistic for each calculated to determine any significant difference in the possible sources. By this, the most likely source of a particular misconception was determined.

An example of the items in the instrument is shown below:

Problem 3

A metal ball is given a push to move it along a smooth horizontal table. Point P is a position on the path of the ball. Predict the subsequent motion of the ball.

Task 7

Indicate and explain the forces acting on the ball as it passes through the point P.

Task 8

Indicate your source(s) of information.

Problem 9

A man can cut down a tree by using (i) a cutlass or (ii) a big saw or (iii) an electrically powered saw. In each case, it takes him 1 hour or 30 minutes or 5 minutes respectively to complete the job. What can you say of the energy used in each case?

Task 19

If the energy used in cutting the tree by process (iii) is E, estimate the energy used in the remaining processes and explain your answer.

Task 20

Indicate your source(s) of information.

In task 7, the misconception detected is the association of force with motion. Similar explanations to those in Ivowi (1986b) were obtained. These include such explanations as "since point p is not a barrier that can stop the ball from moving, then the ball would pass through p with the force acting parallel to the horizontal plane" and "force due to the push as well as force due to the weight of the metal ball will be on the ball." They point to the misconception stated.

In task 19, explanation such as "energy is most in (iii) and least in (i) since power needed will be much more in (iii) than in any other, and least in (i)" was typical; and this indicates a misconception of equivalence of power and energy.

Table 1: Summary of students' responses to tasks

Tasks	N *	a	b	c	d	e	f	χ^2 **
1	97	25	17	14	34	6	1	45.36
3	84	21	10	6	39	8	0	70.42
5	103	26	21	4	42	8	2	79.58
7	95	27	18	2	40	7	1	76.12
9	70	20	9	3	32	5	1	61.83
11	117	17	33	5	50	11	1	89.42
13	86	29	11	5	33	8	0	63.45
15	74	17	10	3	36	7	1	67.59
17	87	28	10	7	31	8	3	48.66
19	79	38	7	5	25	3	1	84.43
21	94	39	13	6	29	4	3	71.27
23	69	34	7	4	21	3	0	76.30
25	97	24	23	2	39	9	0	70.46
27	72	22	10	5	31	4	1	58.20
29	88	19	21	4	39	5	0	73.13
31	81	18	15	5	34	8	1	41.33
33	74	17	11	5	29	9	3	36.86
35	55	15	5	7	22	5	1	33.15
43	43	8	6	4	21	3	1	35.95
39	24	4	2	2	13	3	0	26.50
41	37	13	5	3	13	3	0	24.65
43	38	12	3	3	15	4	1	25.91
45	41	9	4	0	19	4	5	32.17
47	84	15	17	6	40	6	0	62.14
49	49	4	1	2	26	3	1	60.83

* The different values of N arose from the fact that subjects were free to choose more than one source for each task.

** Values of χ^2 significant at 0.05 level for df = 5.

Results:

Table 1 shows a summary of the response of the students to the tasks. Each identified misconception has the frequency of occurrence of the sources and the calculated χ^2-statistic. Although the instrument contained seven categories of possible sources of misconceptions, only six are analysed in table 1 because the responses on the seventh category were not helpful. They either contained repetitions of some of the sources in categories A - F or were left blank by many respondents. The highest frequency recorded for category G was in task 19 and this was only 2. In twelve of the tasks (e.g 1, 3, 7, 45 and 49) no response was recorded by any of the subjects. In all the remaining twelve tasks, only one response was recorded for each of them.

In table 1, all the calculated values of χ^2 are greater than the corresponding value of 11.07 for df = 5 at 0.05 level of significance found in tables. This shows that the misconception identified in each task had a most probable source. An examination of all the tasks in table 1 indicates that the most probable source of misconceptions is D (textbooks); this is followed by A (personal experience/intuition), and then B (teachers). The influence of C (peers), E (other books) or F (instructional television/general television programmes) has been found to be very minimal with regards to misconceptions.

Table 2: Order of Occurrence of Sources of identified misconceptions

Serial No.	Misconceptions	Related tasks for possible detection	Likely Sources in order 1st	2nd	3rd
1	Heaviness increases with height.	1, 3	D	A	B
2	Association of force with motion.	5 - 9, 35 - 45	D	A	B
3	In free fall motion, vertical motion affected by horizontal motion.	5, 21 - 27	D	A	B
4	Energy change always proportional to velocity or horizontal displacement.	9, 21 - 29	D	A	B
5	Two instead of three types of collisions.	13 - 15	D	A	B
6	In all non-elastic collision, kinetic energy decreases.	15	D	A	B
7	In conservation principle, total often disregarded.	15, 17, 21, 31, 33	D	A	B
8	Equivalence of power and energy.	19	A	D	B

In table 2, the identified misconceptions and their possible sources are ordered. The order of occurrence of the sources of the identified misconceptions is D A B for all except one of the misconceptions. In this misconception, the order is A D B and this relates to task 19 where the misconception of regarding power as energy was detected. The same order of occurrence of sources of misconceptions is obtained for tasks 21 and 23 in table 1 although in deciding on the order of occurrence of the misconception associated with tasks 21 and 23, the order is dominated by the results obtained for the other tasks for which the identified misconceptions were detected.

Discussion:

Like the previous studies (Ivowi, 1985, 1986a and b), seven misconceptions were again identified and their sources investigated. In most of the tasks and misconceptions listed in tables 1 and 2, the order of occurrence of sources of misconceptions is D A B which indicates that the most possible source of misconception is textbooks. As is common in schools, both teachers and students depend highly on textbooks. One of the reasons for this authoritative nature of textbooks is the process through which textbooks development normally undergoes. Apart from consultation with standard reference books, experts are usually given a chance to assess the books before publishers put these books out for school use. Under this condition, factual mistakes are greatly minimised or completely eliminated.

By the result of this study, a suggestion appears to be made to the fact that many physics textbooks used in Nigeria contain misconceptions or contribute towards the misconceptions held by students. The literature on the subject had already indicted textbooks as sources of misconceptions; the present result has therefore confirmed this suggestion empirically. Since most of the physics textbooks used in Nigeria are foreign and are standard books used in developed countries, it is reasonable to suggest that the likely cause of misconception through the use of these books may be due to language difficulty. The students investigated in this study have English as second language and may therefore encounter difficulty in using books written by authors whose mother tongue is English.

The second source of personal experience/intuition may relate to cultural factors prevalent in the society. Most students come from home background devoid of educational toys and technological gadgets. Their experiences of the scientific and technological world are very limited; they virtually depend on school teaching and very limited laboratory activities for most of their experiences in physics. Under such a condition, proper understanding of physics concepts may be impaired and misconceptions may arise. Very closely related to this problem is the inherent traditional beliefs and superstitions which may appear logical and are at variance with scientific concepts. The association of thunder and lightning with electrical charges derivable from merely rubbing two electrically neutral substances is a case in point.

The third common source of teachers is not surprising. Misconceptions have been found to persist with years of teaching and learning and that teachers did not appear to be aware of these misconceptions in students (Ivowi, 1985 and 1986a). Teachers themselves have been found to exhibit some misconceptions in physics (Ivowi, 1986b). Teachers were once students and any misconceptions formed then persisted unless some effort was made to erase them. Such effort would most likely arise from attempt to teach concepts to ensure proper understanding. Such would be an objective of a teacher education programme. But the dearth of physics teachers in schools has resulted in both trained and untrained teachers teaching physics in schools. The result of Ivowi (1986b) however indicated that no significant difference was obtained in the contribution of trained and untrained teachers towards the existence of misconceptions in student probably because of the dependence of teachers on textbooks and the popularity of the lecture method in teaching.

Since both students and teachers depend on textbooks for the teaching and learning of physics, the predominance of textbooks as a source of misconception is logical. Teaching requires the use of textbooks, teachers' knowledge and strategies to cause meaningful learning in students. For teaching and learning of physics to improve, the identified sources of misconceptions need to be elminated. This is particularly significant especially at this time of developing new textbooks for the new system of secondary education in Nigeria. Authors who are themselves speakers of English as a second language are likely to write at a level of easy communication to the students. This and non-verbal communication strategies need, however, to be more utilised in the circumstance in order to ensure improvement.

Conclusion:

This study has identified three sources of misconceptions in students; the most common of which is the textbooks used for teaching and learning physics in schools. Since both students and teachers depend on textbooks, efforts at minimizing or eliminating students' misconceptions in physics concepts need to be directed at improved textbooks writing. This is particularly revealing and relevant as new textbooks are being developed for the new system of secondary education in Nigeria.

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ASPECTS OF THE UNDERSTANDING AND TEACHING OF THE LAWS OF SCIENCE

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1. INTRODUCTION

The following question was posed to a group of fourteen graduate student teachers[1] in a class test:

State Newton's third law of motion in words without using the words action and reaction.

The answers given included the following:

- *For every force exerted, there exists an equal and opposite force which is exerted to maintain equilibrium.*
- *For every operation/movement there exists an equal and opposite operation/movement so that the body will remain at rest or continue to move in a straight line with uniform speed.*
- *When two bodies are in contact, each exerts a force on the other. This interaction causes no motion. The forces are balanced. (Translated from Afrikaans)*

These are three examples taken from seven cases of students who held misconceptions about this law. Being familiar with some of the research and literature about misconceptions, this was not too big a surprise, yet the idea that those students were to be "let loose" amongst pupils who knew even less (or who had even more misconceptions) was disturbing! This and similar experiences with other laws/principles as well as the occasional research reports roused my interest to research pupils' and students' understanding of laws in general.

Before sharing ideas, it is necessary to state some of the underlying assumptions which will be serving as a frame of reference. Most of the assumptions are very well known and are based on the ideas and views of the leading exponents in this field.

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[1] All these students had at least completed a first year course in Physics which included Newtonian Mechanics.

In this paper we attempt to synthesize current insights on the understanding of the laws of science, and also report some further ideas and information ensuing from my present research in this field.

2. UNDERLYING ASSUMPTIONS

2.1 *Knowledge acquisition is a constructive or generative process and each student's knowledge is personal and idiosyncratic.*(Fisher and Lipson 1986:784) This implies that students use their existing knowledge to make sense of new experiences/ideas - or in the words of Posner *et al* :
An individual's current concepts, his conceptual ecology, will influence the selection of a new central concept. (Posner, Strike, Hewson & Gertzog 1982:214)

2.2 **Due to their different conceptual ecologies, different students can "incorporate" the same new experiences/ideas differently in their conceptual structures/frameworks.** This implies that even if students are exposed to the same experience (say a demonstration to verify Boyle's law), they will not necessarily come to the same desired conclusion.

2.3 **In order to accommodate a new conception, the following conditions should be met:**

- There must be **dissatisfaction** with the **existing conception**.
- The new conception must be **intelligible**
- The new conception must appear initially **plausible**
- The new conception should be **fruitful** (Posner *et al* 1982:214)

2.4 **Misconceptions may originate as a result of the student's interactions/experiences with the real world and/or because of his misinterpretation of the presented world of ideas.** Driver and Easley (1978:62) refer to these two kinds of conceptions using the terms *alternative frameworks* and *misconceptions*. In this paper the terms *alternative conceptions* and *misunderstandings* will be used to distinguish between these two kinds of conception. The term *misconceptions* will be used collectively to denote either or both these kinds of conception.

2.5 **Misconceptions (especially alternative conceptions) are tenacious and** *are not likely to change simply because pupils are exposed to teaching strategies in which they are given the physicist's explanation.* (Terry, Jones & Hurford 1985:162; Stepans, Beiswenger & Dyche 1986:65-69) To complicate matters further students may adopt a strategy which Gilbert, Watts & Osborne (1982:63-64) call the *two perspectives outcome*. According to this view the pupil may retain his own misconception , but also memorise the teacher's viewpoint as something to be learnt for an examination. The learned science viewpoint, however, is not applied outside the formal classroom situation.

The situation is complicated even more if we take into account that students' misconceptions are often *quite adequate for interacting with a limited domain of the world and that through such interaction these preconceptions are procedurally encoded.*(Hashweh 1986:232) Hashweh continues by stating that certain situations can be understood by using any of a number of conceptions that seem to account for those situations. (*op cit*)

2.6 *Learning is a process of successive approximation and continuous refinement of our mental models of the world* (Fisher & Lipson 1985:51).

2.7 **The student's ability to process information is limited due to the finite capacity of the working (short-term) memory** (Fisher & Lipson 1985:53; Johnstone & El-Banna 1986 & Novak (1984:11) It is believed that the working memory capacity is limited to 7 ± 2 "chunks" of information. Johnstone & Wham (1982) discuss different applications of the limited capacity of the short term memory for science teaching, but in particular the implications for practical work which they mention are important for the learning and teaching of laws/principles.

2.8 **In accordance with information processing theory, it is assumed that knowledge is organized into frames and schemas** (Fisher and Lipson 1985:54-55)

It must again be emphasised that these are only **some** of the assumptions which will act as a frame of reference for this paper. Being one of the central concepts in this paper, it is also necessary to present my views on *understanding* or on *What does it mean to understand?*

3. WHAT DOES UNDERSTANDING MEAN?

Nickerson (1985:215) points to how difficult a question this is:

What it means to understand is a disarmingly simple question to ask, but one that is likely to be anything but simple to answer.

This view of the elusiveness of what it means to *understand* is shared by other authors (e.g. Ormell 1979:32). I will try to clarify the meaning which will be given to the term "understanding" in this paper.

Instead of trying to give a definition, the main characteristics of understanding as described and explained by a number of authors, and which coincide with my own views, will be discussed briefly.

- The act of trying to understand some phenomenon can be seen as the process by which the student (learner) tries to find one-to-one correspondences between aspects of the phenomenon in the *real world* on the one hand and the corresponding elements and relations of the schema(ta) of the *world of ideas* on the other hand. Hashweh (1986:232) declares that .....*understanding occurs when one realizes that a certain situation is a particular case of a certain generalization.*

  Fisher and Lipson (1986:785) have similar views which they express as follows:
  Understanding lies in the extent to which one's internal mental models successfully predict and explain events in the external ("real") world.

- Understanding however is not restricted only to the extent in which the *real world* can be mapped onto the *world of ideas.* There are numerous situations where the learner has to relate some aspect(s) of the *world of ideas* to some other aspect(s) of the same world. A person might be able to map *real world* phenomena correctly to Newton's First and Second laws, yet still not realise that the First law is a special case of the Second law. In this paper the attention will be focused largely on the first type of understanding.

- Understanding is very often context-dependent. Depending on whether, for example, a term such as **force** or **weight** is used in an everyday or scientific context, the meaning can change. But the meaning of a term or symbol can even change within different scientific contexts. The meaning of the well known "=" sign can vary considerably; compare its meaning in $x = x+1$ (Computer Science) and $y = x+1$, $2x+3 = x+10$ or $2(x+3) = 2x+6$ (Mathematics). (Nickerson 1985:217,219; Hewson 1980:398)

- Nickerson (1985:217-221) also convincingly explains what he calls the *nonbinary nature of understanding.* According to this view, understanding is not an all-or-nothing affair, but can vary in degree of completeness. The extent to which a person understands something is dependent on the amount of knowledge which the person has about the related concepts.

  In other words, the degree to which one understands an assertion must depend on the richness of the conceptual context in which the assertion can be interpreted. (*Ibid*:217)

  In the same vein Strike & Posner (1983:54) distinguish between *minimal and full understanding* thereby suggesting the idea of levels of understanding. They use, inter alia, a person's ability to apply the conception as an indication of his/her level of understanding of the new conception. As an indication of minimal understanding, the student should be able to apply the conception to stereotyped and simple problems, while full understanding demands the application of the conception to complex and novel situations.

- An important distinction is made by Richard Skemp (1976). He differentiates between between two forms of understanding, namely *instrumental* and *relational* understanding. Although Skemp might not necessarily agree, we like to think of these two forms of understanding as *levels of control/command.* The difference between these two levels of control can be illustrated by the following example. A child can use a set of rules (algorithm) to write chemical equations without any problems, yet he might still not know where the rules come from or why he uses them in the way he does. This kind of behaviour(understanding) is known as *instrumental understanding*, while a person who also knows where the rules come from and why they are applied in a specific way, displays *relational understanding*.

- Strike & Posner's (1983:26) view that understanding and accommodation are not synonymous is endorsed in this paper. A person can understand a certain conception without necessarily agreeing with the stated idea. If a student states that:
  A bicycle stops when you stop pedalling, because no forces are acting on it
  one may understand the literal meaning of the communication, but that does not mean that one agrees with the statement. For the accommodation of a new conception to take place, the conditions stated in 2.3 should be met. According to this view, the process of accommodation, embraces that of understanding.

- Past efforts to formulate a single all-embracing answer to the question *What does it mean to understand*? have only been marginally successful, partly because understanding is very much *content-dependent*. Understanding a concept such as acceleration is not the same as understanding a concept such as evaporation. These in turn differ markedly from understanding a law of science or a model or an algorithm. This view is also shared by Davis (1978:13) who comes to the following conclusion regarding Mathematics:

  Understanding depends on the type of mathematical knowledge. Understanding a concept is different from understanding a procedure.

  Because knowledge of the different types of content is of such importance to the understanding of the subject (content), a brief overview of a model of types of content is given in the following section.

- Another aspect of the nature of subject matter which might influence the understanding of content, is the extent to which the student has been exposed to sensory and linguistic experiences which in turn could affect the stability of the student's conceptual framework. Penetrating a conceptual shield which was moulded on regular, first-hand experiences might prove a much more formidable task than when the student has had limited exposure to the content. (Driver & Erickson 1983:49)

4. TYPES OF CONTENT IN PHYSICAL SCIENCE

In the literature the following types of content are frequently distinguished:facts, concepts, generalizations and procedures or algorithms (Cooney, Davis & Henderson 1975; Romiszowski 1981; Merril & Wood 1974). This however is insufficient and more detailed distinctions, especially for *concepts* is necessary. A careful content analysis of what passes as *concepts* should reveal differences between *concepts* which call for different teaching strategies and instructional moves to guide the student to a better understanding of these content types. The following *types of concepts* can be distinguished in Physical Science:

- Concepts which describe processes (eg evaporation, interference, ionization, refraction, etc.)
- Material concepts (e.g. acid, voltmeter, ammeter, sodium chloride, etc.)
- Theoretical concepts (e.g. electron, atom, electric field, etc.)
- Variables (e.g. acceleration, velocity, temperature, electron-affinity, etc.)
- Mathematical concepts (e.g. direct and inverse proportion, square, etc.)
- Technical terms (cathode, group, standard cell, ocular, etc.) (Jordaan 1984:36-44)

Over and above these types of concepts, Jordaan also mentions the following: models, theories, **laws**, experimental procedures, algorithms and conventions(including notations). Some of these types of contents are analysed and classified into even more detailed subgroups by Herron, Cantu, Ward & Srinivasan(1977:185-199)

Because a more comprehensive discussion of types of content is beyond the scope of this paper, we will now turn our attention to what it means to understand a law of science.

5. WHAT DOES IT MEAN TO UNDERSTAND A LAW OF SCIENCE?

A law of science is to be taken as an account or elicitation of a regular occurence in nature. This includes well-known laws such as Boyle's law, Newton's laws, Archimedes's law, Coulomb's law, etc. However the description will not be restricted to these well-known laws, but will also include general statements or assertions such as: *Metals expand when heated* or *Light is refracted towards the normal when it moves into an optically denser medium.*

Because a comprehensive discussion of all aspects which could contribute to meaningful conceptual change exceeds the scope of our present discussion, the attention will be focused on

- the abilities which a student should display if he/she understands the literal meaning of a law i.e. if the law is intelligible and
- instructional moves which could contribute to a better understanding of the literal meaning of a law

These two aspects will be discussed similtaneously.

By integrating my own views with those of Davis (1978:15) and Reif *et al* (1976:213) the following general distinctions have been made. For a student to understand the literal meaning of a law (i.e. for a law to be intelligible), it is necessary that he/she should

(i) understand the concepts used in the law;
(ii) understand the mutual interrelationships between the concepts and be able to tell what the boundary conditions are within which the law is valid;
(iii) be able to state the law in his/her own words (paraphrase it) and also recognise an alternative formulation/representation
(iv) recognise and/or give and/or construct simple applications of the law.

5.1 UNDERSTANDING THE CONCEPTS USED IN THE LAW

For the sake of clarity, I will in this section illustrate the points I wish to make by using some of the best known laws. Consider the underlined terms in the following formulation of Newton's Law of Gravitation:

All particles in the universe exert a gravitational force on each other. The force between two particles is directly proportional to the product of their masses and inversely proportional to the square of the distance between their midpoints.

In this formulation we can differentiate between different kinds of concepts, for example:

(i) "everyday" concepts like *particles, universe, exert*;

(ii) mathematical concepts like *directly, inversely, proportional, product and square*;

(iii) scientific concepts like *gravitational force, mass, force, distance (mostly variables)*.

5.1.1 Everyday concepts

To understand a law, each of the concepts used in it must be understood. Although one might expect students to have few problems with everyday concepts, research conducted by Cassels and Johnstone (1985) provides evidence to the contrary. These authors specifically mention that *exert* is confused with *urge, exit* and *exempt*. In my own research it has become evident that even fourth year graduate Science students[2] confuse the meanings of the Afrikaans equivalents of logical connectives such as *unless* and *provided that*. When asked to identify synonyms for *unless* and *provided that*, 25% and 37,5% of a group of 56 of these students respectively could not select the correct alternative from five possible answers. The implication of this confusion becomes evident when these students are confronted with a proposition such as Newton's First Law. The following formulation of this law then becomes quite acceptable to these students:

A body will remain at rest or continue with a constant velocity provided that the forces acting on it are unbalanced.

5.1.2 Mathematical concepts

Cooney and Davis (1976:219) state, for example, that if students do not understand Newton's Gravitational Law, it might be because they do not understand concepts like *directly* or *inversely proportional*. In research which is in progress, it is already evident that students do indeed have a lot of problems with these concepts. Some of the incorrect notions uncovered so far include the following:

- Inversely proportional has the same meaning as reciprocal
- If s is directly proportional to the square of t, then students believe that:
  - # $t = ks^2$
  - # t becomes four times larger if s is doubled or
  - # $s = t^2$ or
  - # if $t = 2$, $s = 4$

The following worthwhile recommendations are made by Cooney and Davis (1976:219) should it be apparent that students are struggling with these conceptions:

If such were the case, examples and non-examples of inverse and direct proportions might be generated for consideration. If this strategy did not alleviate the difficulty, the teacher might then consider reviewing necessary and/or sufficient conditions for the existence of inverse and direct proportions and compare and contrast these two types of proportions.

As in the case of "everyday" concepts, the teacher/lecturer has an important clarifying role. In the case of mathematical concepts the use of examples and non-examples could play a very important part.

Although the above discussion centered around the concepts of direct and inverse proportionality, it is suggested that similar problems probably exist for other mathematical concepts which are used in science.

5.1.3 Scientific concepts

Each statement of a law contains some *scientific concepts*. Most of the words used to represent these concepts are taken from everyday language and given a specific (related) meaning within the scientific context of a law. Sutton (1980:52), however, focuses attention on an important aspect in this respect:

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[2] All these students had taken tertiary courses in both Physics and Mathematics and have majored in one or more of the following subjects: Botany, Zoology, Physics, Chemistry and/or Mathematics.

Words do change in meaning as they are reapplied in new situations, because someone takes a part of their connotation and others accept it as a new denotation.

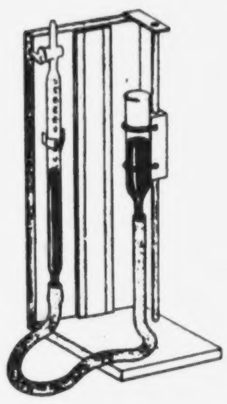
Sutton (1980:53) also makes the important point that despite the efforts of scientists to fix the meaning of words, they often do not succeed. Pupils in turn must also try to construct or reconstruct meaning from the words with which they are confronted. Gilbert, Watts and Osborne (1982) and Gilbert, Osborne and Fensham (1982) have found that pupils very often interpret words in science (such as force) according to their everyday usage. If this is done, it is quite conceivable that pupils could have problems in understanding laws which include these words.

The responsibility of the teacher to guide the students to a clear understanding of the differences and similarities between the everyday and scientific meanings of concepts used in the law can hardly be over-emphasised. One of the most important functions of the teacher is to identify the students' conceptions of these concepts and to rectify the situation when necessary. Methods which can be used to determine the prior knowledge of students are suggested and discussed by various authors. (Sutton 1980; Osborne and Gilbert 1980; Posner & Gertzog 1982; Duit 1984; Jung 1984) Teachers should take cognisance of these methods and should become familiar with the application of at least some of them.

However it is not sufficient to just make sure that pupils understand the correct scientific meaning of the concepts used in the law. They must also be able to apply those concepts within the context of the law. This can be illustrated by using Newton's Laws of Motion. If a student has to calculate the acceleration which a body will experience if different forces act on it, (s)he must be able to identify all the relevant forces acting on the body. [Research done by Marais(1985) reflects a very gloomy picture as to Technikon students' ability to identy forces in simple everyday situations.] If (s)he cannot do this, (s)he will not be able to apply the law even if (s)he understands the literal meaning of the law. To help the student in this it is necessary to identify his/her prior knowledge and to make sure that (s)he actually commands the required knowledge. This forms part of what I believe Strike & Posner (1983:13) refer to as the construction of a framework within which to locate the new idea.

When conducting experiments aimed at verifying or (re)discovering a law, it is also very important that pupils should understand precisely how variables such as pressure, velocity, force, etc. are measured. When performing the experiment aimed at verifying the law of conservation of momentum, velocity is obtained indirectly by measuring the displacements in equal time intervals. To "prove" the law, mass x displacement (and not mass x velocity) is calculated. The need to clarify this indirect procedure should be obvious.

A similar situation is encountered during the verification of Boyle's law using the apparatus illustrated in the accompanying sketch. In this case the procedure to obtain a pressure reading is so complex, that the student cannot, figuratively speaking, see the tree for the forest. The result could well be an overloading of the short term memory (cf. Johnstone 1985). The intended support which the student should have received, is thereby nullified. In this instance a Bourdon pressure gauge which gives a direct reading can help to save the day. However I do believe that it should always be a principle to try to use apparatus which is easily to operate and which gives direct readings.



Pienaar and Walters (1979) p. 506

A last aspect regarding the understanding of the concepts used in a law, concerns the symbolic representation of the law. Because teachers very often and very soon start using symbols to represent the variables, it is of the utmost importance to confirm that students know exactly what each symbol represents. For example in the Gravitational Law it is important that pupils realize that "G" represents an *universal constant* which is independent of the mass and position of the bodies (particles). My own research has indicated that students believe that "G", like gravity, only applies in the proximity of the earth.

Another source of uncertainty and error, is the confusion which exists between "G" and "g". The need to distinguish clearly between related (and confuseable) "entities" included in this and other laws should be self-evident.

5.2 UNDERSTANDING THE INTERRELATIONSHIPS AMONGST CONCEPTS AND SPECIFYING THE BOUNDARY CONDITIONS WITHIN WHICH THE LAW IS VALID

Although it is important that students should understand the meaning of the concepts mentioned in a law, it is just as important that they should understand the relationships amongst these concepts. Posner *et al* (1982:216 - with reference to Bransford and Johnson) capture the essence of what I have in mind in the following words:

Intelligibility also requires constructing or identifying a coherent representation of what a passage or theory is saying.

In an effort to help students to grasp the interrelationships, the teacher/lecturer can conduct a real "experiment" (demonstration) or a thought experiment. When the relationship between the pressure and the volume of a gas is to be explained, the actual performance of the experiment, followed by the processing of the data to show that $p_1V_1 = p_2V_2 = p_3V_3 = p_4V_4 = p_iV_i$ = constant, can help to clarify the exact relationship. Cooney and Davis (1976:218) share this view:

Sometimes examining instances can help students understand a generalization. Other times, instances provide a means for students to believe a generalization.

The latter part of the quoted passage emphasises the role which instantiation (exemplification) can play both in making a new idea (law) intelligible and plausible. This notion is also shared by Strike & Posner (1983:10).

Another form of instantiation which can be employed, is to select a simple, well-known phenomenon which is actually an application or special case of the law, and to use this to illustrate the meaning of the law. This can be done using a real experiment or simply a thought experiment. To elucidate the meaning of the law *The rate of a chemical reaction increases if the exposed surface area of the reactants is increased*, the teacher can *demonstrate the combustion of a thick piece of wood as compared to the combustion of the same mass of finely chopped wood.*

To help students cope with Newton's First Law, it is often a worthwhile mental exercise to ask students to imagine a large, smooth, frictionless plane surface on which a piece of ice is placed. The ice is set in motion and then left alone. What whould happen to the motion of the ice? (The assumption that the surface is frictionless must again be emphasised.) Through discussion the students can be led to see that this is a special case of Newton's First Law. This is but one example of how students can be guided to a better understanding of a law with the aid of thought (*Gedanken)* experiments. I believe that the use of both real experiments and thought experiments are two ways in which the students' conceptual frameworks can be connected to the real world (cf. Strike & Posner 1983).

Another aspect of the understanding of a law with which students seem to have problems, is their (in)ability to state the conditions under which the law applies and to differentiate between the conditions (antecedents) and conclusions (consequents) of a law. To illustrate what is meant by the conditions and conclusions of a law, let us consider Boyle's law:

Conditions	**Conclusion**
If the mass of gas is constant	
If the temperature is constant	then the volume
If the pressure increases	decreases

Cooney and Davis (1976:219) express their views on the importance of this aspect as follows:

The explicit identification of the components of an implication stemming from a generalization is an important pedagogical consideration unique to the teaching of generalizations. Students who cannot identify antecedents and consequents may experience difficulty in proving or applying a given generalization.

Typical errors made by the fourth year graduate students[3] regarding the conditions within which a law applies, include the following:
(i) adding extraneous conditions which do not apply;
(ii) ignoring conditions which do apply.

It was found that:

- 41% of the students believed that momentum is only conserved if an elastic collision occurs;
- 12,5% of them believed that momentum is conserved only if the bodies continue moving in the same straight line after a collision;
- 12,5% of the students believed that momentum is conserved only if the kinetic energy is also conserved;
- 26,8% of them believed that Newton's law of gravitation does not apply to bodies which are far from the earth;
- 14,3% of the same group believed that Newton' third law does not apply to bodies which are accelerated, while 25% indicated that they did not know the answer and 23,3% did not respond.

When given the following incorrect formulation of Boyle's law and asked to identify the errors (if any) not one of these students could identify all the errors.: *The volume of any amount of matter is inversely proportional to the pressure exerted on it.*

Warren (1979) also gives a good example which similarly illustrates pupils' inability to identify the conditions within which a law applies. He identifies the form in which the law is presented, as an important factor influencing the understanding of a law. Warren suggests that Newton's Third Law *is widely misunderstood because of the most unfortunate custom of expressing it in one of the epigrammatic forms, 'to every action there is an equal and opposite reaction' or 'every action is opposed by an equal reaction'*!

According to Warren the first of these formulations implies that there is a time lapse between the action and reaction which is of course incorrect. The second formulation on the other hand could lead pupils to interpret the law to mean that both forces are exerted on the same body (cf. Terry & Jones 1986). 38 (67,9 %) of the group of 56 students mentioned earlier believed that the downward gravitational force of the earth and the upward force of a table form a action-reaction pair if a book lies motionless on a table.

The obvious solution is to reformulate the law so that the correct meaning is clear, for example:

If a body A exerts a force on body B, then body B simultaneously exerts a force of the same magnitude but in the opposite direction on body A.

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[3] This is the same group of 56 students referred to earlier

An alternative strategy could be to place the law in its historical context, i.e. the context in which Newton originally stated it.

Human (in Möller 1980:111) also mentions the possibility of reformulation as a strategy to facilitate the understanding of a proposition. However he adds a practical hint in this respect, and that is to state the law in an "if....then" form, for example (Newton 1):

If no unbalanced forces act on a body, then the body will maintain its state of rest or uniform velocity.

Another strategy (instructional move) which can be employed in explaining the meaning of a law, is to use an analogy (cf. Strike & Posner 1982). In this respect the analogy between electricity and water could be put to good use in explaining some of the generalizations which are encountered in electricity. When using analogies, there are two aspects which call for special attention:

- make sure that the data-base which forms the analogy, is known to the students;
- the positive, negative and neutral (cf. Hess 1974) aspects of the analogy should be distinguished clearly. This view is also shared by Hashweh (1986:239-240).

5.3 ABILITY TO STATE THE LAW IN HIS/HER OWN WORDS (PARAPHRASE IT) AND ALSO RECOGNISE AN ALTERNATIVE FORMULATION/REPRESENTATION

During the assessment of pupils' knowledge of a law, the ability to repeat a text-book formulation is quite often taken as the desired answer. If this is the case, the possibility exists that the students might believe that their ability to regurgitate a few words is also a true indication of their understanding of the law. Anderson and Smith (in Hashweh 1986:242) report that *loose criteria in evaluating students' responses contributed to the persistence of beliefs about light in the elementary school.*

All fourteen students referred to at the beginning of this paper were able to state the epigrammatic form of Newton's Third Law, yet 50 % of them disclosed misconceptions when asked to formulate the law without using the words *action* and *reaction*. I believe that this experience contains three very important messages for the teaching of laws. **Firstly**, during the initial instruction, it is essential to confront students with different verbal formulations of the same law, to explain and elucidate these and to check whether they grasp the meaning of the law by asking them to reformulate the law in their own words. The reformulation could initially also be done as a group activity, but in the end each student should be able to give his/her own formulation. It is also desirable to determine and explain the links between different formulations. Imagine a student's confusion if he were taught the first of the following two statements of Newton's Second Law, but were later confronted with the second as an alternative formulation:

Statement 1: *The acceleration of an object is directly proportional to the applied force and inversely proportional to the mass of the object, and its direction is the same as the direction of the force.* (Meiring, Getcliffe, De Villiers, De Vries & Van Tonder 1982:70)

Statement 2: *Force is proportional to the rate of change of momentum, and the change of momentum is in the direction of the force.* (Pienaar & Walters 1981:68)

(Comment on the correctness (acceptability) of the statements is deliberately withheld)

The **second** important implication is that should one want to test the students' understanding of a law, the least one should do, is to put a ban on the use of certain key words - or in other cases it may be necessary to require students specifically to use certain key words.

For the **third** implication I would like to quote Hashweh (1986:242):

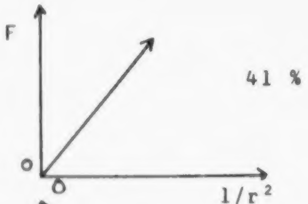
It would help if we could more clearly specify and agree on the level of understanding of a certain concept and the range of contexts in which we expect it to be used.

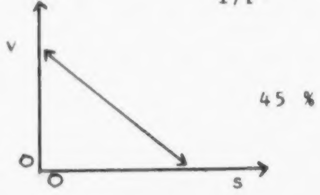
In our case the word concept can be replaced by *law*.

Besides being able to give a **verbal** formulation of a law, students should also be able to recognise and give other formal formulations or representations of the law. For example, students should be able to recognise and give Boyle's law in its graphical form(s) and symbolic form(s) [pV = constant or $p_1V_1 = p_2V_2 = p_iV_i$ or $p \propto 1/V$, etc].

Another, yet more important aspect, is that students should understand the interrelationships between the different formulations/ representations of the law. One should remember that the isolated understanding of the different formulations/ representations i.e. either the verbal or symbolic or graphic form, is no guarantee that the students grasp or can deduce the other formulations/ representations.

To substantiate my claims, consider the following results. When the group of tertiary students were given the accompanying graph and asked to select the best mathematical representation from given alternatives, 41 % chose F/r^2=constant. 45 % of the same group of students indicated that the second graph shown, represents an inverse proportionality. The need to reinforce the underlying mathematics and to pay sufficient attention to these interrelationships should be evident.





The relating of formulations should however not be restricted only to the correspondences between verbal, symbolic and graphical representations. By pertinently emphasing the relationship between, for example, $pV = k$ and $xy = k$, students' understanding of both Physics/Chemistry and Mathematics can be advanced.

5.4 APPLICATION OF A LAW AS AN INDICATION OF A STUDENT'S UNDERSTANDING OF THE LAW

In a previous section we indicated that the application of a law by a teacher can contribute to the understanding of a law. Conversely the ability to apply a law gives us an indication of the extent to which the student understands a law. This view is shared by Strike & Posner (1983:54) and has been mentioned earlier on in this paper (*vide supra*:3).

Although a student's understanding is reflected by his/her ability to apply the law, certain qualifications should be stated. Firstly, it is possible that students could be "applying" their knowledge to what might seem a genuine "application problem". Yet an analysis of the problem and of their prior experience might show that due to their previous encounters with similar problems, the problem has been reduced to what might be called a *plug-in problem*. (As an alternative, I also like to refer to this kind of problem as an *instrumental application problem*.) Many of my students are very proficient in solving this kind of problems, yet they still do not really understand the laws involved.

A second qualification I would like to make, is that a student might appear not to understand a law, because of his/her apparant inability to apply the law correctly. Let us consider the following incident in which a very bright student had to explain the effect of an increase in pressure on the $N_2/H_2/NH_3$ equilibrium using Le Chatelier's principle. The following chemical equation was given:

$$N_2 + 3H_2 \rightleftharpoons 2NH_3 \qquad \Delta H<0$$

She predicted that the equilibrium would shift towards the N_2 and H_2 and explained her answer as follows:

According to Le Chatelier's principle an equilibrium will shift if the conditions affecting it, are changed and the shift will be in such a direction as to diminish or cancel the **effect of the change.**

The essence of the rest of her explanation, given point by point, was:

- The condition which changes is the pressure;
- The **effect of the change** in pressure is an increase in temperature;
- To diminish the effect of the change, the temperature must decrease;
- To decrease the temperature, the equilibrium has to shift to the left (towards the hydrogen and nitrogen). *[Translated from Afrikaans]*

Being familiar with an article by Haydon (1980:318-321), I realised that this student was applying the ambiguous formulation of Le Chatelier's principle with which she was familiar "correctly", just to obtain a wrong answer. To rectify this situation, I did what Haydon suggests, i.e. reformulated the law in an unambiguous form. This incident demonstrates how easily a student might interpret the formulation of a law "correctly" and yet reach an incorrect result time and time again, because of the ambiguity of the statement.

Another strategy related to reformulation and which could be used to remove ambiguity, is suggested by Gold & Gold (1985:83-84). They suggest that the problematic principle of Le Chatelier be replaced by Van't Hoff's laws. Historically these laws preceded Le Chatelier's principle and the latter can actually be described as a generalization of Van't Hoff's laws. Gold and Gold thus in effect suggest a return to two less general laws as a solution to the problem. I believe the same strategy could also be used to good effect with some other laws (e.g. Newton's First Law) - especially as an intermediate step.

6. IN SUMMARY

As a point of departure, I mentioned very briefly some of the basic assumptions which could act as a frame of reference for the discussion which followed. I also disclosed some views on what I believe it means to understand. Inter alia, it was stated that the understanding of Physical Science is very much content-dependent. Hence to understand a scientific concept imposes other demands than merely to understand a law of science. It was suggested that to understand a law a student should:

- **understand the concepts used in the law;**
- **understand the mutual interrelationships between the concepts and be able to tell what the boundary conditions are within which the law is valid;**
- **be able to state the law in his/her own words (paraphrase it) and also recognise an alternative formulation/representation**
- **recognise and/or give and/or construct simple applications of the law.**

In the paper aspects which influence these four factors were discussed and instructional strategies/moves which could facilitate them, were suggested.

One does however realise that for conceptual exchange (accommodation) to occur, much more has to be done than to just lead the student to an understanding of the literal meaning of a law.

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Understanding Students' Understandings: The Case of Elementary Optics.

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1. Introduction

During the last year my group has worked an students' ideas in optics/1/. A paper illustrating the type of work and results /2/ is available in English (from the author). I summarize the main features of this work:

(a) Rather young students (ca. 10 and ca. 14 years)

(b) Concentric approach (combined techniques: interviews, written tests, with picture and/or real experiments, probing acceptance)

(c) The main interest is to understand students conceptions and to describe them (phenomenography, according to F.MARTON's conception/3/).

In this paper I focus on interview material. Studying many interviews, it becomes very clear that it is not easy for a physicist to understand students' talk, because a physicist is used to understand within the "meaning province" of the theories of physics. We think this type of work is useful because it can demonstrate to teachers how easy it is to miss the point in their students' statements. Of course, extracting students' conceptions of optical phenomena from interviews is beset with methodological difficulties. Therefore, in a first part I shall discuss my methodological background, and in a second part report understandings referring to the topic "light and seeing". We investigated other topics, "images", "light and colour", "light and movement". For obvious reasons I restrict myself to one topic.

2. Understanding and interpretation

Part of the difficulties to learn physics is that the meaning of terms depends on theory, or to be more general, on the "meaning province". (Similarly, in some parts of the world nodding means No, in most parts Yes.) Thus learning physics is analogous to getting accustomed to a foreign culture. And one reason why students do not learn physics, or very poorly so, is lack of opportunity to live in this culture: at best it is a "guided tour".

Another difficulty arises from the same situation, this time from the side of the teacher. Living in the "physics culture", he has difficulties to understand the difficulties of his students. This is true concerning talk, written statements, even actions, and experiments. The teacher, and by the same mechanism the investigator, understands students' reactions in his physics meaning province. Thus conclusions from interviews are marked by insecurity - the insecurity of every discourse between persons belonging to different cultures. It is the normal hermeneutic circle-problem. Having extensively worked with interviews as research instrument, we found it useful not only as an object of study in order to find out something about students' understandings - for that reason it was originally devised. We found it useful also as providing paradigm cases of what must happen in class teaching time and again: the real "misconception" arises from an incongruity of understandings, of the student and of the teacher.

Now, what can be done in order to make interpretation, i.e. the understanding by a researcher of an interview as a text, a more controlled enterprise? I see two main possibilities:

(1) Guidance by theory. This is a variety of verificationism: one scans the protocol for verifying passages. Of course, this does not avoid the hermeneutic circle. For verification requires passages already understood. The advantage seems to be that the theory is made explicit, and its functioning as a set of rules of interpretation can be scrutinized and criticized.

(2) Inductive procedures: the understandings to be extracted are not given by a theory beforehand, but arise from the material directly. Of course, this is again no way to avoid the hermeneutic circle. It is a procedure of hypothesis formation and evaluation. One instrument characteristic of this procedure is to find "juicy quotes", i.e. passages which are short, clear, and to the point. In understanding discourse we normally use this instrument intuitively. (We say, e.g., "please repeat what you said just now, I think that reveals to me what you are talking about" - though we may be in error in the end.) By studying interviews, lists of such key-passages emerge. And they are used to understand more cryptic passages, and can be used to reconstruct understandings. Of course, this is no foolproof procedure. But as I have said in the introduction, we use more than one approach. Some think "objective" tests are "hard" science in contradistinction to interpretation/4/. But data are what they are, in interviews and in tests. And also in tests the interesting issue is what test answers mean. You cannot sidestep interpretation, you can only camouflage it.

3. "Light" episodes

In this paper I concentrate on guidance by theory, restricting myself to one promising variety: Language meaning is learned by episodes which have significance to the child/5/. Thus we try to trace the development of meanings of "light" and similar words from discourse about common episodes. (There may be specialities according to culture and language. I do not know if I can successfully convey the German meanings.)

(a) "You cannot see any more, - switch on light" (mach' Licht an). This is a common episode since very early years in development. The ensuing process includes an action by a man, and a complex event: a lamp suddenly flashes up, it is very bright, even dazzling, and objects in the environment are bright, more or less so, and can be seen if looked at. It is safe to conclude that the word "light" has no clear meaning in this episode: light"making" is that process as a whole. Nevertheless, it can be analysed in different ways. Take the following example, phrased in the Schank-Abelson semantic calculus:

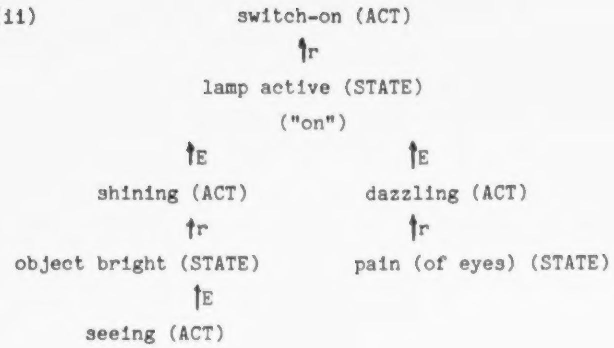
(1) switch-on (ACT)
↑r
bright (STATE)
↑E
see (ACT)

(The arrow expresses "dependency",e.g. the brightness depends on the switching)

The episode is conceptualized as an activity and a resulting state-transition, which makes another activity possible, seeing by looking-at. Interestingly, "light" does not enter this translation. Also the lamp is not seen as an ACTOR (or a cause). But this is a very primitive way of seeing the episode.

We know from interview material that a differentiation arises between very bright sources, especially the sun, but also mirrors, and lighted objects. The first are "dazzling", hurt the eyes, make pain etc. And therefore a shift occurs from the person seeing as an ACTOR to the source as an ACTOR which results in a certain STATE (effect) of the person. Normally lightet objects are never seen that way. They are passive receptors, targets of looking and receptacles of brightness which "comes from" the source as the ACTOR (or cause). Thus the semantic scheme gets more complicated:

(ii)



The branching of the schemes is well illustrated in a passage of an interview, when the student argues, no light must be sent from an object into the eyes in order to see it, because you can see a mirror when it does not dazzle you. That means: the mirror when dazzling is reflecting light - but then you do not see, because you are blinded. When the mirror is not reflecting you can see it. Also one very striking feature of our trials with "Probing Acceptance" (see /2/) was surprise expressed by students when told normal objects send out light.

Though the scheme is more differentiated, still the word "light" can be avoided. And we can trace the source for the vagueness of the word by looking at the scheme: Switching-on results in a state of activity of the lamp, the lamp is "on", the light is "on". But is the "light" the state of activity of the lamp, its dazzling brightness, its effect of letting see objects in its environment, of making objects bright enough? No easy decision is possible and the situation does not demand a decision of this sort. We need to know what light making means, but not light simpliciter.

It is characteristic of this more developed conception that seeing an object results from the coincidence of two activities, of the lamp (source), and of the person seeing. Often the looking-activity expressed by students, also symbolized by arrows, is interpreted as a rival theory of sight-rays. With the analysis given in mind, this is a misunderstanding. The activity of looking is a fact which cannot be disputed. And it would be a serious mistake to talk students out of it by appealing to the authority of science (which often happens). What must be done is to assist meaning differentiation: what is the meaning of light as an object of physics, and what in the province of everyday life? Now, light in the province of physics is a theoretical entity which cannot be seen (but which makes seeing possible under normal functioning of the organism). How can one get into that meaning province? As in similar cases, it is possible by starting from common ground. Physics would be inaccessible and meaningless without such common ground/6/. And here we can see some of it, even in this not very far developed conception: it is the idea of an activity of the source, of certain effects of this activity, even of one activity resulting in another one.

(b) Take another episode from early life, someone saying: "come out of this corner, it is too dark there, come here into the light", or "here's more light", or "it is much brighter here". Now there is no explicit mentioning of a source, though it may be implicit in the situation. The interesting difference here is a reference to parts of a room, to areas where light is present (such that you can see/read better), or is absent. Thus "light" (brightness) can easily be understood as something, an entity filling an area or space.

There are other episodes which focus on light as an entity present, e.g. when we refer to special sorts of light (early in the morning, "Frühlicht", or before a thunderstorm, "unheimliches Licht", or moon-light, "Mondschein"). The locution refers to something present with special perceptual, emotional, or artistic qualities. It does not refer to something invisible and moving, but it has certain effects. (All things are "bathed" in that light, "ins Frühlicht getaucht".)

It is true, sooner or later the child learns that this light "comes from the sun" (or whatever). But this phrase must not be misunderstood in a dynamic sense: it "comes from" means that the sun is the cause (or actor).

(c) Take a last episode, somebody saying "take a light (an electric torch) such that people can see where you are in the night". One may wave the light, and people see it. What do they see? A moving bright spot, not the torch, nor the person. They see it there, far away, and are certainly right to contend that no light comes to them, it is too far away and can no longer make bright. Sometimes interviewees say, one can see the "shine" there around the torch. (Similar locutions refer to a candle light.) In this episode, seeing the bright spot -the "light" - is not thought of as an effect of the torch on the eyes: looking and seeing a bright spot in a distance is the dominant idea. For somebody used to think in terms of physics the idea is quite surprising: You see the distant lamp, but no light comes to your eyes. In this episode the activity of seeing as grasping something far away, and the activity of shining and thus filling space with an entity "light", seem to be completely separate.

Of course, we are eager to ask a lot of questions. (And that is what normally happens in interviews.) For instance, how can you see the spot if no light arrives at your eyes? Well, because it is a bright spot there, (but not bright enough to light things here). We must always keep in mind that the meaning province of everyday life has a different constitution. It is more of a loose group of tribes ruled by ad hoc and inconsistent rules - but the inconsistency does not matter because normally there is no clash of episodes. No decisions are necessary in order to form a coherent scheme: Are the effects of the activity of the source direct - maybe retarded -, or mediated by something send out? In case we decide for mediation: Is it something which continuously comes out of the source, moving away from it, filling space whilst moving through it? Or is it something sent out once (when switching-on the source) and staying there in the space, waiting for objects to be lighted? When reflection occurs, is it the same entity coming from the source and sent on by the reflecting object, or is the object activated to send out something for itself?/7/ Is that something sent out visible or not? Is it bright or not? Does it make bright by staying on the object, or by surrounding it, or by activating the object to send out in turn? In normal life, no decision concerning these, and more, alternatives are made nor required.

In interviews we often find vagueness and inconsistencies. If the theory indicated is correct, or nearly so, we should not be surprised, because in interviewing students feel forced to make decisions on alternatives which were foreign to their mental furniture before. On the other hand, many passages can be understood as resulting from a hard core expressed by the scheme (ii) above.

I should like to decribe now the main features of this understanding of "light" and contrast it with that of physics.

4. Understandings of "light and seeing".

4.1 Physics

(1) "Light" means electromagnetic radiation which transports energy and interacts with matter in quanta; the registered quanta correspond to wave lengths of about 400 to 800 nm.

Thus "light" is looked upon as a physical system mediating the interaction between charged matter by exchange of photons.

(2) The source does not interact directly and instantaneously with other systems, but retarded. Believing in energy and momentum conservation, we must conclude that radiation is an entity which moves through space and exists even if the source ceases to be active. It should be noted that apart from conservation arguments, no experiment compels us to abandon the concept of a retarded direct action of source on object. Of course, this is not the understanding adopted in physics today/8/. It should also be noted that the movement of energy by way of travelling field excitations must not be understood in analogy to the movement of a ball: there are no "paths" of photons. Thus light qua radiation is a queer sort of physical systems. (And students naively denying that light can be localized between source and object may seem nearer to the conception of physics than the conceptions are which physics teaching does offer them.)

(3) Light interacts in a number of ways with objects:

(a) Absorption: transformation of radiation energy into mechanical, thermal, chemical energy.

(b) Re-emission: part of the incoming energy excites higher energy levels, which are lowered by emitting photons. There are two types of this process:

(i) Exciting eigenstates. In that case emission is normally incoherent.

(ii) Excitation far away from eigenfrequencies. In that case classical electrodynamic scattering theory is a very good approximation to what quantum theory predicts: coherent scattering. According to patterns of scattering centres, re-emission varies widely from totally diffuse to sharp "foreward-scattering", i.e. reflection according to the mirror law (which is, of course, never 100%).

(4) The objects scatter incoming radiation, part of which can form a real image on the retina, provided correct accommodation applies.

(5) The physiological processes which are required such that "seeing" may occur, are not part of physics proper.

4.2 Students' basic phenomenological understanding

A This understanding is primarily correlational.

(1) A luminous body ("Source") shines on an object which gets bright (brighter), or illuminated. (Implicitly, the luminous body is unterstood as the "cause" of the illumination.)

(2) An observer directs (head and) eyes to the object and keeps them open.

(3) The observer can see the object if not blind.

Remarks:

(a) "shining-on" implies a direct and uninterrupted connection between source and object

(b) "Shining" is an activity of the luminous body.

(c) The object is a passive receiver of illumination. To be illuminated (= more or less bright) is a state of the object.

(d) Directing the eyes on the object and accommodating is an activity of the observer.

(e) The observer sees the object *there*; he does not see his eyes (or his retina, or even his brain, as some interviewees said) *here*.

B The concept explains a number of special ideas expressed in interviews.

(1) White objects, or objects with so called "bright colours" (especially yellow), can be seen without a source.

We can, hypothetically, reconstruct how students form this idea by taking account of remark (c):

To *be* bright, means to be visible, and without doubt, a bright sheet of paper *is* especially bright. This brightness "comes from" the source, but obviously not from the source alone. For other objects are less bright with otherwise identical conditions, they may even be dark. Thus the white paper must have a special property which makes the difference: they *are* bright, and therefore they can be seen without a source. Of course, this is a chain of reasoning based on playing with the ambiguity of the word "is". But without a theorectical background it is difficult to avoid it. The fact that very bright objects may be felt as "dazzling" or otherwise affecting the eyes, i.e. that they may be sensed as active, may give additional weight to the reasoning.

(2) A source can be seen from a great distance without light "coming from" the source to the observer.

The ambiguity of the word "light" comes out here very clearly. Light does not come to the observer, because the source does not illuminate the observer and his environment. But at the same time he does see the source - its "light" as some explicitly state it -, because the source *is* bright. "Light", i.e. the brightness as an effect, is different from "light" as *being* bright. "Seeing" is not understood as an effect of "light" coming into the eyes, because, obviously, there is no light coming to the observer. Of course, it is easy for us to explain all that within our physics understanding, referring to high absorption and the threshold of optical sensation. But all this requires a different understanding of the situation.

(3) Illuminated objects cannot illuminate other objects.-

This was a strong conviction expressed in interviews. Now, to be bright means to be visible, but not automatically to be a source. To "switch on light" does make bright the source which illuminates the object: but you never "switch on" the object. Also the illuminated object is unterstood as passive, as receptor. "Reflecting" objects like mirrors are a special kind of objects. And what students eventually accept is that all objects are "reflecting", more or less so. They do *not* accept that an illuminated body is itself a source, as electromagnetic scattering theory says -even most teachers do not believe it. Even the fact that an object can *be seen* means that it is a passive object of an active grasping.And even after having demonstrated the illumination by an illuminated sheet of white paper, students denied that other objects do the same, and that this "shining" is a necessary condition for being seen. Of course: "shining" is a special process, subsumed under the category of activity. While *we* see a continuity between more or less intensive re-emission, students'understanding is based on the dichotomous categorization active-passive. For some,it is difficult to leave it.Even if they change mind and admit "reflection" of light by illuminated objects, they phrase it in a characteristic way: they say "light reflects *itself*". Thus in a subtle way they conserve their original understanding: the acitivity of the source is transferred to the "light" - whatever that means now - and the illuminated object is still a passive resistance to the light. (Similar shifts can be observed in mechanics referring to impact processes, see/9/.)

(4) Light stays upon the surface, or surrounds the objects. It is the presence of the source everywhere and the invariable illumination, the brightness of the surface making the object visible(/10/).

(5) It is not the case that new light, more and more light, is continuously streaming from the source to the objects. -
The idea of a continuous "stream" of light is absent from the phenomenological understanding. As long as the source is shining, the effect remains the very same -and in so far as light "comes from" the source, it is "enduring" (es kommt dauernd Licht). An analogy may help: If you paint an object red, you need not add continuously red paint, it is and remains red - till you rub off the paint.

C We can observe many ideas which belong to a transition phase from the phenomenological understanding to the understanding of physics/11/. One example was mentioned under heading B (3) ("reflection"). Another example is the admission of "light" as a kind of instrument, or messenger, or (phrased in a more philosophical tradition) emanation of the source, i.e. as a system with a sort of separate existence. And yet it is understood as bright (hell) - what else? We have the record of two 11 year old bright boys who did not believe that the light rays cannot be seen. The teacher used a LASER for a demonstration in the dark class-room. All students

agreed that the rays cannot be seen, but a small very bright spot on the wall. The two boys spontaneously left their places and rushed near the rays and triumphantly told the stunned rest of the class: We can see the light ray! Poor teacher /12/.

Teaching may eventually lead to admit "light" as "invisible": it is visible when falling on a body. Again in many cases a misunderstanding of what the teacher, hopefully, intends to say occurs: the invisible messenger of the source generates brightness when arriving at the body, and thus it - the bright body and so the "light" - can be seen./13/ No idea of the physics understanding that "brightness" means intensive scattering fo invisible light.

But again, even if scattering and the necessity of "light" coming into the eyes is admitted: in the eyes it becomes visible, at least on the retina light becomes bright!

5. Plus ca change, plus c'est la meme chose?

So it seems to be! What we are facing as physics teachers, especially in the early grades, is the phenomenon of cultural transformation by something like a clash of cultures. Many people reading this paper may get the impression that it blows up a simple piece of teaching to a philosophical problem. Show students the relevant experiments, and they will grasp the physics. I do not agree. Even experiments must be understood, and this can be done in different ways. In fact, we do know very little about that process of cultural transformation. And the aim of this paper is to make a case for teaching to be understood that way. So far, "misconception" research has been inspired by cognitive psychology which is an offspring of the physics culture. Putting teaching and learning within the context of anthropology, we may gain different ways of understanding what is happening in the teaching-learning interaction /14/. The subtle dialectics of conservation and change is well-known to anthropologists, as is the problem of the dependance of meaning on the cultural system. (Missionaries knew it long ago.) In teaching and learning, we observe very similar processes. As physicists, we are ill-equipped to understand their significance and to take it serious, though from the history of quite recent times we should know better/15/. Thus, apart from putting teaching-learning in an anthropological context, I should like to urge investigators to take serious the phenomenological understanding. Before applying allegedly objective testing with hard results, understanding is necessary. This may prove no less difficult for many of us as it is for students to understand our way of thinking. Also I plead for reconsidering what is understood as success of teaching: not merely the ability to retrieve a number of more or less general formulae and principles, but also the concomitant knowledge of changing from one meaning province to another, and the flexibility of doing so according to circumstances - and maybe a little less formulae and principles will do/16/. We are beginning to respect foreign cultures, and see them no longer as inferiors to our own in many respects. It is well in time to consider different understandings of the world in the same way. Referring to optical phenomena, the phenomenological understanding is a good candidate, because the understanding within the theories of physics cannot explain it away/17/.

References

/1/ The investigation is funded by the 'Deutsche Forschungsgemeinschaft', the staff comprises: JUNG,W. (director); MEIER,R.; WIESNER,H.; BLUMÖR,R.; HEISL,W.; MERENU,A.

/2/ JUNG,W.: Untersuchungen zur elementaren Optik; Schriftliche Erhebungen und Interviews, Research Report, 1985. Also JUNG,W.: Untersuchungen über Schülervorstellungen in Optik. Progress Report, 1986, and W.JUNG, Investigations of students' conceptions in optics. Typoscript 1987

/3/ See e.g. MARTON,F.: Phenomenography - Describing Conceptions of The World Around Us. Instructional Science 10, 1971, 177-200

/4/ See e.g. BURGES,R.G.(Editor): Strategies of Educational Research; Qualitative Methods. The Falmer Press: London-Philadephia, 1985
GARZ,D., KRAINER,K: Brauchen wir andere Forschungsmethoden? Beiträge zur Diskussion interpretativer Verfahren. Scriptor: Frankfurt/M. 1983

/5/ See SCHANK,R.C., ABELSON,R.P.: Scripts, goals and understanding. LEA: Hillsdale,N.J. 1977
NELSON,K., GRUENDEL,J: Generalized Event Representations: Basic Building Blocks of Cognitive Development. In: LAMB,M.E., BROWN,A.L. (Editors), Advances in Developmental Psychology, Vol.1. LEA: Hillsdale,N.J. 1981. One of my calloborators, R.BLUMÖR had the opportunity to make interviews in Zimbabwe with Shona speaking students. In Shona language, different words for "light" are used according to the type of episode. It is obvious that students often could not understand the questions asked in English.

/6/ "Modern elementary particle physics is delightfully anthropomorphic: look at the 'life story' of the 'strange particle' in a bubblechamber photograph... To understand the 'annihilation' and 'creation' of particles it is as necessary to have read fairy tales and murder mysteries as text-books of analytical mechanics." ZIMAN,J.: Reliable knowledge. Cambridge UP: Cambridge 1978, p.156. - This quotation, though somewhat overstated, expresses well the necessity of common ground, and the fact that it exists.

/7/ It was one of the main scientific enterprises of the important arab scientist IBN AL-HAYTHAM to prove that the moon emits its own light. He marks an important point of change from Aristotelian to more modern views. See SCHRAMM,M., Ibn Al-Haythams Weg zur Physik. F.Steiner Verlag, Wiesbaden:1963. See also JUNG,W.: Cognitive Science and History of Science, In: Proceedings of the Munich Conference on History of Science and Physics Teaching, München, Deutsches Museum, in press.

/8/ We owe BEVILACQUA a thoroughgoing historical reconstruction of the 19th century debate about action-at-a-distance, retarded action-at-a-distance, and continuous - or better contiguous - action conceptions in electromagnetism. In the end, conservation laws decided the debate in favour of continuous action, or field conceptions respectively. See BEVILACQUA,F.: The Principle of Conservation of Energy and the History of Classical Electromagnetic Theory. La Goliardica Pavese: Pavia 1983

/9/ See, e.g. JUNG,W., WIESNER,H., ENGELHARDT,P.: Vorstellungen von Schülern über Begriffe der Newtonschen Mechanik, Empirische Untersuchungen und Ansätze zu didaktisch-methodischen Folgerungen. Franzbecker,B.: Bad Salzdetfurth 1981, chpt.4.1.2.
See also Appendix 1 (The pushing scheme) of my paper: Uses of Cognitive Science to Science Education, Paper presented to the ATEE symposium, Kiel/IPN, 1985

/10/ In Interviews with German students as with Shona speaking students (see/5/), the idea that light "surrounds" the objects can be found. This comes out strongly in discussions about candle light. Conceptions of "strength" emerged in these discussions, e.g. candle light is not strong enough to reflect at a mirror, in constradistinction to the light of a bright electric torch.

/11/ In the field of anthropology, M.SAHLINS has delightfully described the subtle transformations of meaning ensuing from the clash of two cultures quite recently, see SAHLINS,M.: Historical Metaphors and Mythical Realities. The University of Michigan Press: An Arbor 1981.

/12/ The episode has been reported by HOCK,H.: Eine Untersuchung von Schülervorstellungen über Sehen und Licht. Examination paper, Frankfurt/M. 1980

/13/ Similar integrations of scientific conceptions and phenomenological ones are reported in the literature concerning other domains, e.g. astronomy, see ST.VOSNIADOU and W.F.BREWER, Theories of Knowledge Restructuring in Development, RER 57 (1987) No.1, 51-67. But the difference between astronomy and optics is that the phenomenological understanding in optics is unassailable. It may well be that this is also true concerning astronomy etc.

"Misconceptions" may result from unwarrented generalizations - but these may in turn result from questioning techniques, forcing children to fabricate answers. It is a desideratum to discriminate more clearly between real "misconceptions", e.g. that the sun goes to sleep, and phenomenological understandings which do not fit into the schemes of science because they belong to a different meaning province.

/14/ During the last years the rise of studies from an anthropological point of view may be observed, see, e.g., MENDELSOHN,E., ELKANA,Y. (Editors): Sciences and Cultures. Anthropological and Historical Studies of the Sciences. Reidel: Dordrecht 1981
See also my review article: JUNG,W.: New History and Sociology of Science (NHS) und die Ziele des Physikunterrichts. Physik und Didaktik 13, 1985, 257-262

/15/ I refrain here from using historical similarities, as some like it to do in (mis)conception research, e.g. ANDERSSON,B., KARRQUIST,Chr.: Light and Its Properties, EKNA-report, No.8, 1982. - The difficulty is that we cannot escape the hermeneutic problem. The historian needs to learn to read ancient texts, not with our present-day knowledge in mind, but with the cultural background of the past. It requires a lot of patience and intuition, and the respective control processes, to do that. If one has done it, understanding of students' talk may in fact improve. For examples, and dangers, of historical parallelism, see e.g. my Munich paper, ref./7/.

/16/ I refer to general education, not to training for a special career. Though I am of the opinion that it would do no harm to a future physicist to acquire some meta-knowledge about physics as one culture in conflict with others. The cocksureness of many physicists does not serve physics either. But luckily, there are many exceptions.

/17/ In his book Vision, D.MARR writes: For example, let us look at the range of perspectives that must be satisfied before one can be said... to have understood visual perception. First, and I think foremost, there is the perspective of the plain man. He knows what it is like to see, and unless the bones of one's arguments and theories roughly correspond to what this person knows to be true at first hand, one will probably be wrong. - Quoted from: AITKENHEAD,A.M., J.M.Slack, Editors: Issues in Cognitive Modelling. LEA: Hillscale,N.J. 1985, p.104. My reference to a "phenomenological conception" does not require understanding of the school of philosophy called phenomenology, founded by E.HUSSERL. I should like to avoid transferring the debate between philosophical schools into didactics. Nevertheless, some knowledge of phenomenological writings is useful. My own background derives more from early studies of C.S.PEIRCE, and A.N.WHITEHEAD, than from HUSSERL, see my: Aufsätze zur Didaktik der Physik und Wissenschaftstheorie. Diesterweg Verlag Frankfurt/M. 1979. Decidedly HUSSERLIAN is recent work by B.REDEKER,, see e.g. K.MEYER-DRAWE, B.REDEKER,: Der physikalische Blick. B.Franzbecker: Bad Salzdetfurth 1985

A SYNTACTIC SOURCE OF A COMMON "MISCONCEPTION" ABOUT ACCELERATION

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INTRODUCTION.

In the teaching of physics, which is the context space I want to keep as a focus, the "form-I-see" as a learner is generally natural-language prose, image-schematic descriptions like line drawings and graphs, algebraic expressions, and laboratory models of the "real world". It will be asserted that the meaning of the prose gives rise to syntactical form, and that the syntax constructs and structures "mental spaces".

One of the basic tasks of the learner is captured by the following question: How is the form I perceive related to meaning? Said it other ways, the question might be extended: How is the form I perceive related in any way to what I know? How is the form I perceive related to "the meaning that was intended" by the instructor? How has "the meaning that was intended" been used in constructing the form I perceive?

The data I will present here deals only with a common linguistic description of acceleration, but I will extend the analysis to some other examples. In what follows, the very general questions of form and meaning will be sharpened with specific examples.

COGNITIVE GRAMMARS.

The title of my talk refers to a syntactic source for a mistaken interpretation of a standard definition of acceleration. The definition referred to--*"Acceleration is defined as the time rate of change of velocity."* --occurs in the most widely used high-school textbook in the U.S.A. (Williams, Trinklein, Metcalfe, 1984). In referring to syntax and grammar in this paper, I use as bases the ideas of "cognitive grammars" developed by a number of linguists and cognitive scientists: Lakoff and Johnson (1980), Langacker (1986), and

Jackendoff (1983). I have also used some linguistically-related ideas, like the notion of mental-space construction offered by Fauconnier (1985) and of category construction as discussed by Lakoff (1986) and Medin, Wattenmaker, and Hampson (1987). It is also important to note in this context the work of di Sessa (1985, , 1987) who has developed the idea of "phenomenological primitives" like "force as a mover" and "dynamic balance". He views these as fragments of knowledge that students bring to a study of physics rather than any integrated viewpoint.

How are cognitive grammars related to the concerns of this conference? The relationship is direct, because evidence of misconceptions and misinterpretations often involve either the student's translation of natural language into concepts and/or the rendering of a physical observations or imagistic descriptions into natural language.

The most important basic departure of a cognitive-grammar approach from classical theories of grammar is the rejection of the idea that grammatical constructions are completely independent of the construction of meaning. Classical grammars are considered "autonomous" in the sense that a description of the grammar has no relation to the semantics of the the communication contained in the structure. That is, the classic description of grammar has little or nothing to say about semantics or meaning. For example, this notion of autonomy is a central aspect of the power of purely formal systems like arithmetic and algebra. On the other hand, the arguments of "cognitive-grammar" linguists center on the notion that grammatical constructions and syntax in general can have roots outside of the language facility itself. The major assertion is that grammatical form itself has a conceptual base and that its interpretation can construct a conceptually- based mental space[1].

Fauconnier's (1985) notion of "mental spaces" will be discussed later, but consider saying the following sentences aloud in succession. "Fruit flies like bananas." and "Time flies like an arrow."[2] The first might be considered a factual statement, but the second is a metaphor based on a view of time as a moving object. A sometimes starling shift occurs when the word "arrow" is encountered, and the previously-interpreted noun "flies" must be transformed into a verb that structures the sentence in a fundamentally different way. Consider what you would have done if the second sentence said "Time flies like blueberries."

A assertion related to the notion of conceptually-based grammars is the idea that concepts serve as agents of category construction. This notion involves rejecting the restrictive classic definition of membership in a category as a listing of necessary and sufficient attributes (Lakoff, 1986; Medin, Wattenmaker, and Hampson, 1987). Conceptual knowledge serves to make and rank the importance of certain relationships among attributes that lead to sources of categorization like Wittgenstein's "family resemblances." In cognitive grammars, the notions of "categories" and "reference" are developed more in accord with the constraints of psychological experience and experiment than the constraints of classical theoretical linguistics and the procedures of dictionary makers.

ROBUST "MISCONCEPTIONS"

The overarching question to which my paper is directed is the following. Is it possible to develop a more detailed story of exactly how and why students, across broad ranges of age and background, have difficulty with some linguistic forms of physics-based and algebra-based tasks, particularly when they are structured in certain ways? Previous work on related issues in the learning of physics and the translation of algebra word problems have offered numerous explanations, some pointing to language difficulties as one aspect of the performance difficulty. But in general these explanations have either been vague or have stopped with consideration of the surface features of the syntax. Is it possible to construct a model that might indicate in some detail why it is, when students are faced with tasks which involve translation from natural language, that erroneous outcomes occur so robustly across age, overall grade-performance status in a class, and mathematical skills training? Or, looking in the other direction at the translation issue, what prevents students from easily describing in natural language, in ways acceptable to physics teachers, their observations of physical phenomena? And, finally, one further extension of this same concern: What happens when a student is faced with a constructed graphical image-schema of physical phenomena and asked something about the physical situation that gave rise to it?

As examples of some studies which show these effects, the following are notable for results that are robust across age, educational background, relative physics-course performance (grades) , and other factors. The study of "Mindy's problem"[3] , sometimes known as the "professors-students" problem[4], has been extensive, documenting the robust nature of the "reversed-equation" response (Clement, Lochhead, and Monk, 1981; Sims-Knight and Kaput, 1983; Kenealy,1982). In summarizing investigations of this problem, Kaput (1985) has pointed to an apparent "overriding of a vulnerable understanding of algebraic syntax . . . by natural language syntax and rules of reference". Kaput (1987) later analyzed it further in terms of a default interpretation related to congruence between the adjective-noun syntactical form and the algebraic form. He states that "semantic and imagistic" factors have a strong bearing on the type of translation errors that students are likely to make in linguistically-presented problems. McDermott, Rosenquist, and van Zee (1987) have pointed out the difficulty that people have in connecting graphs to physical concepts and to "real-world" events, noting that the nature of the difficulties[5] is the same across all populations studied; from students in high-school physical-science classes to calculus-based-physics students. Others have worked with children's performance on addition, subtraction, and basic algebra problems and have pointed at the syntax and semantics as sources of difficulty (Resnick, Cauzinille-Marmeche, and Mathieu, 1987; De Corte and Verschaffel, 1987).

The possibility of coherently uniting some of these diverse difficulties by identifying a common source is a highly attractive one. The fact that students "get it wrong" in these studies in such particular ways and in such robust fashion may point to an underlying process that is basically the same in each case. The attempt in this paper applies the ideas developed by Lakoff, Johnson, Fauconnier and others to the particular area of difficulties in decoding and re-presenting physics and related algebra concepts.

DECODING A LINGUISTIC DEFINITION OF ACCELERATION

Before dealing more directly with theoretical notions, I want to present some data which deal with the translation of a particular definition of acceleration from natural-language to natural-language. When presented to novices for interpretation, this task leads robustly to an erroneous restatement of what "acceleration" means. The data is narrowly linguistic in its focus, but illustrates the power of whatever process may be acting to produce the result. The result provides another example of a common and robust misinterpretation of a description of a well-defined scientific concept.

In a previous paper, I have discussed the importance of "acceleration" as a basic concept, one which appears very early in introductory courses and which has been identified by high-school teachers as difficult to teach (Kenealy,1986). I discussed data on a set of 20 problems, which asked subjects to identify in each case an instance of acceleration and its direction. The responses were analyzed by extracting possible rules to explain a pattern of incorrect answers. The three most prevalent rules taken in combination completely predicted the performance of several students on the set of questions. More recently, a earlier version of the following data was presented (Kenealy,1987), but without the possible explanation I am suggesting in this paper.

The study involved the groups shown in Table 1.

TABLE 1. Groups (total N = 513)

1. 8th graders (N = 7). Bright, selected on basis of SAT scores.
2. 9th and 10th graders (N = 91). In general science of biology courses.
3. 11th graders (N = 76). In chemistry courses.
4. 11th and 12th graders (N = 39). In first semester of a physics class.
5. 11th and 12th graders (N = 90). In first semester of a physics class; this group took the task home and most typed it up.
6. College, primarily 2nd and 3rd year (N = 114). Administered as a pretest the first day of an algebra-based physics course.
7. College, primarily 2nd and 3rd year (N = 46). Administered in the 2nd semester of an algebra-based physics course.
8. B.S.in science, not physics (N =15). High-school chemistry, biology teachers in a workshop.
9. B.S., science (N = 35). Experienced high-school physics teachers chosen for workshop on criteria related to excellent teaching.

The populations extend across a range of education and possible preparation to handle the task. The task is essentially a linguistic one, the interpretation of the common definition of acceleration cited above. As mentioned, the definition is one which unfortunately is offered as the first-encountered textual definition of this concept in a significant number of high schools in the United States. The subjects in this study were given the statements shown in Table 2, one on either side of a piece of paper.

Table 2.

1. **We're trying to figure out how people make sense of sentences which are used to define ideas in science.**

   Simply on the basis of making sense, explain what the following sentence says about the meaning of the word "acceleration."

   "**Acceleration** is defined as the time rate of change of velocity."

---

2. "An automobile **accelerates** when it leaves a stoplight."

   What does the word "acceleration" mean in this everyday situation?

The order of presenting the questions was randomized, and, in a simple accounting of correct and erroneous responses to the definitional question, no effect of question order was apparent.

Table 3. Typical "acceleration-is time" responses from Group 8 and associated responses to the second question. Shown is a subject code with physics background and teaching subject.

(HS = high school; question numbers keyed to Table 3; underling added.)

174: (No HS or coll. physics) (Biology, Environmental Science)
1. I would say acceleration is the time it takes an object to pass a measurable set of distances noting if there is a change in time between equal distances.
2. A change in direction or velocity.

661: (Yes HS and Coll. physics) (Biology and Chemistry)
1. tΔv . . . Acceleration is the time an object takes to travel a given distance in relation to the force applied.
2. Acceleration in this situation is the movement of the car at a constant increase in speed through a certain distance in a given time interval.

286: (No HS, yes, coll. physics) (Unified Science; Mathematics and Social Science)
1. Acceleration is the time it takes for you to go from a rest position to the desired speed.
2. From initial velocity of zero rest to some velocity v. Motion of the auto forward.

990: (No HS or coll. physics) (Environmental Ed., Earth Science)
1. . . . the time it takes to go some distance . . .
2. The stoplight is not moving, the car is moving. As it leaves or passes the stoplight, the car moves forward at a given speed or velocity.

The most robust wrong response (see Table 3) consistently identified acceleration as an amount of time required to change a velocity. The answers were coded as in Table 4 and put into broad categories.

Table 4. Coding of responses.

1. a) the speed changes; it goes faster; speeds up; velocity changes.
   b) velocity changes and/with time changes; correct responses.
2. a) the time, or how long it takes, to changes the velocity or speed.
   b) the time, or how long it takes, to change position.
   c) $a = t / \Delta v$; $a = t\ \Delta v$
3. a) blank
   b) don't understand; exact repeat of defining sentence.
4. other responses.

The first category includes responses of "change of velocity, Δv" as an acceptable response. Table 5 lists the statistical results by the four broad categories in Table 4. The number of responses that were judged correct are given in the parentheses in category 1.

Table 5.

GROUP	N	RESPONSE CATEGORY (% OF N)			
		1	2	3	4
1	7	43 (43)	43	0	14
2	91	29 (8)	15	27	29
3	76	21 (8)	28	29	22
4	39	38 (23)	41	5	15
5	90	47 (46)	30	10	13
6	114	45 (30)	38	9	8
7	46	76 (72)	24	0	0
8	15	53 (40)	27	7	13
9	35	91 (88)	3	3	3

. . .

The pattern of correct answers in the various populations generally follows "nominal physics-education level" in an expected way, with one exception which is relatively easy to explain. This exception is the specially selected group of bright eighth-graders (Group 1), who made more right interpretations than nominal educational level would predict. Their group was the smallest (N=7), and there was little scatter in their answers; three got it correct and three missed it by giving the "time" answer. They were tested at the first class meeting of a special summer physics program. One other "exceptional" data point is from a group of high-school physics students (Group 5) who got more right than another similar group (Group 4); their instructor allowed these students to take the task home and gave extra credit for typing the responses. Even so, the number of correct responses was below 50%. The performance of these two "exceptional" groups falls between college students just beginning a pre-med algebra-based physics course and college students at the end of two semesters of that course.

Figure 1 provides a graph of "%-correct" vs. "nominal physics-education level". In Fig. 1, Group 1 has been removed, and the physics background of Group 8 was judged to be between Groups 6 and 7. The background of the teachers in Group 8 included very little physics. The graph has been fit with a curve simply to call attention to the trend of the data.

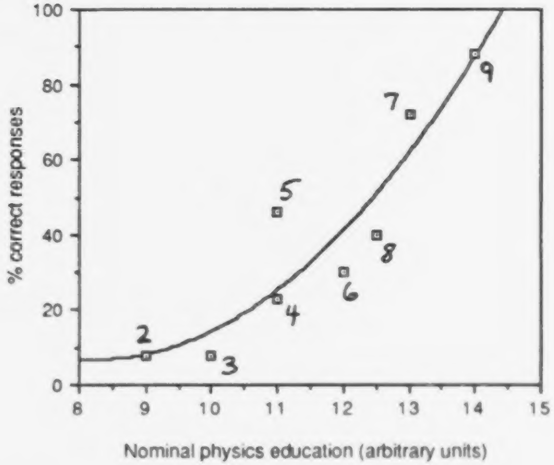
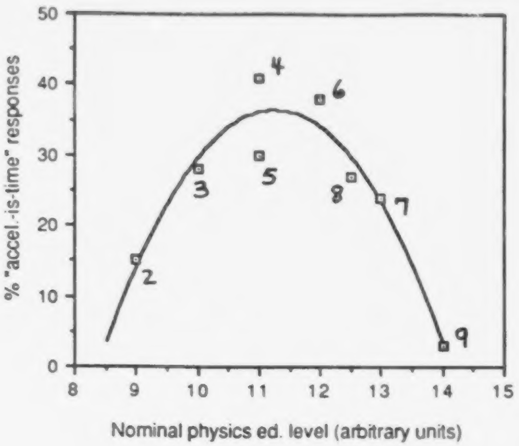


Figure 1.

In Figure 2, the data of Table 5 corresponding to "%-acceleration-is-time" responses vs. "nominal physics-education level" are plotted.

Figure 2.



The Group-5 data on the physics-class high-school seniors who took the questions home is included in order to note that, despite the advantage of taking it home, they still produced "acceleration-is-time" answers in 30% (27 of 90) of their responses. The graph shows an inverse U-shape, with erroneous "acceleration-is-time" answers peaking for the high-school seniors in their first semester of physics and college students beginning the algebra-based physics sequence[6].

Why did those populations most closely associated with physics classes make the most errors of this particular "acceleration-is-time" type? One possible interpretation is the following. The groups that made the "time" error most robustly were the groups that actually paid attention to the instructions and therefore engaged in the decoding task. I will characterize the error-pattern in the data in the following way: When a serious attempt was made to decode the sentence, the erroneous "acceleration-is-time" response occurred in 20 to 40% of the trials within each population group. The two groups at the extremes are relatively easy to account for. The group with the fewest errors at the high-educational level was composed of practicing high-school physics teachers selected for a workshop on the basis of criteria related to "excellent teaching". The group at the lowest educational level were high school freshmen in a variety of courses and the spread of their answers ranged the most widely of any group, with many answers being only remotely related, if at all, to the assigned task. Although we might simply dismiss these two extrema as some kind of ceiling and floor effects, what kind of ceiling and floor are they? Consider the following: the two extreme groups did not or could not decode the sentence in a literal way. The experienced physics teachers accessed many other resources, while many of the freshmen could not make any sense of the sentence. There is support for the latter assertion in the wide scatter in responses in Groups 2 and 3 including about 25-30% who said variations of "I don't know." or simply rewrote the defining sentence exactly.

There is some external support for the suggested interpretation from other studies. For example, one study (Lesgold, Feltovich, Glaser, and Wang, 1981; Lesgold, 1984) plotted errors in the diagnosis of X-ray pictures by beginning, intermediate, and expert radiologists. When diagnostic errors were plotted with axes analogous to those used above, namely, error-rate vs. an experience dimension, the results follow an inverse-U-shaped curve. In another study (Dee-Lucas and Larkin, 1986), the "sensitivity" of readers to a definitional-sentence form in a physics-text passage was plotted vs. a naive, novice, and expert dimension related to physics background. This study also produced an inverted-U-shaped curve, with novice physics students judging sentences, imbedded in textbook passages and identified as definitions, as more important than either experts or those judged as "naive" with respect to physics training. The latter study is of particular pertinence in supporting the rationale in this study that the physics novices, Groups 1, 4, 5, and 6, who were in or just starting classes in physics, paid particular attention to the decoding assignment.

MODELLING RESPONSES TO TASKS

Decoding and re-presentation tasks. The terms "decoding" and "re-presentation", in the view that I am going to discuss, are more appropriate than the term "translation". "Translation" includes both decoding and re-presentation and is too broad for use in this analysis. The goal is to develop a more differentiated and principled account of how various kinds of incorrect responses occur so robustly. A more detailed basis for describing the nature of the robustness may be a guide for developing new teaching strategies and assessing those already in place.

The important "decoding" question is: Given either a linguistic or imagistic description of a physical event, or given a physical event to observe: How does one decode or "construct meaning" from these sources?

The important "re-presentation" question is: How does one describe the experience of the "constructed meaning" in symbolic terms, perhaps either in natural language or mathematical language, so that it can be re-presented to an external audience who has not been privy to the construction process?

The approach that follows leans heavily on the work of linguists: Lakoff and Johnson (1980), Fauconnier (1985), and Lakoff (1986). The work of Jackendoff (1983) , Dinsmore (1987), and di Sessa (1985, 1986) has also influenced this discussion.

Experiential realism. The basic epistemological viewpoint is called "experiential realism" by Lakoff. It may be summarized in the following way. A real world of objects and interactions exists. Humans apprehend and structure their experience of this real world via basic metaphors and idealized cognitive models which arise from their embodied experience of interactions with the real world: UP-DOWN, FRONT-BACK, PART-WHOLE, IN-OUT, CENTER-PERIPHERY, SAME-DIFFERENT, ENTITIES, LINKS, PATHS, CONTAINERS, CONDUITS, SOURCES and activities like PUSH-PULL, CAUSATION AS DIRECT

MANIPULATION, FORCE AS MOVER, COUNTING, MEASURING, SHAPING, GROUPING, and others. (Some brief examples from Lakoff and Johnson (1980) of the structuring of concepts in terms of idealized cognitive models are in the endnotes.[7]) "Experience" includes not only perception and motor activity, but all genetically-acquired properties that contribute to activities and interactions in physical and social environments. *To be a "directly meaningful" reference, a distinction or activity must have been directly and repeatedly experienced.* An individual's experience, though structured by the same processes as others' experiences, is unique to that individual and the meaningfulness of events, situations, and descriptions of situations to that individual will be unique. This does not deny the existence of a real world with real interactions, but it does indicate that there is no uniquely correct description of reality, only, at best, an agreed-upon description between two or more "experiencers".[8]

Categories. The use of categories and prototype structures in organizing knowledge is, in this view, evidence of deeper-lying structures (the idealized cognitive models) grounded in basic experiences. Categorizing is a basic human activity that gives coherence to experience and is the main way we make sense of experience. The real world exists, but, based on evidence from a variety of human sciences, there is some doubt that there are *natural* joints at which to carve up the real world into classical categories in which membership is based on listings of necessary and sufficient conditions.[9]

There is evidence that the basic level of categorization is based on gestalt perception (Mervis and Rosch, 1981). Experiential gestalts tend to be at a central position in hierarchically structured categories. For example, the position of "chair" is relative to a superordinate label "furniture" and subordinate elaborations like "rocking chair" and " desk chair". To paraphrase Brown (1965): while a dime can be called 'a coin' or 'money' or 'a 1952 dime', we somehow feel 'dime' is its real name. "Basic level" does not mean "conceptually primitive level", in that the latter phrase is taken to mean concepts with no internal structure. For example, consider the following list.

Superordinate	Basic	Subordinate
moving	running, walking	ambling, racing
ingesting	eating, drinking	slurping, gulping

The directly meaningful perceptions are in the hierarchy at an intermediate level. They are neither the highest nor lowest level of conceptual organization. Nor are they without structure. But the assertion is that they are the most salient members of the category, the easiest to process, learn, remember, and use. Despite being in the middle of a hierarchy, they are actually cognitively simple.[10] Categories can acquire an abstract structure by projection from the basic-level gestalt to superordinate or subordinate levels or by metaphorical projection from any physical experience.

Mental Spaces. The form of language provides instructions on how to structure mental spaces and how to give meaning to what is being presented. Fauconnier (1985) has viewed the construction of meaning in natural language to be closely tied to the construction of "mental spaces". Two or more spaces interconnected in specific defined ways form a domain for organizing and sharpening certain kinds of incoming information. In simple cases, such domains of spaces can represent the world as it is defined in a painted picture, a movie, or a novel; in more complicated instances, spaces might be constructed to be a particular person's point of view, a situation or event in time and/or space, or a hypothetical situation in time and/or space. There can be mental spaces imbedded in other mental spaces. The structuring of these spaces may depend on propositional models, metaphor, metonymy, or image schemas (Lakoff, 1986).

A simple grammatical structure gives instructions for space construction in context. But the construction process is underdetermined by the instructions. Multiple complex spaces can arise from simple construction principles and simple linguistic structures. One cannot expect the sentence that constructed the space to reflect the complexity of the constructed space.[11]

An aside. Before getting caught here in a kind of infinite regression of spaces too numerous and complex to ever understand, imagine these structures as having one particular aspect of a fractal-generated diagram[12]. Fractal geometry deals with complex forms that are not smooth or homogeneous, but mimic the "constrained randomness" found everywhere in nature (West and Goldberger, 1987). Fractal diagrams can reproduce remarkably real-looking coastlines, mountain ranges, clouds, biological organs, and other natural shapes. One sense in which such structures are constrained is that when examined at higher and higher magnifications, the small-scale structure is similar to the larger scale form. For purposes of my metaphor, this self-similarity at various scales is the salient feature of the fractal model. Understanding the local structure somewhere can help understand the local structure at any level of magnification. I would extend this metaphor to embedded mental spaces, looking first at any connections between two states without worrying much about what is being missed on a different scale.

Fauconnier (1985) asserts that language, as it occurs, "builds up mental spaces, relations between them, and relations between elements within them."[13] The spaces are constructed or evoked by "space builders", phrases such as "Len believes ___" , "In 1952, ___", "If you catch a fish, ___" , "In the movie ___", "In the technical world of physics, ___", and so on. Within spaces, objects may be represented as existing with "true" relationships among them, *regardless of their real-world status* (Dinsmore, 1987). Fauconnier notes that the construction of spaces are not representations of reality or of partial "possible worlds". The construction of spaces represents a way in which we think and talk, but does not say anything about the real objects of this thinking and talking.[14]

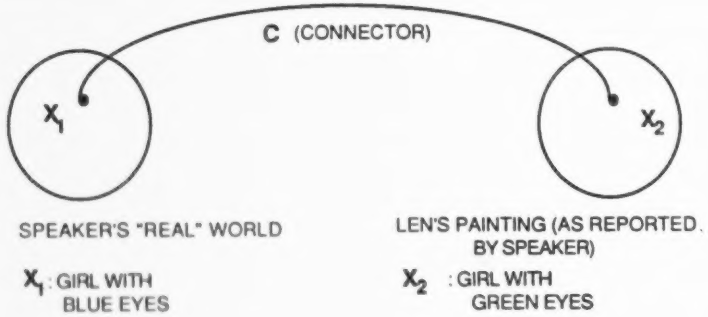
Why is the idea of Fauconnier's mental spaces something worth considering very seriously? The answer is that the idea has been successfully used to handle a variety of classic problems in linguistics and the philosophy of language in a simple, uniform, and intuitively plausible way, problems whose solution has been difficult or impossible within other models of language functioning (Dinsmore, 1987).

Examples. Consider the following sentences in which a speaker or writer is attempting to describe something to you:

"In Len's painting, the girl with blue eyes has green eyes." or
"In physics, acceleration is defined as the time rate of change of velocity."[15]

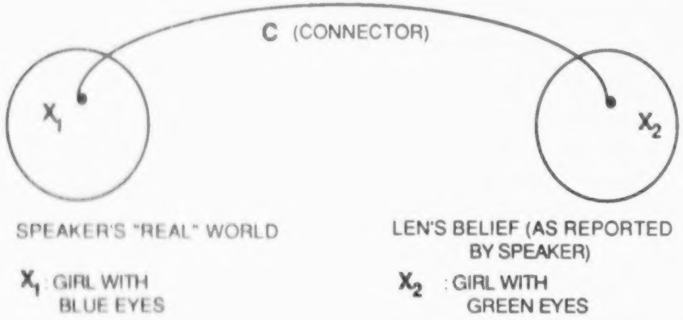
Several features of these two sentences are noteworthy. They both may be precisely true in the real world. But the feature dominating any notions of their truth is fact that both are subject to ambiguous readings. The following paragraphs demonstrate some of the analysis procedures of certain forms of "cognitive grammars".[16]

A. The first sentence might mean several different things. Is it that "the girl with blue eyes" is the real-world model and has been painted with green eyes? Or is "the girl with blue eyes" the one in the painting, while the real-world model has green eyes? Or does the female in the painting have blue eyes and green eyes, being perhaps a multiply-eyed creature from outer space. Arbitrarily choosing one of these interpretations, Fauconnier models this schematically as shown:[17]


C (CONNECTOR)
X_1
X_2
SPEAKER'S "REAL" WORLD
X_1 : GIRL WITH BLUE EYES
LEN'S PAINTING (AS REPORTED BY SPEAKER)
X_2 : GIRL WITH GREEN EYES

The link between the two spaces is called a *connector* and is created by considerations similar to those that produce the basic level of categories, fundamental embodied experiences, and/or broad cultural experience. Whatever the source of the *connector*, it acts in Fauconnier's theory as an identity (ID) principle; the parts so connected may literally stand for one another. For example, "Plato is on the top shelf." most likely means to many people "The books by Plato are on the top shelf." The *connector* gets its power from the pragmatic connection between writers and books, creators and created.

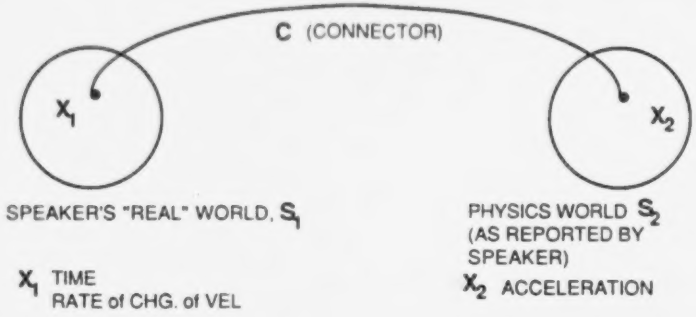
B. Jackendoff (1975) made the important observation that the analysis (not the meaning) of sentences like the first one above are little changed if the initial phrase is of the form "Len believes ___". In this instance the "space maker" phrase constructs a belief space, in which entities, and relationships between entities, can exist as 'true' and used as a basis for reasoning independent of any correlation with the 'real world'. Fauconnier provides an image schema of the following form.[18]



The ID (identity principle) connector (labelled "C" in the diagram) used between two spaces is functional in character, operator-like, rather than being an object or entity. If one has two spaces, S_1 and S_2, linked by a connector, C, and a noun phrase, NP, introduces or points to an element x in S_1, then a) if x has a counterpart x' $(x = F(x'))$ in S_2, NP may identify x'; or b) if x has no established counterpart in S_2, NP may set up a new element x' in S_2 such that $x' = C(x)$.

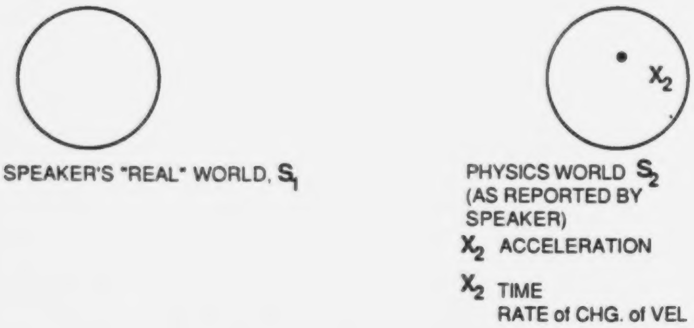
It would be useful to note one further distinction about what the spaces are not. If the following construction occurs, "Len hopes ___", the hope-space constructed is imbedded in the "Len believes ___" space. But the former can be entirely distinct from the latter in the sense of having no elements in common; these spaces, represented schematically by circles, are not like Venn diagrams.

C. In the second sentence listed (the definition of acceleration), the space-making phrase is "In physics ___", or, as it might be seen by a student, "In the scientific world of physicists, ___". For a student, even one relatively sophisticated in another domain, this may set up an empty space or one with fragments of physical experience which are either isolated or have a few, weak interconnections. These fragments could be something like di Sessa's phenomenological primitives (di Sessa, 1985) and/or the basic levels of categories of physical experience (Mervis and Rosch, 1981) and/or simply, on an even more fundamental level, the repeated experiences that order physical phenomena that comes from being embodied in an interactive environment, like the notion of CAUSATION AS DIRECT MANIPULATION, PART-WHOLE, LINKS, PATHS, COUNTING, and SHAPING. Other fragments that one might imagine being activated are algebra equations expressing facts about motion ($v = at$) or Newton's laws ($F = ma$) but without elaboration or strong relations. A simple diagram of it might look like the following:


C (CONNECTOR)
X_1
X_2
SPEAKER'S "REAL" WORLD, S_1
PHYSICS WORLD S_2 (AS REPORTED BY SPEAKER)
X_1 TIME RATE of CHG. of VEL
X_2 ACCELERATION

If an expert reads the defining sentence, one imagines the constructed space to be so rich that whatever phrase follows the space-making phrase is nearly irrelevant in terms of imparting any information or changing the established connections. But a novice faces the enormous task of extracting information from the sentence and relating it to a space with many unstructured fragments. The novice must pay very close attention to the defining sentence and the information contained in its syntax and try to identify and relate it to a known entity or structure. If the latter process fails, any reasonable interpretation of the definition seems unlikely.

Is the diagram above correctly done in terms of the sense of the sentence? That is, is it apparent what part of the definition is in the speaker's "real" world? Any of it? Consider the following diagram.


X_2
SPEAKER'S "REAL" WORLD, S_1
PHYSICS WORLD S_2 (AS REPORTED BY SPEAKER)
X_2 ACCELERATION
X_2 TIME RATE of CHG. of VEL

The first diagram involving the acceleration definition corresponds to what a linguist would call the classical transparent reading of the sentence; it allows the possibility that x_2 does not, or, perhaps, need not, be identified by "time rate of change of velocity". The second diagram is the classical opaque reading: both parts are in the physics space and allow the possibility that x_2 may have no counterpart in R. For example, imagine perhaps that the sentence defined some physical notion related to a space constructed by "In this science fiction novel, ___."

There are some other points to be made about the physics space of the novice. It is generally an isolated space with little metaphorical structuring or structuring of any kind. It is not the space of algebra or algebra classes. It is not the space of the "real" everyday world, where ambiguous language is sometimes to be desired and where approximate expressions of experiences and feelings are valued. It is not the space of history or political science classes, which, for a novice, may be metaphorically structured simply because they are about human interactions. The latter two are spaces where it is rare that a single sentence is as precisely normative as a scientific definition.[19]

DISCUSSION

The definitional sentence of acceleration being examined here gets its importance from its widespread use as the introduction of a basic concept. Many students who attempt to restate the definition, or express an understanding of it in their own words, fall into a consistent error, namely, re-presenting the definition with some form of the "acceleration-is-time" response.

"Acceleration is time." There is much data from studies on problem-solving in various domains that indicate novices will classify problems or tasks on the basis of features mentioned within the statement of the problem (Lesgold,1984). In categorizing physics problems, novices will mention springs, inclined planes, pulleys, and other physical entities mentioned in the statement of the p[roblem.

Mathematics students can recognize various "word problems" by type. In general, novices recognize cues which may trigger some pragmatic rules that allow them to begin to treat a problem (Dreyfus and Dreyfus, 1986). These considerations along with others mentioned in the previous sections, suggest what may occur when a student confronts the isolated definition.

The process may be similar to the following. The student realizes the importance of the defining statement and attempts to parse the sentence. (The mental space perspective, which might indicate whether the student believes there is any connection to the real world, is unimportant here. The parsing task should be possible whatever the initial construction may be.) The student is not attempting to parse standard English, but technical jargon, like the phrase "time rate", which is not well-formed in English. The defining sentence is reminiscent of the grammatically correct imbedded sentences that linguists construct when demonstrating that it is possible to generate an infinite number of grammatically correct sentences. For example:

The barn door the horse the farmer cut loose opened blew shut.

which makes sense when grouped in the following way. If you first read the sentence without the imbedded sentences, and then add them back from the outside in, the meaning becomes clearer.

(The barn door (the horse (the farmer cut loose) opened) blew shut.)

The phrase "time rate of change of velocity" may, to a novice, be grouped in different ways. Some possibilities are:

1) (time) (rate of change of velocity);
2) (time) (rate of change) of (velocity);
3) (time) (rate) of (change of velocity);
4) (time) (rate of (change of) velocity);
5) (time (rate of change) of velocity);
6) (time rate) of (change of velocity).

Of the more than 300 responses in categories 2,3, and 4 in Table 5, only 20 mentioned the word "rate" at all. Here are some verbatim examples of parts of those responses.

a) time or rate to build up speed;
b) the time and the rate and the velocity of something;
c) the rate of time increases;
d) the time it takes the rate to change;
e) the time and rate and change of speed;
f) the time it takes the rate to change;
g) the time and rate it takes for a person to get from one place . . .

Note that in several cases "time" and "rate" and "change of speed" appear as separate nouns or noun phrases, syntactically isolated from each other, perhaps as an initial list of cues for the student. One example of a student's correct response mirrors his struggle with the syntax. By parsing the sentence back-to-front, the student has eliminated the confusion of the initial adjective, "time":

h) "I read it over 4 or 5 times: if something has a velocity, and the velocity is changing, then that velocity must be changing at a rate, and that rate is acceleration."

In many responses, the word "time" is taken as an independent noun. The appearance of isolated nouns and noun phrases indicates an attempt to extract meaningful information, but the result indicates unstructured fragments. The word "rate" , when recognized as an appropriate cue, is confusing because it implicitly contains time in normal usage. Consider the meaning of the phrases "birth rate" and "crime rate." The syntax gives few clues to the novice about how the words are to be grouped.

Another feature exacerbates the difficulty of the task. The word "time" is in the prominent position in a definition usually given to the superordinate level of a category. Consider the following initial phrases of several definitional statements:

> Eyeglasses are optical devices . . .
>
> A pen is an instrument . . .
>
> An eye is the organ of sight . . .
>
> An eyeglass is a lens . . .
>
> Acceleration is the time . . .

The salience of the syntactical position of the word "time", combined with the novice's search for cues, and the difficulty of decoding the way in which the words should be grouped, set the stage for the observed robust error pattern.

A U-shaped curve and the dimension of experience. Assuming the construction or evocation of mental spaces, one way to restate the difficulties novices have is to note that the available structuring for the space, although much greater than for naive subjects in a domain, is not structured strongly or buttressed by much experience. An expert's space, besides being structured, perhaps, by a better grasp on theory, is presumed to be structured by wide exposure to concrete examples. The basic-level experiences, discussed above as central to categories, may have been augmented by gestalt-like sophisticated laboratory or clinical experiences within the expert's domain. This may allow the expert to reason meaningfully with these complex entities in the same manner novices might reason meaningfully with the more basic embodied experiences. Support for the idea of experts reasoning in this fashion can be found in Dreyfus and Dreyfus (1986).

It is important to say that the above account is along the same lines of an account given by Lesgold (1984) in describing the U-shaped curve of errors found while studying the diagnostic skills of novice, intermediate, and expert radiologists. But in that case the explanation was couched in terms of schemas and an information processing model. In that model, the complex gestalt entities mentioned above are identified as compiled procedures. Lesgold comments:

> "Trainees functioning at an intermediate level are in the process of compiling and tuning their ability . . . They are also learning to take into account . . . constraints imposed by film context and variations in film quality. Finally, they are developing their ability to construct a global model of the patient's medical condition . . .their performance suffers in those cases where their new more complete, schemas assert partial control but are insufficiently automated to finish the job; that is, they no longer have the simplistic recognition abilities of the new trainee, but they have not yet automated the refinements they have acquired."[20]

The last sentence states the reasons for the nominally better performances on either end of the experience dimension.

The mental-space account, considered as structured by the basic-level experences and phenomenological primitives, contributes at least two things to the information-processing account:

1) a psychologically plausible explanation of the terms "compiled procedures" and "automated, refined, flexible schemata," and
2) some tools to analyze linguistic difficulties, tools which are absent from the schema account.

Strategies in teaching. As is true for most definitions of scientific concepts in a pedagogical setting, less is not more. Condensed scientific definitions tend to bury the procedures and experiments that underlie them. As a counterexample, consider the textbook definition of acceleration offered by Sears, Zemansky, and Young (1980):

> Considering again the motion of a particle along the x-axis, we suppose the at time t_1 the particle is at point P and has velocity v_1 and that at a later time t_2 it is at point Q and has velocity v_2. The average acceleration of the particle as it moves from P to Q is defined as the ratio of the change in velocity to the elapsed time . . .

The distinction between this description and the one studied in this paper is the distinction between a feature specification of a scientific concept and a procedural specification. Reif (1986) provides an excellent discussion of this point.

The pioneering work of Arnold Arons over many years has consistently emphasized the importance of operational definitions. (See, for example, Arons, 1984.) In this context, it is interesting to note his comments about what I would consider the definitional elements which are somewhat analogous to basic-level experiences.

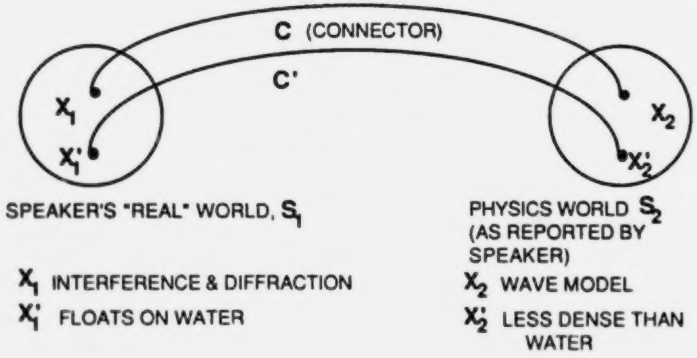
> Finally, it needs to be firmly emphasized to the students that operational definitions of technical terms cannot be "figured out" from the terms themselves and *must* be memorized. . . It is far more important to memorize the vocabulary than to memorize the formulas.[21]

The insight provided by mental spaces in characterizing the distinction between belief spaces and a "real" world may help model a teaching difficulty, related to definitions, that McDermott (1975) has pointed out. In discussing the preparation of teachers for teaching physics, she has emphasized the importance of structuring linguistic definitions in certain ways. Consider the following two sets of descriptions. Each statement may be thought of as prefaced by "In the scientific world of physicists, . . . ":

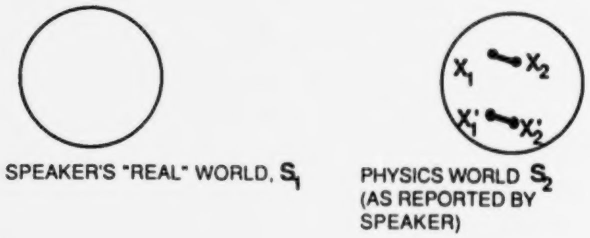
A. 1) Because light consists of waves, it exhibits diffraction and interference.
   2) Because light exhibits diffraction and interference, a wave model is a useful description.

B. 1) Because the boat is less dense than water, it floats.
   2) Because the boat floats, it is less dense than water.

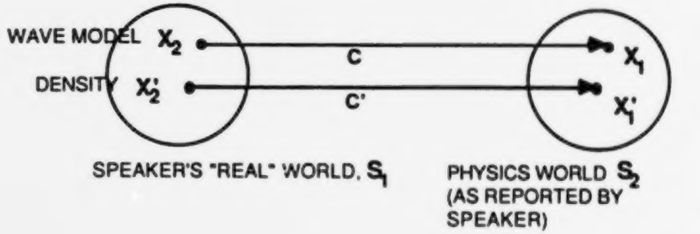
The forms labelled 2) in each set place the observational phenomena as primary and in the "real" world, as shown in the following diagram.


C (CONNECTOR)
C'
X_1
X_1'
X_2
X_2'
SPEAKER'S "REAL" WORLD, S_1
PHYSICS WORLD S_2 (AS REPORTED BY SPEAKER)
X_1 INTERFERENCE & DIFFRACTION
X_1' FLOATS ON WATER
X_2 WAVE MODEL
X_2' LESS DENSE THAN WATER

The diagram makes clear where the vocabulary terms, developed by scientists to talk about the phenomena, are located. The diagram makes explicit why the forms labelled 2) are appropriate explanations. The forms labelled 1) could be confusing to a novice and might appear to be structured in the following way, with everything in the physics world and no connectors to the real world:


X_1
X_2
X_1'
X_2'
SPEAKER'S "REAL" WORLD, S_1
PHYSICS WORLD S_2 (AS REPORTED BY SPEAKER)

Or, even worse in terms of the understanding of the students, they may get confused about what part is defining what terms:


WAVE MODEL
DENSITY
X_2
X_2'
C
C'
X_1
X_1'
SPEAKER'S "REAL" WORLD, S_1
PHYSICS WORLD S_2 (AS REPORTED BY SPEAKER)

where X_1 and X_1', on the right in the "theoretical" space, are physical phenomena more closely associated with observation, namely, "interference and diffraction" and "floats on water." This arrangement is pedagogically unsound.

ENDNOTES

[1] As an aside, it should be mentioned that this doesn't directly address the question of the "innateness" of the grammatical components.

[2] I first heard the sentences from di Sessa during a talk, but the grammatical argument cannot be blamed on him.

[3] This consists of some variation of "In Mindy's restaurant, for every four pieces of cake ordered, five pieces of pie are ordered. Write an equation which tells you how many pieces of cake are needed if you know how many pieces of pie will be ordered."

[4] "There are 7 students for every professor at a certain conference. Write an equation which represents this situation."

[5] The difficulties they cite refer to "translation" problems, attention to the "wrong features" of graphs, the matching of information in a narrative passage to graphical representation, the requirement for active interpretation as opposed to the use of memorized procedures, and students' attempts to construct analytical graphs of motion, v vs. t, which result in reproducing the spatial appearance of the motion, an outcome particularly prevalent if no measurements were made.

[6] As an aside, the comment of a psychologist familiar with graphs of drug dose vs. behavioral measures made on animals was: "Ah, the ubiquitous inverse U-shaped curve." I was startled and asked what she meant. She pointed out that it is often the case that very low and very high doses of drugs produce similar low response rates. There is probably no analogy to be drawn to high doses of education.

[7] HEALTH AND LIFE ARE UP; SICKNESS AND DEATH ARE DOWN
He's at the peak of health. Lazarus rose from the dead. He's in top shape. He fell ill. He's sinking fast. He came down with the flu. His health is declining. He dropped dead. SOURCE: SERIOUS ILLNESS FORCES YOU TO PHYSICALLY LIE DOWN.

HAVING CONTROL IS UP; BEING SUBJECT TO CONTROL IS DOWN
have control over him. I am on top of the situation. He's at the height of his power. He is under my control. He fell from power. He is my social inferior. He is low man on the totem pole. SOURCE: PHYSICAL SIZE OFTEN CORRELATES WITH PHYSICAL STRENGTH

TIME IS A MOVING OBJECT. The time will come . . . The time for action has arrived. Let's meet the future head-on.
TIME IS STATIONARY AND WE MOVE THROUGH IT. As we go through the years . . . We're approaching the end of the month . . .
TIME IS A RESOURCE. Time is money. Time can be spent, wasted, given, stolen, . . .

[8] Theoretical organizing ideas like "frames", "scripts", "schemas" and "scenarios" have been developed previously as cognitive models. They might be characterized as network structures with labelled values that code propositional information. The labels may be "empty slots" that are filled with situation-specific information or with default values. As currently structured they are fairly restrictive and do not seem to be able to address problems dealing with language, metaphors, and metonymic phenomena. See Lakoff (1986).

[9] In fact, if those natural joints do not exist, the hunting-based metaphor of the real world as an dead animal to be butchered at its joints may, fortunately, have to be abandoned.

[10] Lakoff (1986), p. 199.

[11] Fauconnier (1985), p.168.

[12] I offer this model to honor of the long tradition of using whatever is the latest fascinating technology or mathematics concept to model the mind.

[13] Fauconnier (1985), p.2.

[14] Ibid., p. 152.

[15] As pedagogy, this sentence is an unmitigated disaster. However, it is a technically correct piece of jargon, with the origin of the phrase "time rate" used by physicists to avoid confusion with "space rate", by which is meant a change in some variable as a function of position rather than as a function of time. For example, as one heads east from the Rocky Mountains, one's height above sea level changes. The "space rate of change in height" might be expressed as a change in height per kilometer of eastward travel.

[16] Since these arguments are being applied on a restricted problem by a non-linguist, one should take care not to judge their potential power on this very narrow application. I am also synthesizing here the approaches of several different authors, and it is unclear that each would agree with the other, let alone agree with my application. That being said, I have been very careful not do violence to the constraints expressed in their theories, at least as near as I can work with them in the mental space(s) evoked by their writing.

[17] Fauconnier (1985), p. 12.

[18] Ibid., p. 14.

[19] Lakoff (1986), p. 122,123 , comments cogently , quoting Paul Kay, on folk theories of how words get defined.

[20] Lesgold (1984), p. 52.

[21] Arons (1984), p. 22.

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PUPILS ARE ABLE

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The aim of this study was to obtain detailed information on the development of reasoning during a learning process. Four pupils (14 - 16 years old) were observed in depth when working with a number of problems in electricity. Their activities and comments were recorded using audio-visual techniques. The study was approached in a constructivist tradition.
The case study has shown that lower secondary school pupils are capable of formulating problems at their own level, developing knowledge of their own and designing their own experiments to test ideas. They are full of initiative and imagination when allowed to work at their own pace, exploring electric systems. Pupils are able to expand and refine their conceptual structures in dialogues with a teacher or another pupil about electrical experiments.
Now I would like to give some examples of this:

Ed(16 years old) is asked to light a bulb with a battery and a copperwire. In figure 1 you can follow his attempts.

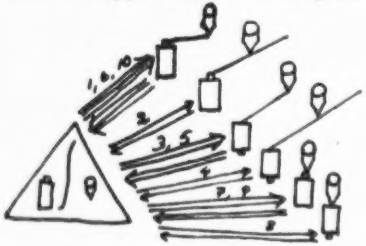


Figure 1

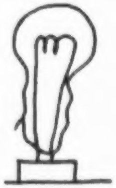


Figure 2

His attempts can be seen as decided by his unipolar model. Ed makes a drawing of what he thinks a bulb looks like inside and explains how it works (fig.2).
Ed is consistent. The bulb is unipolar "from outside" and "from inside". He presents two theories of how the current can 1) enter the bulb in both wires and meet so it will glow, or 2) current will circulate and go back to the battery, but he does not know if current goes back. He says that the first way may cause a short circuit, so the second way is better.
Ed tries to light the bulb with another type of battery and makes the following connections (figure 3).

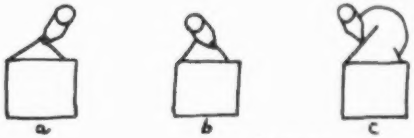


Figure 3

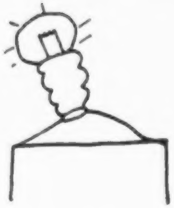


Figure 4

He is convinced that two metal poles on the battery will contact the lowest point of the bulb. So he tries for a long while to get this to function. He is told by the teacher that it is possible to light the bulb with the battery only. Suddenly the bulb flashes. Ed holds the system quite still and inspects it. Then he makes a drawing (figure 4). He sees what he expects to see. His observation is guided by his unipolar model.
When Ed is told to try to use a wire in this last system he discovers the second pole of the bulb (figure 3 c). Now he suddenly speaks of the current circulating in the circuit. When refocusing on the first battery system he immediately connects it successfully (figure 5).

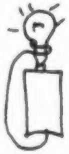


Figure 5

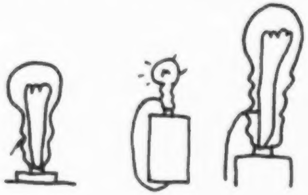


Figure 6

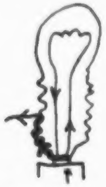


Figure 7

Confronted with his bulb model, he discovers a conflict which he solves in the way shown in figure 6.
In a later session Ed uses another theory of how the bulb functions (figure 7). To the question whether the current has to follow the way through the filament, he replies "yes, otherwise it wouldn't light" Only after the concept short-circuit is introduced does he show a full understanding of the way the bulb is constructed and functions.

Ed now formulates a problem on his own. He reflects over his desk lamp and wonders how it is constructed. He thinks there must be two wires in the flex. He is offered different kinds of wires and chooses a two-conductor-flex. He starts the following experiments (figure 8).

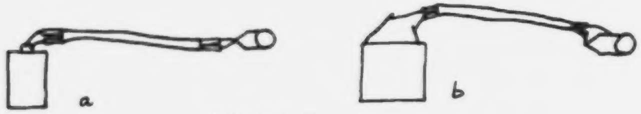


Figure 8

Different batteries (contexts) activates different models. Ed says about the system in figure 8b:"Though now it goes in the same way in a circle, though it now goes closer, like"
This example shows how Ed tries to combine his everyday experience with the new experience and his own idea about the circulating current. He succeeds in solving his problem and he also deepens his conceptualization of the circulating model.

The second example shows how a girl of 14 develops knowledge as she works with electrical equipment. Margaret starts with a unipolar model, advances to a two-component model and ends up with a circulating current model.
When she explains how her bulb functions, she speaks of plus and minus that enter the bulb. Therefore the bulb has one pluspole and one minuspole. She develops her thinking while she works with different bulbs (figure 9).

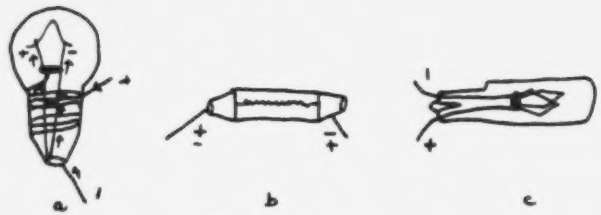


Figure 9

Margaret connects her bulb b to the battery. Then she spontaneously turns it around, that is, changes poles. She is very astonished when it still lights and tries the same turning experiment with bulb a. Margaret has found a problem on her own. In a long discussion with herself where she consistently uses the two-component model, she finally concludes that it probably doesn´t matter how it is connected since every pole can conduct both plus and minus charges to the bulb.

Then Margaret tries the experiments on the c bulb and is satisfied when she constructs an explanation for her experimental results: Then it can be so that perhaps copper conducts everything then not only plus or only minus but it conducts both, she says.

Margaret´s two-component model enables her to explain that two bulbs are lit by one battery (figure 10).

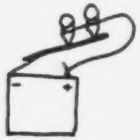


Figure 10

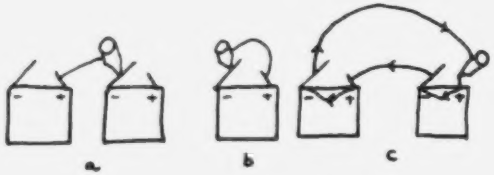


Figure 11

The two bulbs get both plus- and minus charges and therefore they shine.
But her two-component model fails when she tries to explain the result of a connection between two batteries and a bulb (figure 11a). Comparing her two experiments 11a and 11b, she cannot understand why the bulb does not shine in a when it does in b. Both are supported by plus and minus. After being reminded of the concept closed circuit, she tries a third connection c and suddenly starts talking about a circulating current and accepts that model as an alternative to her two-component model. Through some more experiments Margaret´s confidence in the circulating model grows and she gradually abandons the old two-component model during the following months.

This study shows what problems are specific to the pupils, and at what level these problems are. Ed was delighted to find that his two-conductor flex entered the circuit so it formed a circle, though it was "denser" in that part. Margaret´s problem was the nonpolarity of the bulbs.

As soon as the pupils got to know the rules, that is what was expected of them,their self-confidence grew greater. The study shows that it is possible stimulate learning by challenging the original conceptions that the pupils hold and training their operative readiness so that they themselves can verbalize questions, formulate hypotheses, test these and go on developing their own knowledge. This method will allow the pupils´independence and creativity to be utilized.

To have an operative readiness in electricity means to use both process- and content-oriented conceptions with confidence in a new situation. In constructivistic education the pupils´ own thoughts are requested. When the pupil verbalizes his/her own opinion, his attention is directed to his own thinking and an awareness of its structure will be possible. When you formulate a hypotheses, you put forward your own ideas in an attempt to apply them to a new situation. You will try to fit the new into what you already know.This endeavour to integrate promotes deeper learning. To dare to formulate a hypotheses out of your own thoughts demands cognitive audacity.Pupils who train themselves to put their ideas to the test, train themselves to be brave. Audacity is an indication of selfconfidence. The pupils will learn to trust their own ability to find out things for themselves and be less dependent on the teacher as the one who knows everything. Selfconfidence is an essential ingredient of operative readiness.

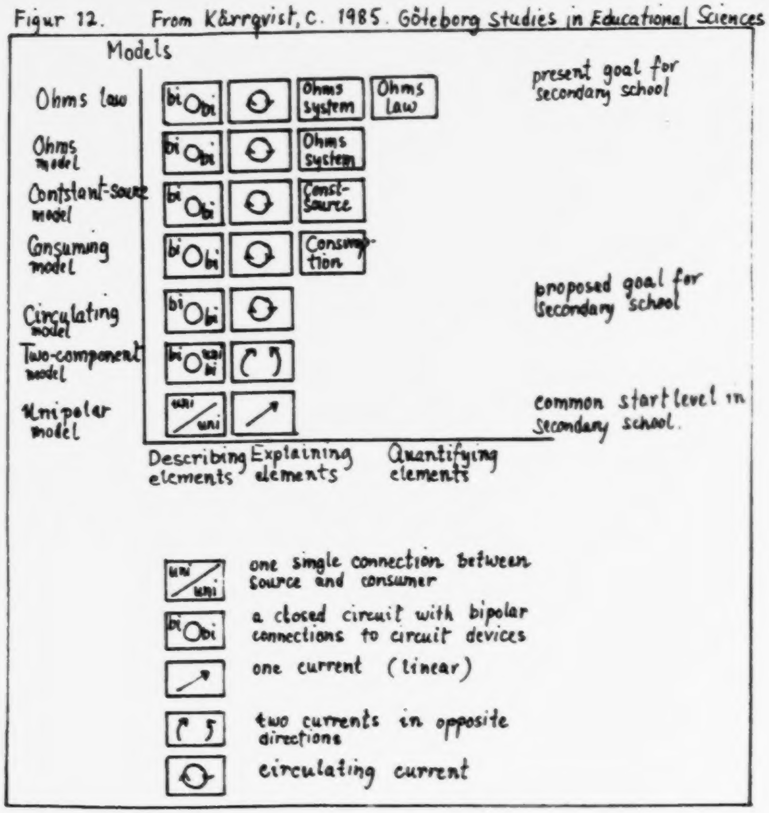
If pupils are allowed to work with their own problems and at their own pace, they can stand still and "know what they already know" for a while. Maria Montessori (1936) described a three-year-old girl who was quite absorbed in fitting a series of wooden cylinders of different sizes into corresponding holes. As soon as she had succeeded she turned the box over and started again. She repeated this exercise 42 times, before stopping suddenly for no apparent reason. Montessori thought that the girl´s activity, which was exactly the same each time, was quite meaningful to the girl, mainly because she gained time in which to develop selfconfidence. Perhaps you need to feel sure about what you already know, without being forced to hurry on to the next thing, before you can have the self-confidence required for daring to change your model of explanation.

Pupils are able. They are able to develop a deep understanding of the idea of the closed circuit. They are able to set up experiments on their own, put forward hypotheses, test these, formulate their own questions and develop explanation models. With the help of some concepts that the teacher introduces they are able to go on and widen the significance of these concepts. In a constructivistic type of education the demands on the pupils will increase, while the pupils have to think and act independently.

New didactic goals in electricity:

1. A functional Action Theory
   This action theory is composed of The Circulating model (fig 12) and the concepts: shortcircuit, resictance, conductor, current and "the simplest way"
2. raised ambition lewel for Operative Readiness.

Figur 12. From Kärrqvist, C. 1985. Göteborg Studies in Educational Sciences 52.



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A Research Strategy for the Dynamic Study of Students' Concepts Using Computer Simulations

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Microcomputers and appropriately designed software can help students learn science concepts. They can also provide media for investigating how students develop science concepts. In this first paper in the set, we describe a research strategy we have used to examine how students' concepts develop during interaction with computer simulations, and we discuss two pilot studies examining concepts of gasses. The second paper in the set to be presented by Patricia Simmons provides more explicit details about a study examining concepts in genetics. While the research strategy can be used to study students interacting with a variety of computer related materials, the focus in this paper will be on simulations containing interactive visuals.

Visuals & Research on Learning

Interactive visuals may enable computer software to become an especially powerful tool in science teaching. They go beyond conventional instruction that emphasizes only the verbal and the mathematical attributes of scientific models. Appropriate simulations and tutorials can help students learn about the natural world by having them see and interact with underlying scientific models that are not readily inferred from first-hand observations. For instance, this paper describes visuals that represent microworlds and that allow students to explore models of atoms, molecules, and charge in ways not possible with conventional instructional materials.

Current theory and research in cognitive science pictures the learner as actively using higher order mental activities including such executive processes as reflection, planning, analysis, and self-regulation in constructing. knowledge (Osborne & Wittrock, 1983; Garner, 1987). Schuell (1986) wrote "What the student does is actually more important in determining what is learned than what the teacher does." Hence, one fundamental task of instruction is to get students to engage in learning activities that help students develop science concepts. Perhaps appropriately designed instructional software that contains interactive visuals and dialogue can help students become more active learners helping them to construct concepts that are more consistent with those of scientists. While various educators have made this claim (Jay, 1981; Bork, 1981; Krajcik, Lunetta and Simmons, 1986 and 1987), there has not been a spectrum of experimental work performed sufficient to validate this position.

Computer Visuals

Developments in software and hardware now enable authors to include a variety of visuals in instructional software. The various uses of visuals and their instructional effects in computer environments need to be carefully described and examined. The strategy described in this paper can help us perform this much needed research.

Visuals can be defined as pictures, line drawings, bar graphs, cartoons, line graphs, animated images and dynamic graphs. Many of these visuals are at different points on a static to dynamic continuum. For instance, pictures and line drawings are static visuals while animated images and interactive graphics are dynamic visuals. With dynamic visuals the student can change variables and conditions and receive instantaneous information by observing on the computer monitor how the change affects the graph or animation of the simulated system (Lunetta and Krajcik, 1987). While each of these domains deserves careful elaboration and study, our work focuses on the instructional effects of dynamic visuals.

For clarification, here are some examples of *dynamic visuals*. These examples relate to research we will describe later in the paper. Students observing and interacting with representations of gas particles are using one form of dynamic computer visuals. Figure 1, Animated Particles from the "Model of a Gas" lesson in the *Molecular Velocities* disk (Krajcik & Peters, 1987)., shows a screen display of the particle animation representing gas molecules. In this program students develop a mental image of an ideal gas through observing and interacting with a particle simulation. Students can increase and decrease the temperature of the gas and observe the subsequent change in the movement of simulated gas particles. The linear movements of these animated particles are described by the Boltzmann equation. Some particles move slowly, some move fast, but most move at intermediate speeds. By using this program students obtain primary information from interacting with the dynamic visual.

In the "Temperature and Pressure" lesson in the *Gas Laws* disk (Krajcik & Peters, 1987), students predict the effects of temperature on pressure, then collect data on the temperature - pressure relationship, and simultaneously observe the data being plotted. This is an example of *dynamic graphics.* Figure 2, Temperature vs. Pressure, shows a dynamic graphics frame from the "Temperature and Pressure" lesson.

In *Models of Electric Current* (Lunetta, Lane & Peters, 1986) students can observe and interact with a model of electrons migrating through a simulated crystal structure within a wire in a closed circuit. By increasing and decreasing the voltage, students can observe changes in the simulated electron movement. They can also sample current readings at different voltages and observe graphs of voltage versus current.

Interactive computer visuals like these can help students learn science concepts by having them see and interact with underlying scientific models not readily inferred from first-hand observations of real world phenomena or from verbal or static visual presentations. Dynamic visual representations of microworlds allow students to explore models of electrons, atoms and molecules in ways not possible with conventional materials. For instance, various lessons in the *Gas Law* disk permit students to actively interact with a model of the kinetic behavior of gases by increasing and decreasing temperature and observing the changes in motions of the simulated gas particles. The dynamic visuals in this software present information central to understanding important science concepts. Such simulations may promote conceptual learning that is difficult to achieve with conventional instructional materials and methods and may serve as a vehicle for more effective science instruction. However, the influence of dynamic computer visuals such as those in *Gas Laws* and *Electrical Interactions* on the learning of science concepts has not been carefully examined.

The remainder of this paper describes a research strategy for examining the effects of dynamic computer visuals on the learning of science concepts and for examining changes in the students' conceptual knowledge as they interact with instructional software. Although the focus in this paper is on software containing dynamic visuals, the research strategy can be used with a variety of instructional software. (A more general description of this research strategy can be found in Krajcik, Simmons and Lunetta, 1987.)

A Dynamic Research Strategy

Our studies have included prompting students to think-aloud as they interact with software containing dynamic visuals. We have interfaced the microcomputer with a video cassette recorder (VCR) allowing us to make a permanent record of the video output from a microcomputer displaying the students' responses as well as their verbal commentary via microphone input. (In our studies, we have used Apple IIe and IIc microcomputers.) We have recorded and begun to interpret students' comments about their observations, perceptions, predictions, explanations, and decisions simultaneously with the display on the computer monitor. The video tape records can be analyzed carefully following the sessions with subjects to assess concepts and problem solving processes. They can also be shown to the student subject and further analyzed via "stimulated recall". This technique can provide the researcher with further insight into the student's perceptions of what he or she was observing and thinking. In addition, the think-aloud nature of this method provides one vehicle for inferring underlying psychological processes (Garner, 1987).

We have referred to this research strategy as "structured observations" (Krajcik, et. al., 1987). It enables us to study:

1) the nature of students' concepts;
2) how concepts change and develop with time and experiences;
3) how students apply concepts to solve problems;
4) how concepts can be influenced by instruction;
5) how different kinds of visuals influence learning;
6) how the verbal dialogue in which visuals are embedded influences student learning, e.g., the nature and placement of questions;
7) how students interact with instructional software;
8)..how to design and employ software more effectively;

This research medium provides more control over certain instructional variables than is possible in many conventional research environments. Treatments can be identical across a large number of students without the changes in treatment that are common when instruction is under the control of human teachers.

This research strategy can also provide information about the design and use of instructional software. For example, our observations have revealed that novice computer users do not respond to computer prompts in the same ways that experienced computer users do. We have also learned that most students have difficulty interpreting information presented in a computer frame which is divided in several sections. We have learned that some students have a preference for information presented in visual form while others prefer alphanumeric presentation.

While the specific directions in the various studies we have performed have varied slightly, they have had similar characteristics. Typically, the student came to a room in his or her school and sat at a microcomputer interfaced with a VCR. The investigator greeted the student and explained that he or she (the investigator) was present to study the effectiveness of the instructional software. The investigator then suggested that the student think aloud while interacting with the computer simulation. The student was told that reading the screens aloud did not constitute thinking-aloud. Expressing ideas and decisions, interpreting what the screens said, and making predictions did constitute thinking aloud. Generally, students were told that the investigator knew that thinking aloud could be awkward at first since in school and at home we don't usually do that. However, the investigator strongly encouraged students to think-aloud in non-directive ways.

When a student did not verbalize thoughts and observations, the investigator prompted with comments like: "Please share your thoughts" or "Please say what you are doing." The investigator allowed the student to work through the programs without interference but encouraged thinking aloud while working. The investigator occasionally asked the student questions such as "What do you see on the screen?"; "What does it mean?"; "What are you thinking now?" to prompt the verbalization of perceptions, thoughts, and decisions. Help was given only when the students had technical problems with the hardware or software. If the student requested help with the science content of the program, the investigator replied: "I am here to evaluate the software and to help only if the equipment malfunctions. I do not want to influence your decisions or learning. Please do what you feel is most appropriate and then say why you chose to take the action you did." The investigator refrained from engaging in extensive dialogue with the student until he completed the work with the software.

We have found getting some students to think aloud to be a real challenge. Currently, we plan to get students to describe what they think it means to think-aloud before we tell them a researcher's definition. We also plan to imply that for some people thinking aloud seems to facilitate learning. This, of course, will be a subject for investigation in its own right. The "structured observations" research strategy should be especially appropriate for initial efforts to assess the effects of thinking aloud on learning.

Sample Investigations and Results

While we have examined students' interactions using CATLAB (Kinnear, 1982), Models of Electric Current, (Lunetta et al., 1986), Molecular Velocities (Krajcik & Peters, 1986) and Gas Laws (Krajcik & Peters, 1987), in this paper we will cite a small number of specific examples from two pilot studies that have used Molecular Velocities and Gas Laws.

Pilot Study I

The placement of probing questions in an instructional program may have a significant influence on learning. The nature and placement of questions are among several mathemagenic variables (Rothkopf, 1970; Wilson and Koran, 1976) having potentially important implications for the teaching and learning of science concepts in a computer environment. The effects of these variables were the subject of analysis in the first pilot study outlined here. In this pilot study we wrote prompting questions into "Model of a Gas" on the Molecular Velocities disk to encourage students to think aloud. For instance, the program prompts: "Predict what will happen to the movement of the molecules as the temperature increases. Say your prediction aloud, then press RETURN." Using such questions throughout the program enabled us to assess students' concepts and understanding of the particulate nature of matter; we can also observe whether students' concepts changed during interaction with the program. The program entitled "Model of a Gas" focuses on the following attributes of the kinetic molecular theory:

1) gas particles are uniformly distributed, on average, in a closed system;
2) gas particles are in constant motion
3) at a specific temperature the gas particles have a range of molecular speeds;
4) changing the temperature causes the velocities of gas particles to change.

Student volunteers from an introductory college chemistry course were used in this pilot study. Two preliminary generalizations can be made from the analysis of their tapes. First, the tutorial dialogue and dynamic visuals helped students focus on the important attributes of the kinetic molecular model presented in the software. For instance, most of the students did not verbalize that the simulated particles had a range of molecular speeds until they were directed to do so by dialogue in the program. One student in the sample did not make this observation even after being prompted to observe more closely. Second, the students gave a more extensive verbal description of the behavior of gas particles at the end of the session than when they began. At the beginning students typically said that a gas consisted of small moving particles. However, after students interacted with the software and the dynamic visuals, students also verbalized that the gas particles moved in random directions and that temperature affected the speed of the gas particles. While all students identified more attributes of a gas at the end of the session, most students did not verbalize that gas molecules had a range of velocities following the instruction presented in that study.

Pilot Study II

In the second pilot study reported here, the effects on learning of dynamic visuals related to the relationship between gas temperature and pressure were examined. In the experimental treatment students used one of two versions of computer software designed to teach the concept of an ideal gas and the qualitative relationship between pressure and temperature. Version one included tutorial dialogue and version two added visuals in the form of particle animations. Both versions covered similar concepts. Each version was developed so that students would spend approximately equal time to complete the lesson. Preliminary analysis of the videotapes indicate that students spent approximately equal time on each version.

Each version of the software had the following instructional objectives:

1.) gas particles are uniformly distributed in a closed system;
2.) gas particles are in constant motion;
3.) at a specific temperature gas particles have a range of speeds;
4.) changing the temperature causes the velocities of gas particles to change;
5.) at constant volume the velocities of the particles are directly proportional to the pressure;
6.) at constant volume pressure is directly proportional to temperature.

The pre and post-instruction clinical interviews were combined with the think-aloud strategy. Students were asked to speak aloud while interacting with the software and their commentary via microphone input and video output was recorded on a VCR. In addition, change in students' concepts are being measured in pre and post-treatment interviews. The combination of these techniques allows the researcher to examine change in students' conceptual knowledge. Interview protocols, modified from those used by Novick and Nussbaum (1978 and 1981), probe students' understanding about the particle nature of matter. Examples of the questions in the interview protocols included: "If you could see the air inside a flask, draw how it would look."

While the video tapes and interviews have not yet been thoroughly analyzed, preliminary results suggest that students who interacted with the visual version developed concepts more consistent with scientific models than did those students who interacted with the non-visual version. The pilot has been conducted most recently with tenth graders who were taking biology and who had completed one or more physical science courses in the preceding two years. In the sample, all but one were scored as exhibiting a "continuous model" prior to work with the software. In the posttest interview, the majority were scored as "particulate". In a control group receiving only verbal software treatment, only one student moved from continuous to particulate following the instructional treatment.

Limitations of the Research Strategy

The pilot studies outlined in this paper must be viewed as tentative and preliminary, and it would be unwise to make broad generalizations from them. Nonetheless, they suggest some promising research and development that has the potential to contribute to what is known about concept learning. The research methodology has considerable promise, but several problems need to be addressed:

1) Getting students to think aloud is not a simple task;

2) Systems for coding student behaviors must be perfected, and intercoder reliability improved;

3) Systems for meaningful reduction of the large quantities of data gathered with each subject and program must be perfected;

4) There must be greater access to appropriate hardware in school environments to properly support instruction and research;

5.) There must be greater availability of appropriate instructional software. Science educators should apply what they know about concept learning in designing appropriate instructional software to take advantage of opportunities presented by new technologies for concept development.

Implications for Instruction, Curriculum, and Research

Structured observations can serve as important sources of information about students' concepts and about how students learn science concepts as they interact with instructional software. While we have cited examples directly related to computer visuals, the technique is appropriate for a range of studies involving science instructional software. For instance, currently at the University of Maryland a graduate student is using the technique to investigate how students learn the concept of pH using interfacing equipment.

Research of the kind described in this paper can enhance understanding of how students develop important science concepts, and problem solving skills. It can provide valuable information on important variables in computer environments such as placement of questions and the use of computer graphics. Such studies can also provide an empirical and theoretical foundation for the synthesis of instructional software, for improving teaching strategies and for improving school learning environments.

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The development of the concept of speed: A case of different frames of reference.

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Our study examined the nature and development of children's understanding of the concept of speed as revealed in the way children talk about the concept. We analyzed the evolution of the changes in the way children talk about this concept by interviewing subjects who ranged in age from 8 years to adulthood. Although the focus of this presentation is on analyses of verbalizations, we have not ignored more traditional assessment techniques. In fact, some of the research with Gardosh as well as a series of studies with Bob Siegler use a variety of standard non-verbal techniques to examine the development of the concept of speed. While the topic of this conference focuses on children's misconceptions, our presentation sheds light on the richness of children's conceptions. This is, indeed, the other side of the same coin.

Like many other concepts, our ideas about the speed of an object vary as a function of frame of reference. To illustrate: Imagine a man walking on a travelling train. How fast does he walk? At least three different answers, answers which depend on the perspective of the observer doing the judging of speed, can be given to this simple question. (1) From the point of view of someone sitting in the train, the man is walking quite slowly. (2) From the point of view of someone standing on the platform, the man is moving rather fast, this because his speed is seen as an additive function of the man's rate of movement plus the train's speed . (3) From the point of view of someone outside of our planet, the relative speed of the man is fastest of all, for the speed at which he walks now is an additive function of the

One child is sitting near the center while the other is sitting near the end. Is one of them going faster than the other? The answer depends on our frame of reference.

From a linear point of view, the outer one moves faster since he moves along a longer distance in the same time. From a rotational point of view they move in the same speed, since they cover the same angles or rotations, in the same time. You might wonder why I belabor the role of frame of reference, something this audience surely takes for granted given that I have cribbed from Einstein. My reason is straightforward. I believe that conclusions about children's understanding of speed suffer for their failure to take what we might all assume is obvious. But before I develop this argument, I should deal with some matters of background.

Linear speed, in the sense of distance covered per time unit, is taught repeatedly in school, at different levels of formalization from the final years of elementary school to the end of high-school. Rotational speed, in contrast, is taught at a highly formal level, and only at the end of high school to mathematically advanced students. At least this is the state of affairs in Israel. These educational facts may lead us to believe that:

a) Young children conceive of speed solely within a linear framework;
b) The rotational framework is acquired only during adolescence or adulthood, if at all; and
c) The acquisition of the rotational framework is mediated by schooling, presumably because our regular, day to day transactions with the physical world do not afford the inputs required for the development of an understanding of rotational speed.

It seems that Piaget (1970), as well as others who studied the development of the concept of speed, adopted these assumptions, either explicitly or implicitly (Ehri & Muzio, 1974; Levin & Simons, 1986; Lovell, Kellett & Moorhouse, 1962; Tanaka, 1971). Piaget's studies and analyses of the concept of speed are based almost exclusively on linear motion. So for example, he presented children with two trains going in and out of two linear and parallel tunnels. The tunnels were obviously of a different length, and the trains entered the tunnels and left them simultaneously. The children were asked if the trains were running at the same speed, or which one was faster. Until the age of 7 or 8, children tended to answer that the trains had the same speed, this because they shared simultaneous starting and stopping times or points. When the tunnels were removed though, so that the children could see that one train passed the other, they correctly attributed a greater speed to the faster train. Findings like these led Piaget to conclude that young children's intuitions about speed is related to a sense of overtaking. When one object passes another, it is taken to be faster. If so the children's concept of speed is not related to the overall distance travelled; nor is it related to the overall duration of a trip. It's conception is more ordinal in nature, one based on the relative spatial ordering of objects at different points in time.

To illustrate the dominating effect of passing on children's responses to questions on speed, Piaget used concentric motions. Children were shown two cars travelling side by side around the same circle and hence the same center. Note that in this situation the paths of both moving cars were visible. As before, children were asked if the cars moved at the same speed, or if one of them went faster. Since they ran side by side, neither of the cars passed the other. In line with Piaget's expectations, young children tended to answer that the cars had the same speed, even when it was pointed out to them that the outer car covered a longer distance. Interestingly enough, with the concentric motions display, the error of relying on passing (or actually, on lack of passing) and ignoring overall distance, persisted up to the age of 9 to 11 years.

How should we explain the discrepancy between the age at which the reliance-on-passing error is overcome in linear motion and the age at which the error is overcome in rotational motion ? Does it really mean that a concept of speed is tied into notions of passing, until a late point in development? Or is it instead possible that the "equal speed" response that is given on the rotational motion task stems from the children's use of a rotational frame of reference? We cannot rule out this possibility, since it is indeed the case that, from a rotational framework, the speed of two cars moving along two concentric circles is equal.

I must tell you that Piaget raised this possibility but dismissed it. He concluded that young children who used the "equal speed" response, were

not applying a rotational framework, since with age, they give it up, in favor of the "outer faster" linear response. Moreover, when shown and told that the cars covered different distances, they tended to come up with a completely wrong response, this time that the inner car was faster.

In contrast to these conclusions, we have developed the idea that such data do reflect the fundamental facts about frame of reference that I brought to your attention when I started my talk. It is important to point out that Piaget's studies and analyses of the concept of speed are based almost exclusively on linear motion. We argue that there is no reason to believe that the concept of speed is limited to the linear framework. Linear extension is a very salient dimension in certain motions but not in others. To get a feel for what I mean ponder an egg-beater. Whatever you are thinking, I doubt that you are thinking about linear motion at this moment. It is not relevant to your representation of the motion of an egg beater and hence not salient. This leads us to conclude that linear motion is salient when it is relevant to the motion under consideration; otherwise it is ignored.

This discussion of relevance introduces our more general claim, this being that, children have, from a young age, a core concept of speed, in the sense of amount of output per time. Various kinds of outputs, ones that set up different frames of reference, are processed in terms of this core concept. They use the frame of reference dictated by the context of a problem to select the kind of motion that they should focus on when solving the presented problem. The idea is that, when considering the speed of clapping of hands, they will refer to rate rather than to linear speed. Of course if they do so they will ignore the distance covered by the hands, and attend to the number of clapping. In the same vein, when considering the speed of the rotating of a hula-hoop, they will refer to the number of rotations and not to the distance covered by some point on the the hula-hoop. To be specific:

The hypotheses of our study were:

a) At least by middle-childhood, children conceptualize speed in both linear and rotational terms.

b) From this age, they can differentiate between different motions, and therefore apply linear terms to linear motions and rotational terms to rotational motions.

c) The ability to coordinate and relate different frameworks, may emerge later. This is because such an ability requires skill at transforming speed from one framework to the other.

The study

Subjects

Two hundred children and university students participated in the study. All the children came from schools in Israeli cooperative settlements called Kibbutzim, and were selected from the 3rd, 5th, 7th, and 9th grade. The Kibbutz schooling system is known to compete favorably with the best schools in Israeli cities. The university students were from humanistic and social sciences, mostly studying psychology or education. None of them had a university course in physics. Each age group had an equal number of females and males.

Method

Data were collected through interviews about rotational and linear motions: The part of the interview I will describe here dealt with the motions of 5 familiar objects. The objects were never shown to subjects, so we can say they were presented verbally. Two of the objects represented linear motions - a car and an ant - and three represented rotational motions - a mixer, a drill and a record player.

The participants were asked to determine: (1) the speed of each of the target objects ; and (2) how they could know this speed. If they seemed to have trouble with these questions, they were further encouraged to invent a way to determine the speed of the object concerned.

To give you the flavor of the interview, consider the following translation which is about the speed of a mixer. The interviewee is a boy in the 6th grade.

E: What is the speed of a mixer?
S: Don't know.

E: Try and guess.
S: Can I say the speed in ...
E: Whatever seems to you logical.
S: Fast.
E: Fa-a-a-st...
S: Fast, but fast relative to something; relative to a car, which goes fast, it's slow; Relative to.........
E: How do we know the speed of a mixer?
S: I imagine there is a way to measure it. I don't know...
E: invent something...Let's pretend you are sent to the kitchen in order to come back with an answer to the question: "what's the speed of a mixer?" What would you do?
S: To measure... To define the speed of a mixer... To say how much?
E: To say exactly what is the speed of a mixer.
S: Perhaps...How many rotations it makes in a certain time.
E: Yes...?
S: And...and... I don't know...To divide it...also.
E: To divide in what? what to divide?
S: One in the other. The number of rotations to divide in the time... it does it.

A coding system was used to categorize the responses. The specific categories were further classified into four major categories:

1. Pre-formal; 2. Linear Alone; 3. Rotational Alone ; and
4. Both Linear & Rotational or Transformational.

Since children could give explanations that fit in more than one of these categories, we coded their overall performance according to the following decision rule. First, we assumed that the 4 response categories represented three different levels, with category 1 considered the lowest level of response, and category 4 considered the highest. Categories 2 and 3 were placed at the intermediate level. This assignment done, the child's best response (as indexed by the level of explanation) was used in the analyses I will present today. But before I go to these, I should give you a bit more detail on the nature of the categories themselves.

1. **Pre-formal responses**: consisted primarily of 4 kinds of answers:
   - i) Tautological responses, based on what you might call "eye testimony". Example: To determine the speed of an ant, you should see how fast its legs move.
   - ii) Ordinal responses. Example: An ant is faster than a turtle;
   - iii) Responses referring to non-spatial outcome. Example: A drill is fast, otherwise it could not make a hole in the wall.
   - iv) Responses referring to psychological qualities. Example: An ant is fast since it is diligent.

2. **Linear responses**: referring to linear distance and time. Examples: To determine the ant's speed, we let it walk along one meter and measure the time with a stop-watch; We know the ant's speed by making it walk for 10 seconds, and measure in centimeters the length of road it covered.

3. **Rotational responses:** referring to number of rotations and time, or to time per rotation. Examples: To know the speed of a drill, I count how many times it comes to its starting position in 5 seconds; I let the mixer make a full round and measure the time.

4. **Both Linear & Rotational or Transformational responses**: Mostly, giving both a linear and a rotational response. Otherwise, referring to number of rotations per time transformed into linear distance. Example for transformational responses: I count the number of rotations a mixer makes in 5 seconds, and multiply it by the length of the circle it made in a rotation.

OK, now for the results. Table 1 shows the frequencies of children using the different response categories for linear and rotational motions per age group. Remember, only the best response per child was taken into account.

Table 1: Number of Children Using Each Explanation Type for Objects That Characteristically Move in Either a Linear Or Rotational Fashion

		GRADE					
Motion Item	Explanation Category	3rd Grd.	5th Grd.	7th Grd.	9th Grd.	Univ. stud.	Total
Linear	Pre-formal	11	3	0	0	0	14
Rotational	Pre-formal	16	5	5	0	0	26
Linear	Linear	26	35	39	39	38	**177**
Rotational	Linear	0	0	0	0	1	1
Linear	Rotational	1	0	0	0	0	1
Rotational	Rotational	21	27	23	34	28	**133**
Linear	Both or Trans	2	2	1	1	2	8
Rotational	Both or Trans	3	8	12	6	11	40

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* Total for each of rotational and linear motion items is based on an N per age group of 40; therefore each child is represented twice in the above table, once on the basis of their best score on linear items and once on the basis of their best score on rotational items.

The following findings deserve mention:

a) Pre-formal responses are not all that frequent to start with. Still, they decline with age for both the linear and rotational motion items. Although we expected a comparable number of pre-formal responses for the linear and rotational motion items, they seem to be used somewhat more frequently for rotational motions.

b) Linear speed explanations overwhelmingly dominate talk about linear motion items and rotational speed explanations overwhelmingly dominate accounts of the rotational motion items. This well differentiated pattern of responding occurs for all age groups, including the youngest.

c) Despite the fact that all age groups differentiated between item type as indexed by the way they talked about their speed, there was an effect of item type on the ease with which children accessed the suitable explanation type for that item. Thus, children were better able to talk about linear motion when considering linear items than they were able to talk about rotational motion when considering rotational items.

d) Very few children talked about both linear and rotational speed when pondering a given object type. If children did this, they tended to do so more often for rotational motion items; an outcome which makes sense since it would be nonsense to talk about the rotational speed of linear motion items like ants. (Where it would be acceptable , as in the case of cars, a few subjects said they could measure its speed by counting the number of rotations of the wheels.)

So far the data reveal that the large majority of all of our age groups differentiated between rotational and linear motions by virtue of the fact that they apply the appropriate frame of reference when answering questions about speed. Our next analyses address the questions of (a) whether the children realized that they were altering their frame of reference as a function of item type; and (b) showed any ability to transform their assessments of linear speed into rotational speed or vice versa. The pertinent data come from that part of the interview where we asked the participants to determine which of two motions is faster, where one was linear and the other rotational. After they answered this question, they were asked to justify their answer. In order to be brief, I'll present the results from analyses of Ss justifications for the ant and mixer pair of items. Results were similar for other comparisons.

To illustrate this part of the interview, I'll translate one paragraph of an interview with a university student.

E: What do you think is faster, an ant or a mixer?

S: There is no...they are different speeds.

E: Is it impossible to compare?

S: What do you mean?

E: Is there no way to know what is faster?

S: No, I..

E: Try to find a way.

S: Visually, the mixer shows more speed. But there is no connection between the speeds.

E: Try to make up an objective criterion that will enable to compare.

S: It is possible to put something on the end of a beater ... I don't care what... a red dot and to see what distance it traces in a certain time, and to see what is the distance that an ant covers in the same time.

The responses provided by the children were classified into six separate categories:

1) **Visualizations** : these responses refer to "mere seeing". Example: I can see that the mixer is faster.

2) **Source of power**: The child provides a physical mechanism to explain why one of the objects is faster. Example: the mixer has electricity, so it must be faster.

3) **No coordination**: The child uses two different criteria for each of the items and makes no effort to coordinate these. Example: The mixer is faster, since it can beat ten eggs and the ant can only walk.

4) **Absence of common measuring frame:** The child describes the two speeds in terms of distance per time, but fails to find a common measurement frame. Example: The mixer is faster since it finishes 10 circles when the ant covers only 6 centimeters.

5) **Rotational transformation:** The linear item type, e.g. the ant, is placed in a rotational context , e.g going around in circles,. The comparison between the linear and rotational item then takes place within a rotational framework. e.g:

how long it takes an ant and a mixer to complete a circle or how many circles each completes in a given time.

(6) **Linear transformation**: In this case the rotational item type, e.g. the mixer, is placed in a linear context, e.g., The distance covered by the mixer is imagined as stretched out, and a comparison is drawn between the time taken by the ant and the mixer to cover the same distance. Alternatively, the comparison is drawn between the distances they cover in the same time.

(7) **Other**

The following table presents the number of children in each age group who used the different explanation categories.

Table 2: Number of Children in Each Age Group Who Used Each of the Explanation Categories to Compare the Speed of Linear and Rotational Items

	GRADE					
Response Category	**3rd Grd.**	**5th Grd.**	**7th Grd.**	**9th Grd.**	**Univ. stud.**	**Total**
Visualization	18	13	9	1	2	43
Source of power	11	6	5	1	0	23
No coordination	2	3	2	5	3	15
Absent M. Frame	4	4	4	3	2	17
Rotational transformation	2	2	7	9	11	31
Linear transformation	1	10	10	21	22	64
Other	2	2	3	0	0	7

Three findings stand out in this table:

a) An effort to coordinate the two frameworks (by performing either linear or rotational transformation on an item) increases systematically with age, and becomes the predominant response by the 7th grade. Half of the children in this grade provide either linear or rotational coordination.

b) Linear transformations occurred more frequently than the rotational ones.

c) The tendency to compare the two motions without considering the need to do some transformation, occurred rarely. In those cases where the child was unable to coordinate the motions, she relied on less formal response modes, e.g. visualization.

What can we say about the conception or misconception of speed?

Our final conclusions are:

a) By at least the age of 10, children have a core concept of speed. This concept is concerned with the relation of output to time and captures the fact that different dimensions can serve as the appropriate output. In linear motions, the appropriate output is typically distance. In rotational motions, it is typically the number of rotations. In discrete motions it may be rate.

b) From the same age on, children are aware that the different kinds of speed cannot be compared unless at least one is transformed. The knowledge how to proceed with the transformation comes later, emerging only in the 7th grade.

c) The tendency to take linear speed as the common frame of reference, is more dominant than is the tendency to view rotational speed as the common frame of reference. The more dominant status of the linear framework could be due to at least two factors: First, although every rotational speed has a linear component, the opposite is not true. Second, for years after year school children encounter speed defined within nothing but a linear framework. Together these factors may serve to make the linear frame of reference psychologically privileged.

d) In general: to say something is privileged is not to say that it is unique. In particular, the fact that children assign linear speed a somewhat privileged status does not mean that they cannot work with rotational speed. Nor does it mean that they cannot adjust their frame of reference. Given the writings of those who study children's concepts of speed, I might have to conclude that the children are better able to be relativists than are the adults who study them.

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CONTEMPORARY COSMOLOGICAL BELIEFS

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Introduction

Observations of the spectra and distances of galaxies by Vesto Slipher, Edwin Hubble and others during the years 1915-1930 showed for the first time that the universe is expanding, with all the galaxies moving away from each other[1]. Measurements of the rate of expansion and other considerations have led astronomers to conclude that the universe began expanding from a highly dense state, called the Big Bang, about 10-15 billion years ago. These conclusions are now almost universally accepted by astronomers and physicists.

While the work of Slipher and Hubble had to await modern telescopes and instruments, theories of cosmology have been invented by human cultures through the ages[2]. Cosmological speculations have evolved from myths like the Babylonian *Enuma Elish*, which used highly anthropomorphic analogies and imagery, to physical models, which applied whatever scientific arguments were available at the time. Regardless of scientific content, however, all cosmologies are human world views and thus partly mirror the religious, psychological, and philosophical preconceptions of their authors. For example, Isaac Newton, whose own theory of gravity naturally suggested a universe in motion, equated space to the "eternal, infinite, absolutely perfect" body of God[3,4], implied in his *Opticks* that God prevented the stars from falling together[5], and presented a rational but faulty argument to the theologian Richard Bentley as to why the universe had to be static[6].

Gerald Holton has emphasized that scientific theories through history have been partly motivated by a small number of ideas and counter ideas that operate outside the domain of empirical or analytical evidence[7]. One of these pairs of ideas and counter ideas (called themata and antithemata by Holton) is constancy versus change. Using this dichotomy, we can divide cosmological theories into two classes: static and nonstatic. In the first class, the universe is imagined to be unchanging and eternal. In the second, the universe evolves and changes in time. Aristotle's cosmology[8], with its fixed and ageless heavenly spheres, is of the first class. Lucretius' cosmology[9], with its happenstance swirling of atoms, continually forming and unforming structures, is of the second. In historical papers on cosmology, constancy and change were often associated with order and disorder, respectively. A changing or unbalanced universe was frequently viewed as a universe headed toward disorder. For example, Immanuel Kant, in his "Universal Natural History and Theory of the Heavens" (1755), saw the universe as governed by "a single law, with an eternal and perfect order" and argued that the gravitational attraction acting on the fixed stars of the Milky Way "would draw them out of their positions and bury the world in an inevitable impending chaos, unless the regularly distributed forces of rotation formed a counterpoise or equilibrium with attraction ... and mutually produced the foundation of the systematic constitution"[10]. The concept of a static cosmology has had a strong grip on modern scientists. Albert Einstein assumed a static universe in his 1917 theoretical paper on cosmology[11] and found a static solution to his cosmological equations. A few years later, he resisted serious consideration of Alexander Friedmann's time-evolving solutions to his (Einstein's) own cosmologial equations[12,13] and acknowledged the relevance of such solutions only after the definitive observations of Hubble in 1929. As recently as 1948, astrophysicists Hermann Bondi, Thomas Gold, and Fred Hoyle proposed a non-evolving model of the universe called the Steady State Model[14,15], in which all changes arising from the outward motion of galaxies are exactly cancelled by the creation of new galaxies.

We hypothesize that static cosmologies partly reflect the deep human desires for order, constancy, stability, and control, all of which have been noted by anthropologists and psychologists[16–18]. If so, then these associations might be revealed in the cosmological beliefs of contemporary society. To begin an exploration of this hypothesis, we undertook two studies. One study was a national survey of adults and contained a series of multiple choice questions about cosmological beliefs and the reasons for those beliefs. The other study posed an open-ended essay question to high-school students, asking their reactions to an expanding universe. The psychological associations of adolescents are particularly interesting, since these young people are in the process of forming their belief systems.

For perspective, the national survey contained a few general astronomical questions not directly related to cosmology. We summarize some of these results also. As well as we can determine, our study is the first large national survey of astronomical knowledge in the general public[19], and our results should help extend the understanding of scientific literacy into this area. Scientific literacy must certainly be taken into account in the study of the psychological motives underlying cosmological beliefs. The nonstatic nature of the universe is now considered an established scientific fact, and some portion of the general public has been exposed to this fact.

The National Survey of Adults

Our national survey was conducted by telephone by the the Public Opinion Laboratory at Northern Illinois University, in late June and early July, 1986[20]. A total of 1120 adults aged 18 and over participated in interviews that averaged 20 minutes in length. Questions Q2-Q13 are given in Tables 1-6. Question Q1, omitted in the tables for brevity, was: "Would you say that the Sun is a planet, a star, or something else?"

Astronomical Literacy

We consider first the issue of astronomical literacy. Fifty-five per cent of people correctly identified the sun as a star; 25% said it was a planet; 15% said it was something else, and 5% either said they didn't know or refused to answer. In contrast, only 24% of American adults correctly responded that the universe is increasing in size (Table 1). Evidently, the expansion of the universe is a fact not nearly so well known as other basic astronomical facts. (Even among people with graduate degrees, only 43% correctly answered Q2.) Since the aggregate figure of 24% includes people who guessed their response, a better estimate of the fraction of people actually informed of the expansion of the universe is 19%, which includes only people who gave the right response to Q2 *and* based their response on scientific findings they had heard about (Q3, table 2). We note that neither the nature of the sun nor the expansion of the universe are facts widely known by the general public.

To search for possible associations between astronomical knowledge and age (A), gender (G), education (E), and membership in a church or religious organization (C), we performed a number of logit analyses[21,22]. Such analyses take into account the mutual associations between A, G, E, and C when looking for the individual effects of these variables on the response to Q1 (or Q2). We will report a detailed description of these analyses elsewhere. The main results are that correct understandings of the nature of the sun and of the expansion of the universe are both significantly associated with age, gender, and education. Younger people, males, and more educated people are more knowledgable in these areas than older people, females, and less educated people, respectively. These trends are in qualitative agreement with previous studies of scientific interest and knowledge in the general public[23,24]. An understanding of the expanding nature of the universe is also significantly associated with membership in a church or religious organization, with non-church members more knowledgable than church members. There is no evidence for this religious association with a correct understanding of the nature of the sun. Our interpretation of this last result is that an individual's religious beliefs have more influence on his cosmological concepts than on his general astronomical concepts.

Reasons for Cosmological Beliefs

We turn now from the issue of scientific literacy to the possible psychological, philosophical, and religious factors associated with cosmological beliefs. The above results already suggest a possible role of religious factors in cosmological beliefs. Question Q3 (table 2) allowed us to identify the subset of respondents whose cosmological beliefs were based on their personal view, rather than on knowledge of scientific findings. As shown in table 3, 75% of these respondents believe that the universe is constant in size. *In the absence of knowledge, most people believe in a constant universe*. Furthermore, the associations between cosmological knowledge and age, gender, and education found for the full set of respondents fall below a significant level for this subset of respondents. The only association that remains significant is that between Q2 and membership in a church or religious group, and this association is not highly significant. We conclude that while the likelihood of being informed about the expansion of the universe is much increased for younger people, males, and better educated people, the belief in a constant universe is independent of these factors for people whose belief comes from personal opinion.

Table 4 gives the respondents' own acknowledgment of the factors determining their personal view on change in the size of the universe. These four possible factors were not presented as mutually exclusive, and, on the average, each respondent acknowledged about two of them. As can be seen, the most popular factor was a respondent's own observations. The visible night sky certainly *appears* motionless, and this observation is clearly a strong motivating factor underlying the belief in a static universe. However, personal preference was acknowledged as a factor by over half of the respondents. Additional insight into the reasons underlying the belief in a static universe comes from an open-ended pilot survey of about 60 adults in the Boston area. For the adults who said they believed in a constant (static) universe, the reasons given included the following: "learned in school," " religious views, the universe stays the same always," "basic energy principles," "seems like it," "absolute guess," "read it somewhere," "a feeling," "newpapers, meditations on life," "just my belief," "because the heavens are endless, so they can't change," "it always stays the same, God made it all at once," "taught in Sunday school." These comments confirm that religious views underly some people's belief in a static universe.

Table 5 gives the fraction of respondents reporting that they would be troubled to learn of a constant universe or troubled to learn of an expanding universe. Significantly more people were troubled by an expanding universe than by a constant universe. Table 6 gives the respondents' own acknowledgement of the reasons they would be troubled by an expanding universe. Note that a large majority acknowledged being troubled by a possible danger to earth and by some unknown change in the universe. These two concerns were also prominent in the open-ended student essays, discussed below.

The Student Essays

To provide a deeper understanding of the associations with a nonstatic universe, an open-ended essay question was given to 83 high school students, in May, 1986. The students were selected from two high schools: Arlington High School in Arlington, Massachusetts, a suburban community bordering Cambridge; and Old Saybrook High School in Old Saybrook, Connecticut, a small town about 40 miles from New Haven. Although both of these schools are public, middle-class schools, their location suggests that the students might have a better than average knowledge of astronomy. The students in the sample were divided about equally between boys and girls, ranging in age from 14-18. The essays were administered in English classes during regular school hours. The students were asked: "If astronomers learned that the universe is increasing in size, with all the galaxies moving away from each other, how would this make you feel?" and invited to put their thoughts on paper. No further words were spoken until after the essays were collected. Essays ranged from a few words to 150 words.

The essays were independently analyzed by two of us (AL and BL), in a manner described in this pararaph. First, we read through the essays and, without mentioning particular essays, agreed upon a set of 7 content categories that encompassed all of the ideas expressed. Next, we each independently assigned content categories to each essay. An essay could be assigned as many content categories as fit, and every essay was assigned at least one category. (The smallest number of categories assigned to any essay was one and the largest was three.) We then compared our assignments. One of us made a total of 113 assignments and the other a total of 106 assignments, for a grand total of 219 assignments. There were 182 overlaps, or 83%. By comparison, if the assignments had been done randomly (given that each of us, on average, assigned 1.3 categories per essay and there were 7 possible categories), the fractional overlap would have been 1.3/7=19%. Satisfied with our ability to categorize essays in a reliable way, we then discussed the disagreements in category assignments and arrived at a final list of assignments. Table 7 gives the 7 content categories, the fraction of essays mentioning ideas in each category, and some representative quotes. An essay was placed into category 6 only if it mentioned some *specific* form of impending disaster; if it only expressed a vague fear or insecurity, it was placed into category 5. An essay placed into category 7 could not be placed in any other categories.

We now consider the results, shown in table 7. Categories 1 and 3 represent a positive reaction to an expanding universe. Nineteen essays, or 23%, fit into one or both of categories 1 and 3.

The strength of response in category 2 surprised us. Evidently, high school students today are so conscious of space travel, space exploration, and the possibility of extraterrestrial life that any issue relating to space and the universe may bring up these associations. We feel that this result reflects the fairly recent exploration of space by human beings. It is hard to imagine such a response 30 years ago, before Sputnik and the beginning of the space age.

Categories 4, 5, and 6 reveal negative psychological associations with the expansion of the universe. Thirty-six essays, or 43%, fit into one or more of these categories. This is a substantially larger fraction than the 19% of adults who reported that they would be troubled to learn of an expanding universe (see tables 5 and 6), probably because the students were asked to reflect on their feelings in an open-ended essay while the adults were only allowed a yes or no response. Categories 5 and 6, which may be more closely related than category 4, express fears of unknown change, loss of control, and possible destruction and death. Twenty-one essays, or 25%, fit into one or both of categories 5 and 6. Aside from the numbers, the students' detailed comments and own choices of words give insight into what disturbs people about a changing universe.

Finally, we note that religious ideas were not mentioned in the student essays, in contrast to the definite religious associations in the adult survey. We offer several possible causes of this discrepancy, none of which are mutually exclusive. One possibility is simply the difference in formats of the two studies. The students were not asked explicitly to comment on a possible religious component to their feelings, nor were their responses tested against their church membership. Another possibility is a cohort effect, like that mentioned in regard to category 2. The understanding of religion today, and its relationships to science and world view, is different from what it was even as recently as 25 years ago. Cosmological questions today lie less in the religious domain and more in the scientific domain than they did in the past. In a classroom discussion with the 18 students from Arlington High School after the essay was handed in, most of the students made a clear distinction between their view of a spiritual heaven, where souls went after death, and the physical heaven, in which galaxies were located and moved about. Another possibility, which we consider less likely, is an aging effect. With age, each individual may increasingly incorporate religious notions into his or her world view. One or both of these last two possibilities is somewhat supported by results from the adult survey, where it was found that the fraction of church members basing their response to Q2 on religious beliefs (Q4 in table 4) increased with age of the respondent. (In a dichotomized test, 30% of younger church members based their response on religious belief and 39% of older church members did, with a confidence of 95% in the significance of this difference.)

Conclusions and Prospects

In a national survey of American adults, 55% correctly identified the sun as a star, and 24% correctly responded that the universe is expanding. This astronomical knowledge is strongly associated with age, gender, and education, supporting and extending previous studies of scientific literacy. Younger people, males, and better educated people are more knowledgable in the area of astronomy. We have also found that members of a church or religious group are less knowledgable in specifically cosmological areas. Education by itself, although clearly important, is not the most critical factor in astronomical literacy. Age has the highest association with a correct understanding of the nature of the sun; gender has the highest association with knowledge of the expansion of the universe. The greater scientific knowledge of younger people suggests that pre-collegiate teaching of astronomy has improved over the last several decades, particularly in the comprehension of the universe as a whole. The importance of gender and church membership suggests that astronomical literacy is entwined with social institutions and values, as well as with education. These results have implications for science education.

A majority of people believe that the universe is constant in size, including 75% of respondents basing their belief on personal opinion. Nineteen per cent of the adults in a multiple choice survey and 43% of the students in an open ended essay expressed negative emotional reactions to the discovery of an expanding universe. These reactions included fear of unknown change, fear of loss of control, a sense of helplessness, feelings of insignificance, and concern over immediate danger to earth and possible death. We speculate that some of these factors may partly explain the historical attraction of a static and stable universe. In this regard, it is significant that increased education does not diminish the prevalence of belief in a static universe among uninformed people, who might be likened to people living before the discovery of the expansion of the universe.

We encourage additional studies to test our results and conclusions, particularly those that might be sensitive to the form of the questions. Future work on the role of emotional factors underlying cosmological belief might include extended interviews with individuals and include surveys in other countries, to explore possible cultural factors. In such studies, it would be valuable to include scientists as well as nonscientists. For scientists, however, it might be harder to reach the emotional level, which may have been buried under the weight of acquired knowledge and scientific tradition. Our work here has concentrated on a specific scientific question - the static versus nonstatic character of the universe - because of its historical importance and its identification with the basic human conflicts between change and constancy and between order and disorder. Further psychological studies of related scientific questions may increase our understanding of the psychological and social environment in which science is done.

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Notes

1. A good history of modern work in cosmology, leading to the discovery of the expansion of the universe, is J.D. North, *Measure of the Universe* (Oxford: Oxford University Press, 1965).
2. An historical selection of original papers on cosmology, from Babylonian times to the present, is in M. K. Munitz ed., *Theories of the Universe* (New York: The Free Press, 1957).
3. The quotation is from Isaac Newton, *Principia* (second edition, Cambridge, 1713), General Scholium, Book III, trans. in *Sir Isaac Newton's Principia* (Berkeley: University of California Press, 1962), vol. 2., pg. 544.

4. E. A. Burtt, *The Metaphysical Foundations of Modern Science* (New YOrk: Doubleday, 1954), chapt. VII.
5. Isaac Newton, *Opticks* (New York: Dover, 1952), pg. 369.
6. Newton's first letter to Richard Bentley, Dec. 10, 1692 in *Opera Omnia*, IV, 429; reprinted in *Theories of the Universe*, op. cit., 211-212.
7. G. Holton, *Thematic Origins of Scientific Thought: Kepler to Einstein* (Cambridge MA: Harvard University Press, 1973), intro. and chapt. 1.
8. Aristotle, *De Caelo (On the Heavens)*, trans. W. K. C. Guthrie (Cambridge MA: Harvard University Press, 1971).
9. Lucretius, *De Rerum Natura*, trans. W. H. D. Rouse and M. F. Smith, in the Loeb Classical Library Cambridge MA: Harvard University Press, 1982).
10. I. Kant, *Universal Natural History and Theory of the Heavens* (1755), trans. W. Hastie *Kant's Cosmogony* (Glasgow, 1900); reprinted in part in *Theories of the Universe*, op. cit., pp. 231-249. The quotations are taken from pages 238 and 239 of the latter reference.
11. A. Einstein, *Sitzungsberichte der Preussischen Akad. d. Wissenschaften* (1917), p. 142; trans. W. Perrett and G. B. Jeffery in *The Principle of Relativity* (New York: Dover, 1952), pp. 177-188.

12. A. Friedmann, *Zeit. Phys.*, **10**, 377 (1922).

13. Einstein published two replies to Friedmann's paper. In the first, *Zeit. Phys.*, **11**, 326 (1922), Einstein claimed that Friedmann's paper had made an error and that it actually *mathematically* proved that the cosmological solutions had to be static. In the second reply, *Zeit. Phys.*, **16**, 228 (1923), Einstein acknowledged that he (Einstein) was mistaken in thinking that Friedmann had made an error and that Friedmann's work presented a correct alternative to Einstein's own earlier static cosmological model. However, in the hand-written draft to this second reply (The Collected Papers of Albert Einstein, Boston University, document 1-026) is a crossed-out sentence fragment saying that, to Friedmann's time-dependent solution of the cosmological equations, "a physical significance may hardly be ascribed." ("denen eine physikalische Bedeutung kaum zuzuschreiben sein durfte.") This quotation is reprinted with the permission of Hebrew University of Jerusalem, Israel.

14. H. Bondi and T. Gold, *Mon. Not. Roy. Astron. Soc.*, **108**, 252 (1948)

15. F. Hoyle, *Mon. Not. Roy. Astron, Soc.*, **108**, 372 (1948).

16. J. G. Frazer discusses the various ways that human cultures have expressed the desire for control - from primitive magic to religion to science - in *The Golden Bough* (1890); reprinted in (New York: Avenel Books, 1981), chapter 1.

17. Theories of the tension between constancy and change have been fundmental to psychological explanations of the human experience. Sigmund Freud describes his "principle of constancy," which later became his "pleasure principle," as "the effort to reduce, to keep constant or to remove internal tension due to stimulation" *Beyond the Pleasure Principle* (1920), trans. in (New York: W. W. Norton, 1961), p. 49. On page 2 of the same work, Freud quotes G. T. Fechner, saying "every psycho-physical motion arising above the threshold of consciousness is attended by pleasure in propoprtion as ... it approximates complete stability."

18. According to the developmental psychologist J. Piaget, a tendendey toward a state of mental equilibrium operates from infancy to adulthood. See *The Origins of Intelligence* (New York: International Universities Press, 1952), pgs. 5-6.

19. We have searched the indices of all surveys done by the Harris and Gallup organizations, including *The Gallup Poll: Public Opinion, 1935-1971* by G. H. Gallup (New York: Random House, 1972) and individual annual volumes from 1972 to the present of *The Gallup Poll: Public Opinion* by G. H. Gallup (Willmington: Scholarly Resources Inc.). No questions have been asked on the physical nature of the solar system or universe, although questions have been asked on UFOs and on the likelihood of extraterrestrial life.

20. The POL sample frame chooses persons drawn randomly from 150 primary geographical sampling units, which include the 33 largest metropolitan areas in the United States and 117 counties. Forty-two states are represented.

21. L. Goodman *Analyzing Qualitative/Categorical Data: Log-Linear Models and Latent Structure Analysis* (Cambridge MA: Abt Books, 1978).

22. S. Fienberg *The Analysis of Cross-Classified Categorical Data* (Cambridge MA: MIT press, 1980).

23. J. D. Miller, *The American People and Science Policy* (New York: Pergamon Press, 1983).

24. *Daedalus, 112:2* (1983).

Table 1. Belief in constant versus changing size of universe.

Q2: "Do you think the Universe is getting bigger in size, getting smaller in size, or remaining the same in size?"

	Bigger	Same	Smaller	Don't know/ No Answer	N
All U.S. adults*	24%	59%	10%	7%	1120
Gender					
Female	18	63	12	7	636
Male	37	50	7	6	484
Age					
LT 35	32	55	9	4	394
35-54	28	57	10	5	428
GT 54	16	60	12	12	298
Education					
LT HS	17	59	10	14	118
HS	21	62	11	6	577
AA	29	61	8	2	87
BA	34	50	9	7	211
Grad	43	43	10	4	127
Member of church or religious group					
Yes	21	63	10	6	748
No	37	46	9	8	372

* For this row, and only this row, percentages were weighted in accordance with national demographics in age, gender, education and race.

Table 2 Basis for belief in size of universe.

Q3: "Is your answer to the previous question based mainly on scientific findings you've read or heard about, or on your personal view?"

	Scientific	Personal	Don't know/ No Answer	N
All U.S. adults*	35%	59%	6%	1120
Gender				
Female	30	64	6	636
Male	50	45	5	484
Age				
LT 35	37	60	3	394
35-54	43	52	5	428
GT 54	34	56	10	298
Education				
LT HS	27	59	14	118
HS	35	61	4	577
AA	28	68	4	87
BA	46	47	7	211
Grad	61	36	3	127
Member of church or religious group				
Yes	37	58	5	748
No	42	52	6	372

* See Table 1.

Table 3 Personal belief about size of universe.

"Do you think the Universe is getting bigger in size, getting smaller in size, or remaining the same in size?" [tabulated just for respondents answering "personal view" to Q3]

	Bigger	Same	Smaller	Don't know/ No Answer	N
All respondents	12%	75%	11%	2%	626
Gender					
Female	9	77	12	2	406
Male	15	73	8	4	220
Age					
LT 35	13	75	11	1	235
35-54	14	75	10	1	224
GT 54	8	76	11	5	167
Education					
LT HS	17	70	9	4	70
HS	11	76	11	2	353
AA	15	75	10	0	59
BA	8	76	13	3	99
Grad	9	80	11	0	45
Member of church or religious group					
Yes	9	78	10	3	434
No	18	69	11	2	192

Table 4. Basis for personal view on change in size of universe.

[Asked only to people answering "personal view" to Q3]

Q4: "Is your personal view based on your religious beliefs?"
Q5: "Is your personal view based on your own observations?"
Q6: "Is your personal view based on what you would like to think?"
Q7: "Is your personal view based on just a guess?"
(Can answer yes to more than one.)

	Religious Beliefs	Own observations	Would like to think	Guess	N
All U.S. adults*	31%	79%	53%	59%	626
Gender					
Female	27	77	53	59	406
Male	26	81	47	55	220
Age					
LT 35	24	78	47	56	235
35-54	25	76	52	61	224
GT 54	32	81	55	55	167
Education					
LT HS	46	81	61	63	70
HS	26	80	50	58	353
AA	27	83	59	51	59
BA	20	75	47	60	99
Grad	13	64	42	56	45
Member of church or religious group					
Yes	35	79	52	55	434
No	9	78	48	64	192

* See Table 1.

Table 5. Likelihood of being troubled by constant versus expanding universe.

Q8: "If astronomers discovered that the universe is constant in size, would this trouble you?"

Q9: "If astronomers discovered that the universe is expanding, with all the galaxies moving away from each other, would this trouble you?"

	Troubled by Constant Universe	Troubled by Expanding Universe	N
All U.S. adults*	7%	19%	1120
Gender			
Female	6	21	636
Male	7	15	484
Age			
LT 35	7	16	394
35-54	5	19	428
GT 54	7	21	298
Education			
LT HS	14	26	118
HS	5	20	577
AA	7	22	87
BA	5	11	211
Grad	8	15	127
Member of church or religious group			
Yes	6	20	748
No	7	15	372

* See Table 1.

Table 6. Basis for being troubled by expanding universe.

[Asked only to people answering yes to Q9]

Q10: "Would you be troubled because there might be danger to the Earth?"
Q11: "Would you be troubled because the Universe would not be peaceful?"
Q12: "Would you be troubled because the Universe would be changing in an unknown way?"
Q13: "Would you be troubled because God's creation would be coming undone?"
(Can answer yes to more than one.)

	Danger to Earth	Unpeacefulness	Unknown change	God's creation undone	N
All U.S. adults*	92%	59%	92%	55%	205

* See Table 1.

Table 7. Content analysis of student essays on feelings about an expanding universe

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"If astronomers learned that the universe is increasing in size, with all the galaxies moving away from each other, how would this make you feel?"

Category 1: Expression of intellectual curiosity about the universe, with no negative feelings (Fraction of essays in this category: 14%)

Representative quotes:

"I'd feel that we were increasing our knowledge and technology." (essay 35)

Category 2: Concern over the increased difficulty of exploration of the universe, space travel, or communication with extraterrestrial life (20%)

"I really wouldn't want the galaxies to move farther away from each other. I feel that there is so much out there to discover. If the galaxies kept moving it would be harder to get to them, and explore." (essay 8) "If the galaxies moved farther apart then it would be more difficult to study them and learn if there is life in them." (essay 61)

Category 3: Fantasy of solving some problem on earth by the new discovery (12%)

"I feel that it would be great because then we'd have more room. Cities wouldn't be so dense and there wouldn't be such a heavy population." (essay 33)

Category 4: Personal feelings of insignificance, smallness, loneliness, or isolation (19%)

"If I learned that the galaxy was increasing, this would probably make me feel very small and unimportant, even more than I feel already... If the universe gets bigger, it will make us seem smaller, and this would probably be hard on us, because we are used to thinking that we are the most important things in the whole galaxy." (essay 2) "It would make me feel very small and unimportant." (essay 5) "It would make me feel weird because ... it would just not be a galaxy anymore with planets or anything around us anymore and it would feel that the earth is alone and no one is around us anymore." (essay 26) "The earth and earth people would become continuously less important in the Universe. We would serve a smaller purpose, if we are sure we have a purpose now." (essay 38)

Category 5: Vague feelings of insecurity and anxiety associated with unknown change or lack of control (18%)

"It makes me feel a sense of impermanence about the universe." (essay 4) "Scared." (essay 10) "It is quite frightening to think that the universe is increasing in size." (essay 18) "It would worry me because of the unknown change to me." (essay 68) "...it's a feeling that it's gonna happen anyway and can't be controlled by anyone. People in the world think they have control over everything and this would be a shock." (essay 82) "It sends chills up my spine to think about the whole thing, and whatever is happening or going to happen to the universe is inevitable." (essay 13)

Category 6: Fantasy of a scenario or catastrophic event leading to the end of the earth or solar system (10%)

"I feel that if the universe was growing in size, and the galaxies were moving apart, that could only go on for so long and then something would explode or destruct. It seems like every major movement of the universe has a negative reaction - like the sun eventually burning out and the gamma rays hitting the earth." (essay 1) " ... the sun would probably move farther away and we would probably die." (essay 20) "I would feel a little nervous because it could be from gases and may cause the world to blow up. It just wouldn't be comforting to me." (essay 68) "I just hope it's still around for many years to come, and that by increasing in size it will not break apart." (essay 27) "This would make me feel helpless because if we drift away from the sun, then there will be no life on earth." (essay 54)

Category 7: Lack of concern, lack of response, or feeling that there would be no effect (31%)

"I wouldn't really care, it would never affect me." (essay 42) "I probably wouldn't feel anything because I don't know enough about the universe." (essay 44) "It really wouldn't matter to me ... I live here on earth and that's all that I'm concerned about." (essay 34) "If I found out that the galaxies were moving away from each other and expanding, I probably wouldn't worry. Hey, McDonalds had to expand, and so did Wendys, eventually, so it probably wouldn't phase me." (essay 81)

TERTIARY PHYSICS: A CASE STUDY IN STUDENTS' CONCEPTIONS OF SOUND

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1. INTRODUCTION

The conceptual problems which tertiary physics students typically encounter can be both complex and daunting. These conceptual problems are often compounded by sets of implicit, and occasionally explicit, pedagogical assumptions predominant in many tertiary physics teaching environments. Interviews with recent physics graduates in a teacher education programme presented an insight into some of the conceptual problems which are associated with the teaching of sound at the introductory level. Transcripts from these interviews are used to present a case that many conceptual problems are associated with pedagogical assumptions which appear to have their roots in what many science educators consider to be an inappropriate philosophical perspective - a perspective based on a commitment to metaphysical realism.

2. AN EPISTEMOLOGICAL BASE TO PEDAGOGICAL ASSUMPTIONS: Metaphysical Realism.

The dominant view of the nature of science which has permeated with uncanny consistency through all major universities in the western world is one or another form of philosophical empiricism/positivism - this has been true for most of this century (Finley, 1983; Gauld, 1982; Glasersfeld, 1984; Hodson, 1985; Marquit, 1978; Mulkay, 1979; Norris, 1984; Otero, 1985; Schon, 1983). In order to appreciate why this philosophical view is considered by many science educators to be inappropriate (for example, see Finley, 1983; Gauld, 1982; Helm, 1983; Hodson, 1985; Holton, 1979; Marquit, 1978; Norris, 1984; Otero, 1985) the main epistemological theme of empiricism/positivism needs to be examined.

Although there are many shades of epistemology under the banner of philosophical empiricism/positivism it is possible to "discuss the main theme without going into the details of variations and still provide a useful analysis" (Nersessian, 1984, p. 5). A useful way of viewing the basic scientific meaning/truth/epistemological theme pertaining to the empiricist/positivist umbrella is to describe it as "metaphysical realism" (Glasersfeld, 1984; Putnam, 1981). A metaphysical realist is a person who is commited to an epistemology where knowledge is a one-to-one mapping (accurate reflection) of nature's independent reality which exists in our "objective world"; something is "true" "only if it corresponds to an independent 'objective' reality" (Glasersfeld, 1984, p. 20).

3. EDUCATIONAL PROBLEMS ASSOCIATED WITH METAPHYSICAL REALISM.

Now the question arises: Why should metaphysical realism's epistemological themes present a problem for physics education at the tertiary level? The answer is framed from a constructivist perspective. Recent research (for example, see Driver and Bell, 1986; Erickson, 1984; Fensham 1983; Novak and Gowin, 1984; Pope and Keen, 1981; Pope and Gilbert 1983) has indicated that many of the conceptual problems which physics students encounter may have their foundation in teaching which fails to take any significant account of the prior knowledge, intuitions, beliefs and understandings that the students invariably bring with them to the instructional setting. Essentially there is a lack of cognizance that "meaningful learning" (Novak and Gowin, 1984) involves a process whereby individuals construct their own meanings in an effort to make sense of their experiences and the information that they receive. (For a discussion on a constructivist view of learning see Erickson, 1987).

Metaphysical realism inherently downplays any personal subjective contributions to science, in attitude, historically, and epistemologically. For example, in his criticism, Marquit (1978) wrote:

> The emphasis which modern empiricists place on the logical structure of scientific knowledge has greatly reduced the dependence of modern sciences on ... speculative and intuitive methods ... to undervalue the contributions of conscious mental or theoretical activity to the process of acquiring an understanding of the physical world. (p. 785)

This pervasive "undervaluing of theoretical activity" in physics is described by Einstein (1973) as follows:

> If you want to find out anything from the theoretical physicists about the methods they use, I advise you to stick closely to one principle: don't listen to their words, fix your attention on their deeds. To him who is a discoverer in this field, the products of his imagination appear so necessary and natural that he regards them, and would like to have them regarded by others, not as creations of thought but as given realities. (p. 264)

When philosophical perspectives in teaching effectively dismiss knowledge and understanding as being constructed by autonomous thinking human beings (recognising the consequent relativistic nature of knowledge) it is not surprising to find physics students who have successfully completed their undergraduate physics with conceptual understandings which are in a very real sense "alternate" to the accepted position of current physics (for example, see Faucher, 1983; Helm, 1980; Hewson, 1982; Sjoberg and Lie, 1981; Villani and Pacca, 1987). These alternate conceptions are constructed by students both prior to formal teaching and during the teaching process - see Driver and Erickson (1983) and Driver and Oldham (1986).

Some physics conceptual problems which appear to have their roots in metaphysical realism will now be considered using students' conceptual understanding of sound to support the discussion. The participants in this study were students who had majored in physics and were currently in a teacher education programme.

3.1 Encouragement to rote-learn facts.

Since metaphysical realism has "truth" corresponding to an "objective reality" (Glasersfeld, 1984), the knowledge of "facts" becomes important and the pedagogical assumption that "the knowledge of facts provides an adequate psychological foundation for concept learning" (Otero, 1985, p. 364) naturally follows. Teaching which gives overriding importance to "facts" which "match" ontological reality (Glasersfeld, 1984) has foundationally supported the dominant Western teaching perspective termed "cultural transmission" (see Pope and Gilbert, 1983, and Pope and Keen, 1981).

Cultural transmission is a pedagogy which presents knowledge as "bundles of truths" in a presumed "logical" order into "tabula rasa" minds - the metaphorical sponge absorbing knowledge. Knowledge, as culturally defined, is to be internalised by younger members of the culture in their educational apprenticeship (see Benjamin's 1939, classic "Saber-tooth Curriculum" for an excellent satire of the process). Cultural transmission is naturally open to support from the psychological paradigms of behaviourism and neo-behaviourism. "Facts" and their associated conceptual understanding are best learnt by studying hard and the best stimulus-response reinforcement for studying hard is provided by examinations, tests and quizzes (the positive reinforcement is a good grade and vice versa). In such a scenario it is likely that, for many students, introductory physics becomes an extremely difficult subject of laws, formulae and problem solving algorithms, all to be "learnt" to pass the final examination; a conceptual swampland. Thus it becomes possible for students with a deficit of appropriate current physics conceptual understanding to be successful in examinations by memorizing sufficient material (for example, see Arons, 1979, 1983; Dahlgren, 1979; Dahlgren and Marton, 1978; Lin, 1982; Lundgren, 1977).

If learning is viewed as an activity which primarily involves "relating what one has encountered (regardless of source) to one's current concepts" (Strike, 1983, p. 68) then memorization on its own cannot contribute to conceptual growth (see Driver and Oldham, 1986; Posner et al, 1982; Hewson, 1981). For example, consider the following transcript extracts taken from interviews, with education post-graduate students who majored in physics, on the phenomena of sound. The first transcript extract starts when interviewee "Tom" first began to discuss sound propagating in a vacuum:

(I = interviewer, R = Tom's response).

I: Are you talking about in a vacuum now?

R: Yes, I'm talking about in a vacuum because, well it doesn't have to be in a vacuum. (Long pause and sound of Tom mumbling to himself). So in other words I was wrong about my initial idea about having to have particles to, umm, propagate sound waves, because sound in a vacuum is something like 3 times 10 to the 8 whatever .. m/s. (Long pause). Sound travels slower in air than it does in a vacuum.

Then Tom went on to discuss sound travelling from a vacuum into a medium. The transcripts resumes at this point:

I: And when light travels from a vacuum to air is that the same?

R: No.

I: What is the difference then?

R: (Long pause while Tom appears talks to himself) Oh yes! It is different because .. in a vacuum the speed of light is 3 times 10 to the 8 or something like that and when it goes into air it has to move particles as well, because I know in water the speed of light is less. That's why you get diffraction and things like that.

From his replies it appears that when Tom studied introductory physics he adopted a learning style based upon memorization which has conceptually left him with "a mass of logically and psychologically inconsistent fragments" (Dahlgren and Marton, 1978, p. 34). The inconsistent fragments in Tom's explanation can be clearly seen as he confuses poorly memorized facts about light and sound. For example, he recalls that sound propagates at "3 times 10 to the 8 or something like that" in a vacuum - an obvious recall confusion with the speed of light in a vacuum. In general it appears as if he cannot conceptually distinguish between light and sound. Tom is clearly giving the matter a lot of thought and reflection (perhaps for the first time) and struggling with the concepts, but his memory is not good enough for him to consistently structure his thoughts.

Later on during the interview Tom again revealed a learning style based on memorization. The section of the interview from which the second extract is taken dealt with how the students conceptualized the sound wave equation(s) with which they were familiar:

(I = interviewer, R = Tom's reply)

I: What laws govern the propagation of sound? Where do the equations of sound propagation come from?

R: (Long pause) .. Maxwell's equations are electromagnetic waves so sound is not part of that. Sound waves are .. they propagate differently. They always use water waves as an analogy to sound, umm, its because water waves require molecules in order to propagate and its analogy with sound is that they require molecules to propagate, they need a medium.

I: What would happen if you had done sound and were teaching Newton's laws and a student asked you: "Sir does sound obey Newton's laws?" What would your answer be? What laws govern the propagation of sound?

R: Well I am not sure, I am sure there are analogous equations to light and sound but sound is different because it requires a medium. What do you mean by Newton's laws? Do you mean gravitational laws? (After clarification) .. Sound obeys Newtonian mechanics, I think they found, so it must obey Newton's laws.

Tom's replies seem to imply that he cannot remember how, or on what principles, the wave equation was derived but he can remember an analogy, water waves. A strange analogy since water waves are transverse in nature, however, the analogy did help Tom to recall that sound requires a medium for propagation - something which had confused him earlier.

Helm (1983) pointed out that an understanding of physics equations implies the "ability to recognise the contexts in which those relationships are valid" (p. 42). Tom displayed none of this and was seemingly unsure of what was meant by Newton's laws - another example of inconsistent conceptual fragments resulting from rote learning. In his replies Tom also provided an insight into his view of the process of physics, "Sound obeys Newtonian mechanics, I think they found, so it must obey Newton's laws." Tom appears to have an epistemological commitment where physics is a set of discovered rules - which he tried to memorize.

Cultural transmission and examinations which reward or require memorization, illustrate how the tenets of a philosophical commitment can permeate throughout an education system. In university physics it is not that many instructors do not recognise the shortcomings of the type of examinations used (for example, see Arons, 1979; 1983), it is just that we are inextricably bound together by the common "cultural transmission" elements of our respective apprenticeships.

3.2 Problem solving and conceptual understanding.

Another physics pedagogical assumption which appears to be based on a commitment to link studying hard to the generation of conceptual understanding is found in the area of tutorial problem solving. An extrapolation of the metalearning commitment to studying hard usually has introductory students preoccupied with solving sets of standard tutorial problems. The link between students successfully solving sets of standard physics tutorial problems and their consequent conceptual understanding appears to be an underlying physics pedagogical assumption which is very widely held by physics educators (Chi, Glaser and Rees, 1981; Larkin and Reif, 1979; Larkin, McDermott, Simon and Simon, 1980). For example, Van Harlingen (1985) offered the following advice to introductory physics students:

> A solid understanding of physics includes the ability to solve a variety of problems .. your ability to successfully solve problems is one way that you (and your teachers) can measure your understanding. (p. 30, emphasis mine)

However, especially at the introductory physics level, the physics problems assigned as homework tutorials (and which subsequently form a major part of physics examinations) may be characteristically described as "stereotyped quantitative examples" (Gamble, 1986). For example, cross-reference many of the chapter problems in popular introductory physics textbooks such as Giancoli (1985), Halliday and Resnick (1986), Sears, Zemansky and Young (1985), and Weidner (1985).

It is not being advocated that physics tutorial problems play little or no role in helping students construct the kind of conceptual understanding which is desirable from the physicists perspective, and should be abandoned. Certainly problem solving in introductory physics has a vital role to play (see Arons, 1983, 1984a, 1984b). However, it is being claimed that physics teachers' unilateral linking of conceptual understanding with the ability to successfully solve such physics problems is a doubtful pedagogical assumption.

Hewson (1980, p. 398) has reported that even when different physics students "can solve the same problems satisfactorily they can have different conceptions". Confirmation of this was reported by Reif and Heller (1982) in their research on physics problem solving in basic university-level physics. In this research Reif and Heller also noted that:

> .. the cognitive mechanisms needed for effective scientific problem solving are complex and thus not easily learned from mere examples and practice .. Too little attention is commonly paid to the organization of the knowledge acquired by students .. students are given little help to integrate their accumulating knowlege into a coherent structure facilitating flexible use. (p. 125, emphasis mine)

Writing about student patterns of thinking and reasoning, Arons (1983, p. 577) wrote, "Without practice in giving verbal interpretations of calculations, students take refuge in memorizing patterns and procedures of calculation." To give verbal interpretations of calculations a student needs to create a world of "hand waving physics" which essentially involves exploring the essence of a conceptual idea both from within oneself and within the current physics theories, models and concepts. Hewson P. and Hewson M. (1984) have noted that conceptual conflict, a prelude to conceptual change, requires epistemological commitments to internal consistency and generalizability. The following transcript extract from the "sound" interviews illustrates a student's lack of such epistemological commitments and the consequent type of incompatibility which can exist between a student's intuition and a learnt physics model used for solving problems. In such cases the transcript indicates that students prefer to rely on their intuition.

During the "sound" interview the participants were presented with several tuning forks and encouraged to "play" with them. The tuning forks' frequencies were stamped on the forks and this was pointed out to the students. After the students had "played" with several forks the interviewer picked one up, struck it to produce a tone, and then asked the subject for an order of magnitude estimation of the wavelength of the sound being produced. "John's" reply was typical of the replies given by all the students interviewed:

(I = interviewer, R = John's reply)

R: Well I would tend to say something like a millimetre.

I: O.K. if that's 500 cycles per second (referring to the tuning fork in question), approximately, and we take the speed of sound to be 300 m/s, what would you calculate the wavelength to be?

R: Umm, cycles per second .. so you take velocity in m/s divided by wavelength, now its per second. (John declines the offer of of paper and pencil). Just trying to work with the dimensions. O.K. so you take velocity and divide by wavelength, 'cause velocity is frequency times wavelength. So wavelength is velocity divided by frequency, so that is 300 m/s divided by 500, which is say 0.6, so it would be 60 cm.

I: O.K., how would you explain that?

R: (Student laughs.) Umm, the formula works. (More laughter.) That is scary ... I now realise my millimetres was wrong because with something like a guitar string you can get .. well you don't really get frequencies that high but .. you can certainly get an A 440 .. umm .. mmm (John appears puzzled)

I: Why did you feel that this (referring to tuning fork in question) was within the millimetre range?

R: Umm, I guess I know that it is vibrating very quickly, 500 times per second - you can't see it really so .. and the mechanism is molecules, which are tiny.

John's intuition evoked a millimetre estimation for a wavelength which was approximately half a metre in length. John, however, had no trouble correctly calculating the wavelength. To obtain a correct answer appeared to be sufficiently important to John that he reassured himself using dimensional analysis. Why then did he so spontaneously offer his millimetre estimation? Why did he not do a quick mental calculation if he was the least bit uncertain of his answer? These questions give an indication of the confidence John had in his intuitive model, they also indicate that John had never correlated his intuition with his calculation model (as a physics major he must have completed many wavelength calculations). For John, it appears as though such physics calculations form part of a set of memorized procedures. The discrepancy between his intuitive estimation and his physics calculation appeared to astound him ("That is scary"), another indication that he had never made any connections between these types of calculations and his intuition. John's immediate efforts to reconcile his intuition and his calculation were not very coherent.

Another student, "Simon", revealed that although he recognised and accepted the isomorphic nature of wave equations in physics he had difficulty in developing a coherent conceptual perspective of the various concepts. Consider the following interview extract, Simon and the interviewer had just completed some experimentation on the reflection of light and sound using parabolic mirrors and found that the same laws seemed to apply to both.

(I = interviewer, R = Simon's reply).

I: O.K., well I wonder if you see sound being in any way similar to light?

R: Yes, umm, I guess because of the theory I have done. I .. it is hard to say. I can relate and deal with sound and light in a similar way by writing down similar equations.

I: Where do the light wave equations come from?

R: Well if you want to go all the way down to Maxwell's laws: electric fields and magnetic fields.

I: And sound waves?

R: Sound waves would simply be pressure variations.

I: Could I apply Newton's laws?

R: (Pause) yes, yes because at a molecular level it is just collisions. Conservation of energy and momentum. So in everyday life I see them as completely separate phenomenon .. light has different sources than sound, and you can, umm, sense it with your eyes rather than with your ears. I see it as completely separate, but in physics I can see the exact same equations apply to both, it is just you have different variables. In one case, or anyways for that matter, on paper you can do the same thing. You can have the Doppler effect with sound waves but it is not very often you have any reason to see a Doppler effect with light waves. So I guess how I relate to them on paper they are interchangeable, in real life I don't see them as being so.

Here we see Simon indicating that he had been unable to conceptually reconcile his daily experience of light and sound with what he had learnt in physics. The very nature of his very articulate reply seems to indicate that although he may have given the matter some thought, he chose to avoid conceptual conflict by compartmentalizing his everyday experience and what he learnt in physics. Strike (1983) reported a similar finding:

> Many undergraduates are not exactly overburdened with a need to have what they learn in science class be consistent with other scientific ideas or their own experience. Somewhere they have gotten the idea that science is allowed to be paradoxical and is not supposed to have anything to do with their everyday experience. (p.74)

From this perspective, physics, then, becomes something one "does" rather than something one "understands". Thus it appears that it is of questionable value for physics educators to assign tutorial problems for homework without having an insight into how students are constructing their conceptual understanding of the physics being taught (also see Arons 1983, 1984a, 1984b; Clement, 1981; Faucher, 1981).

3.3 The rate of instruction.

The metaphysical realist influence on traditional science instruction and epistemology, in its neglect in valuing students' existing conceptions, purposes and motivations, has covertly added another dimension to the conceptual difficulties which tertiary physics students have to face. Instruction which views the lack of "correct" understanding to be a consequence of a lack of "studying hard" (or alternatively lack of required intellect) may proceed at an enormous rate. The amount of course content covered then takes on overriding importance (English language introductory tertiary physics textbooks all offer the same basic curriculum). This rate of instruction is typically so rapid that any in-class reflection becomes impossible (see for example, McMillen, 1986; Tobias, 1986). Arons (1979) captured the nature of this typical instruction pace with a "length contraction" metaphor (from Einstein's Special Theory of Relativity):

> It is the premise of the vast majority of introductory physics courses ... that if one takes a huge breadth of subject matter and passes it before the student at sufficiently high velocity, the Lorentz contraction will shorten it to a point at which it drops into the hole which is the students mind .... The students have been 'told' but they have not made the concept part of their own .... they get 'credit' for going through a memorized calculational procedure which happens to give them the 'right' numerical answer ... while they have no understanding of the physics. (p. 650)

A rapid rate of instruction (machine-gun instruction) has a special place in the hidden curriculum. It emphasizes a positivistic epistemology and actively encourages memorization. What makes matters worse is that most physics teachers and many physics undergraduates appear to resign themselves to a rapid rate of instruction in the name of protecting some set of unwritten standards. There is simply too much to do and not enough time to do it.

A general theme which emerged from the interviews was that the students were never conscious of having "kicked physics ideas around". As one student put it, "In class there was never time for anything but more physics .. and more homework problems". In the interviews many of the students seemed to be reflecting on the topic of sound for the first time, for example:

"The more I think about it the less sure I am."

"I am just trying to formulate this, I have never been asked that question before - it makes it tougher."

"Just wondering if I do know ..."

"I think I am confusing myself."

One reason for the students' apparent lack of reflection may lie in the typical rapid rate of physics instruction discussed earlier. In rapid teaching there is little or no time to engage in, and encourage, reflection - that is left for the student to do on his own. This lack of reflection may in turn be linked to a broad belief based on the epistemological themes of metaphysical realism which were discussed earlier. The belief is that science and humanity students construct their conceptual understanding in different ways. This type of metalearning assumption was recognised by Tobias (1986) and McMillen (1986) in their studies done in physics departments at the universities of Chicago and Indiana respectively. In these studies they sought to obtain an insight into the nature of the problems which many students have with university introductory physics, in particular non-science majors. This was done by inviting faculty peers from the humanities at the respective universities to attend typical introductory physics lectures and comment on the difficulties that they experienced in the lectures. McMillen (1986, p. 18), citing Tobias, described the belief as follows:

> Scientists learn through problem solving, reaching understanding through step-by-step analysis. Many learn best by themselves. Humanists, however, "play" with concepts, learning by writing and restating their ideas.

For a constructivist this kind of assumption is painful. It reflects no appreciation for the interactive process that takes place within a person's conceptual repertoire as they try to make sense of new concepts; and, no appreciation for the importance of relating what a teacher is trying to teach to what the learner already knows (see Ausubel, 1978; Driver and Erickson, 1973; Driver and Oldham, 1986; Novak and Gowin, 1984). Instead, learning is presented as a mechanical process; "step-by-step" collecting and building up knowledge, like a squirrel collecting nuts.

Constructivist learning and conceptual change models (for example, see Driver and Oldham, 1986; Hewson, 1981, 1982; Posner et al, 1982) postulate that in order for a person to construct conceptual coherence, his or her existing conceptual conflicts need to be resolved. When conceptual conflicts are not resolved and a person wants, or needs to, retain different conceptual models then these conceptual models tend to be compartmentalized (Claxton, 1983). A way of looking for evidence for conceptual compartmentalization is to look for contextual explanations - different contexts may draw on different conceptual models. In the "sound" interviews several students unconsciously provided different conceptual explanations when presented with different contextual sources of sound. For example, consider the following extract taken from "Greg's" interview.

The background to Greg's explanations was as follows: At different stages in the interview the students' macroscopic and microscopic models of the propagation mechanism of sound were probed using three different contextual sources of sound. These were: a source not common to the classroom yet common to everyday experience (bursting a balloon); a source taken from the students' personal experience (their own example); and, a source recognisable from a teaching context (a tuning fork).

(I = interviewer, R = Greg's reply)

First Context: A balloon being burst with a pin.

I: (Sound of balloon bursting) Could you relate to what happened?

R: Physically?

I: Yes.

R: Well you have the air in the balloon and it was under pressure. When the balloon was popped there was an expansion of the air in the balloon out into the outside which was under less pressure and it generated pressure waves. The movement generated waves in the air which travelled to the ear .. creating the sensation of noise I guess, in the ear.

I: How would it travel?

R: Well just the general outward motion of the air from the balloon made ... disturbed the air so much there was just this outward motion and because the force outward was greater .... because the force outward was so great that it created a partial vacuum in that space where the air was, the air had to rush back in again and it basically created a vibration that travels outwards. It is making the air molecules move and it is not just happening in one place, anything that happens in one place tends to affect everything around it so that sound basically travels outward and of course the wider the area it travels outward in the less intensity there is so the sound sort of fades off into the distance or the farther away from the sound the less there is to it. Umm ....

Second Context: Interviewees' own example.

R: My favourite example is down on the beach on a summer day when the air pressure is really high, umm, sort of when it is really low, umm, you see somebody picking up a stick and throwing up a rock in the air and hitting it and then you hear the sound a couple of minutes later, or a couple of seconds later. There is just a slight delay, that means the sound is being produced when the stick hits the rock, but those molecules are only agitated there, or only start moving there, and it takes a while for this idea that these waves have to travel and reach your ear.

I: Could you follow up on that? O.K. someone hits the rock with a stick and there is an agitation of molecules? (Student agrees) And then?

R: They, umm, it is like a pulse, there is a disturbance there in the air and ... the pulse it travels through the air. The molecules in the air are ... pick up the vibration and transmit it along.

I: What were the molecules doing before that?

R: Normally they are just at rest, they are just maybe moving around with wind.

Third Context: Sound produced from a tuning fork.

R: Normally they [the molecules] are at rest .... they have a small amount of energy but it is randomly distributed, they are sort of moving around, quite randomly, not in any specific direction. When a sound happens they start moving back and forth in the direction that the sound is travelling. They are moving back and forth, at a certain rate, they are oscillating, like these waves are going in and out, this high pressure business again. So depending on how .. I think I am confusing myself ..

The balloon bursting was done at the beginning of the interview session and Greg offered only elements of a basic macroscopic model. This conceptual understanding probably has its origins in the "ideal fluid approximation" which is often used as a physics model to explain the propagation of sound in liquids and gases. By the time Greg offered his own "favourite" example he had begun to think microscopically using his own terms - a conceptual model where air molecules are normally at rest. He did this despite being fully acquainted with the basic concepts of the kinetic model. When the context changed to one strongly associated with classroom physics, the tuning fork, Greg spontaneously went to a microscopic descriptive level based on the kinetic model. He then made an attempt to begin to reconcile his kinetic and ideal fluid models and at that point admitted confusion.

The above example provides an insight into the capacity which people have for holding and using conflicting conceptual models. In this example it appears that it was the context of the problem which determined whether Greg would use his scientific or intuitive knowledge to explain phenomena - what compartment of knowledge he would use. In a case such as this, the rapid rate of instruction and metalearning beliefs which discount the necessity for physics students to "play" with new concepts to achieve consistent conceptual understanding, can only encourage students to avoid conceptual conflict by compartmentalizing their conceptual models.

4. UNINTENDED PEGAGOGICAL ASSUMPTIONS: an example.

The Tobias and McMillen studies referred to earlier indicated that university physics instructors had little insight into the conceptual difficulties which their students typically encounter. A major problem often lay in making conceptual connections between the verbal (or written) explanations, demonstrations, and mathematical representations. As an illustration of the kind of conceptual construction that can take place when confronted with such a problem consider the model of sound propagation in air which "Peter" had constructed. When considering Peter's model bear in mind that he felt quite confident about it, having just completed a teaching practicum where he taught "sound":

> You see waves travel in sine waves or cosine waves or something like that, its a wave propagation. Now I think the wave propagates with the use of molecules but its not the same molecule that is going through, its like this molecule will hit another one that will go up here and hit another one that will go down here (Peter is tracing out a sinusoidal path of molecular collisions on the desk surface with his finger).

The interviewer was so intrigued with Peter's model he asked him to draw it on a piece of paper; he did so as follows:

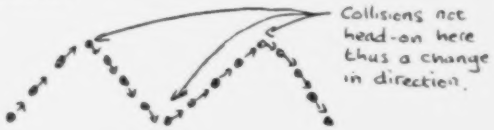


Figure 1: Peter's sound propagation model.

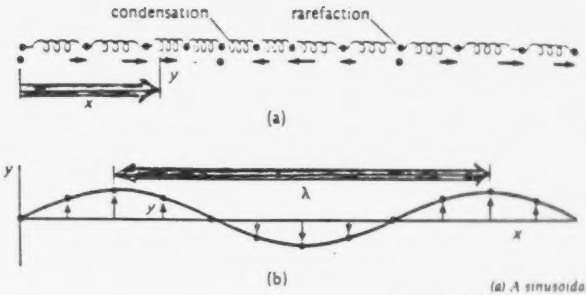
To give some explanatory perspective to Peter's model, an examination of typical physics instruction in this area is necessary.

Sound waves are longitudinal waves in an elastic media. They can be conceptually represented as displacement, pressure or density longitudinal waves, however the wave equations are simplest to solve when mathematically represented in sinusoidal format. This is the beginning of a potential conceptual swampland.

Longitudinal waves are typically distinguished from transversal waves as follows:

> We can distinguish different kinds of waves by considering how the motions of the particles of matter are related to the direction of propagation of the waves themselves .... If the motions of the matter particles conveying the waves are perpendicular to the direction of propagation of the wave itself, we have a transverse wave ... While it is true that light waves are not mechanical they are nevertheless transverse. Just as material particles move perpendicular to the direction of propagation in some mechanical waves, electric fields and magnetic fields are perpendicular to the direction of propagation of light waves ....If the motion of the particles conveying a mechanical wave is back and forth along the direction of propagation, then we have a longitudinal wave ...Sound waves in a gas are longitudinal. (Halliday and Resnick, 1974, p. 300)

An extremely common physics instructional demonstration of longitudinal waves are the longitudinal waves produced in a horizontally stretched helical spring (a "slinky"). The waves are generated by creating alternating compressions and extensions at one end of the spring. The helixes of the spring vibrate to and fro and the longitudinal wave is vividly seen propagating along the spring. After this demonstration the compressions and extensions of the spring are commonly represented by the crests (maximum forward spring displacement from equilibrium position) and troughs (maximum backward spring displacement from equilibrium position) of a travelling wave of sinusoidal form. An example of this type of representation is given in Figure 2. (taken from Physics by Richard Weidner, 1985, p. 386).



(a) A sinusoidal longitudinal wave. (b) The corresponding wave shape, with displacement from equilibrium plotted as a function of distance.

Figure 2.

Following a definitional distinction between transverse waves and longitudinal waves (particle displacement either perpendicular or parallel to propagation direction) a particle displacement graph of a longitudinal wave plotted in a way that it resembles a transverse wave (as shown in Figure 2.) must generate confusion for many students.

To fully appreciate the type of complexity involved an insight into the plotting-explanation is given. Consider the transverse-like representation in Figure 2. The horizontal x axis represents the direction of propagation i.e. along some positive x direction. Suppose that the undisturbed position of certain particles at a given instant in time is represented by the positions C,D,E,F,G ... on the illustration sketch in Figure 3.

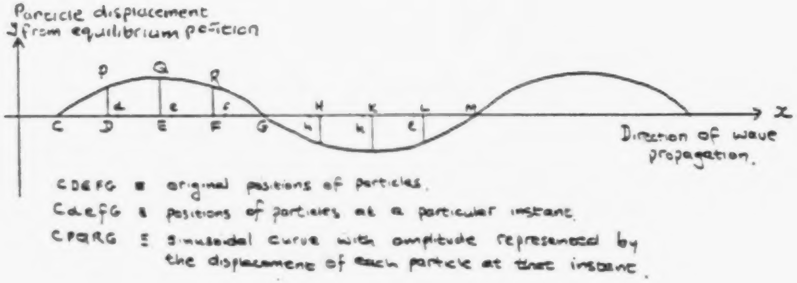

Particle displacement
y from equilibrium position
P
Q
R
d
e
f
H
K
L
M
x
C
D
E
F
G
h
k
l
Direction of wave propagation.
CDEFG = original positions of particles.
CdefG = positions of particles at a particular instant.
CPQRG = sinusoidal curve with amplitude represented by the displacement of each particle at that instant.

Figure 3. A longitudinal wave particle displacement representation.

Suppose that at a particular instant when a sound wave propagates through the section of medium in question these particles occupy the positions C,d,e, f,G,h,k ... Now the transverse-like representation is obtained by drawing a perpendicular from each point originally occupied by a particle. The perpendicular is in the plane of the paper, perpendicular to the direction of propagation, with a length which is proportional to the displacement of the particle at the chosen instant in time. The perpendicular is drawn upward if the particle's displacement is forward, and downward if the displacement is backward. If this procedure is continued for every particle along the direction of propagation then the ends of the perpendiculars form a sinusoidally shaped curve i.e. a transverse-like representation. This is a mathematically "convenient" representation of a longitudinal wave with the particle displacement being represented at right angles to their actual displacement - the definitional-distinction of a transverse wave!

From the perspective of the type of potentially conflicting models described above, an explanation for Peter's conceptual model of sound propagation in air may be that he made an effort to combine similar types of conceptual models. On the one hand he had the view of the mechanism of sound propagation being air molecular collisions, however, on the other hand the sound wave was probably presented to him in a sinusoidal representation similar to that described above. A sinusoidal representation of a longitudinal wave which looks like a transverse wave could easily blurr the two wave models into one. Now combine this hazy sinusoidal wave model with a model of molecule collisions and we have Peter's model - collisions which take place along a sinusoidal path.

5. CONCLUSION.

Using examples of physics students' conceptions of sound it has been illustrated how philosophical commitments can influence physics teaching and its outcome. An effort has been made to link these conceptions to typical university physics teaching practices, not to criticize or judge them but to offer some real insight into the kinds of conceptual problems which they may be generating. Naturally not all introductory physics teaching has been a failure but some of our assumptions need to be looked at again as some have the potential of generating more confusion than conceptual understanding. Students' conceptual models, such as those described in this paper, are perturbing, especially when the student has majored in physics with good grades and intends to become a school physics teacher. I believe they emphasize the importance for physics instructors to have an appreciation of constructivism; to engage in a process of teaching where knowledge is not a bundle of logical connections to be "exchanged" but rather a conceptual building which takes place upon the beliefs and understandings which the students bring to class; and, to encourage students to verbalize and "kick ideas around" - encouraging the growth of epistemological commitments to internal consistency and generalization.

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RULE USAGE ON RELATED TASKS: PATTERNS OF CONSISTENCY

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Introduction

The study reported here was a preliminary investigation into the consistency of the rules subjects used on related Rule-Assessment task sets. For the purpose of this study consistency will be defined as using the same or a closely related, rule on different task sets.

The Rule-Assessment technique was developed by Robert Siegler (1976, 1978). The basic hypothesis of the technique is that everyone presented with a problem or task will construct a procedure to use in working the task. Different individuals' procedures will vary in sophistication depending on their background knowledge of the domain of the task. The procedures individuals construct are called rules. They are constructed from the interaction, in the subject's mind, among the information provided and the knowledge and mental processes employed by the subject.

The essence of the Rule-Assessment technique is to build a task situation where the pattern of the subject's responses to the task can be used to infer the strategy (rule) he/she used to work the task. That is, the tasks sets which subjects work have a very specific structure which allows for identification of the different ways subjects use the given information to respond to the task. In addition to generating a sequence of responses, subjects are asked to explain their reasoning for the procedure they use. These two items of information, pattern of responses and explanation, provide strong evidence for the rule used, and frequently allow for inferences about the conceptions behind the rules.

Maloney (1984, 1985, and 1986) has shown that college students use rules to make predictions about the behavior of a variety of physical systems. But all of these studies looked at students' rules on isolated tasks. Recently several investigators have become interested in the consistency of students reasoning with science concepts. Clough and Driver (1986) examined subjects alternative frameworks about several science topics to determine how much consistency, if any, these frameworks exhibited. Aguirre and Ericksen (1987) investigated subjects procedures for dealing with vector quantities to see what types and level of consistency were present. Hejnacki and Resnick (1987) conducted a study of the consistency and coherence of subjects reasoning in dealing with problems involving vector additivity.

The goal of the current study was to explore how the rules subjects used for related tasks compared. That is, would subjects perceive the relation between the task sets and use appropriate rules? If not, would the rules they did use be related in any identifiable way?

Procedure and Subjects

There were actually three different studies involved. Parts of the results from these will be combined and presented here. All of the studies used the same basic rule-assessment procedure, and involved similar subject pools. The particular task sets and the number of task sets worked varied. For purposes of this paper I will present the general characteristics of the procedure and the subjects, and then detail one of the three studies to provide a specific context for the "common-sense" patterns of consistency. The three patterns presented will, however, be a composite drawn from all three studies.

The materials used in these studies were paper and pencil rule-assessment task sets. Each task set had either 28 or 32 items. Each item within a task set showed two arrangements of the same basic situation. The top illustration in Figure 1 shows a sample item from a task set involving toy boats which are coasting to a stop. The two arrangements could vary in none, one or both of the two variables involved. The same question was asked about all of the items in a task set, e.g. which boat goes farther. In a similar way the same three answer posibilities were available for all items. The subject's complete response sequence from the 28 (or 32) items was used to determine the rule he/she used. Subjects were also asked to explain their reasoing after they answered all of the items in a task set. This written explanation was used as an additional guide in identifying the rule the subject used.

Insert Figure 1 about here.

All of the task sets used in these studies had situations involving two variables whose specific quantitative values varied. For example, one task set had a toy truck sitting at rest on an incline, the truck was going to be released, roll down the incline, along a short horizontal stretch and then hit and compress a spring. See the top illustration in Figure 2. The two variables in this task set were the masses of the trucks and their initial heights on the incline. Examples of the rules someone could use to predict which truck would compress the spring more are: (1) to make all predictions on the basis of only the mass of the truck, or (2) to use the mass as the primary determiner, but if two trucks have the same mass to use the initial height on the incline, or (3) to multiply the mass and the height together and use the product as the basis for predicting. All of these procedures, and a number of others, could be used to make predictions in this task situation.

Insert Figure 2 about here.

The subjects for these studies were high school and college students. Some of the subjects had taken a high school physics course, but many had not. None of these subjects had taken a college physics course. Most of the subjects were paid volunteers, the rest received extra credit for a course in which they were enrolled. The college students were primarily non-science majors.

In the specific study that will be the primary focus of this paper all of the subjects worked two pairs of task sets. Some worked the pairs in one sitting while the rest worked them in two sittings. Calculators were available for all subjects. Subjects were asked whether, when they were working the second set in each pair, they remembered what they had done on the first set of the pair. They were also asked if they used any part of their ideas or procedures from the first set when working the second set.

Task Sets Used

There were four ways the task sets in the different studies were related. One relation was that task sets could involve different objects which were actually undergoing the same motion. As an example Figure 3 shows sample items from two task sets that shared this relation. One of the task sets had spheres thrown off cliffs, and the other had liquids flowing horizontally out of pipes. The second relation between task sets was for them to share one variable, e.g. mass, but differ in the second variable, e.g. speed in one and height in the other. For example, this was done with both the spheres off cliffs and flowing streams task sets just described. The third relation between task sets was for them to involve the same variables, but ask for different predictions. For example, the top boats set shown in Figure 1 could be paired with a second set which asked about the time for the boats to stop. The fourth relation between task sets will be the focus of the primary study of this paper and will be described in detail below.

Insert Figure 3 about here.

The relations among subjects' rules were investigated for three pairs of task sets which were related by being the "reverse" or "opposite" of each other. In the first set of each of these paired task sets the subjects were given information about two aspects of the situation and asked to predict a third aspect of the situation. In the second task set in the pair the subjects were given that third aspect of the situation as a variable along with one of the first two and asked to predict which of the two arrangements would have the greater value for the other one of the original aspects. These paired task sets are labeled "reverse" or "opposite" because they have the form: Set 1--given A & B, predict C; Set 2--given A & C, predict B. This will be clarified by talking about the specific task sets used.

One of the three pairs of task sets involved toy trucks, inclines and springs. One sample item from each of these two task sets is shown in Figure 2. One of these two task sets has a toy truck sitting at rest on an incline, the truck is going to be released, roll down the incline, across a short horizontal section, and then hit and compress a spring. The variables for this situation are the masses of the trucks and their heights on the incline. The question for this task set is which truck will compress the spring more? The second task set in the pair shows the trucks pressed up against the springs. They are going to be released and the truck will be propelled along the horizontal section and then up the incline to some point where it will stop for an instant. The variables in this set are the compressions of the springs and the masses of the trucks. The question for this set is which truck will go higher on the incline.

These task sets are "reversed" in the sense that an appropriate way to answer the first set would be to compute and compare the product of the mass and the height of each truck, while an appropriate way to answer the second set would be to compare the ratio of the compression to the mass of the two trucks in each item. Consequently, they involve "reverse" mathematical procedures.

A second pair of task sets with this reverse relation had toy boats on a pond. See Figure 1 for example items from these two task sets. In one set of the pair the boats had just had their motors turned off and were coasting to a stop. The variables were the masses of the boats and their speeds at the instant the motors were turned off. For each item they worked, subjects were asked to predict which of the two boats in the item would coast farther while stopping. The second task set of the pair had boats which had stopped. The variables were the masses of the boats and the distances they had coasted while stopping. For each item subjects were asked which of the two boats had been going faster when the motor was turned off. This pair was also a reverse pair from the mathematical perspective, since a reasonable way to work the first set is to calculate the product of the mass and speed squared, while it would be reasonable on the second to take the ratio of the distance to the mass.

The third pair of "reverse" task sets used springs. See Figure 4 for example items from these two task sets. In one task set of the pair the springs were pulled down a certain distance and released. The stiffness and the distance the spring was pulled down were given, and subjects were asked which of the two springs would have the longer period. (Period was defined and explained on the cover sheet of the task set.) The other task set in this pair had springs which were already vibrating. The amplitude of the motion, i.e. the distance pulled down, and the period of the vibration were given, and subjects were asked which of the two springs had the greater stiffness.

Insert Figure 4 about here.

This pair of task sets is not related in the same way mathematically as the other two. Here the appropriate rules for both task sets involves ignoring the amplitude. That is, in the first task set the period is related to the stiffness only, and in the second set the stiffness is related to the period only.

Subjects "Common-Sense" Forms of Consistency

As mentioned above the three "common-sense" forms of consistency are really composites that are drawn from the results of all three studies. They apply in slightly different ways, and to different extents, to the different relations between the task sets. Most of the discussion below focuses on the results from the "reverse" paired study, but some of the sample reasoning statements will come from the other studies as well.

The first result of the "reverse" pair study is that few of the subjects used rules which exhibited the appropriate form of consistency. Overall approximately 13% (50/394) used appropriate rule pairs. All of these were on the toy boats and toy trucks sets, no one used the appropriate rule pair on the springs sets.

Almost a fourth of the subjects, 23% (91/394), stated that they did not use any of their ideas from the first task set in a pair when working the second set of the pair. These subjects responded yes to the question about whether they remembered what they had done on the

first task set when doing the second set. But then they said they did not use any of their ideas, or any part of their procedure. (It is important to point out that these statements, about how subjects thought to relate the task sets, were retrospective reports.) There were actually two subgroups among these subjects.

One subgroup said that they did not think the two tasks were related, or were not strongly enough related to require linking what they had done on one to what they did on the other. Examples of these subjects reasoning are:

"Did not use ideas, there is some connection but not a very strong one."

"No, different thing, working out a different problem."

"Remembered, but it had no bearing on this."

"Did not use because I was supposed to find two different things in the two task sets."

These subjects clearly saw the two tasks sets as having at most a tenuous relation. They expressed a belief that they did not relate what they did on the two task sets.

The other subgroup, within the group of subjects who stated they did not use ideas or procedure from one task set on the other, was composed of subjects who said they did not think the task sets were related, but then stated a relation in their explanations. For example:

> "No did not use ideas, although both tasks dealt with the same principle, they were separate experiments to be treated differently, although you keep the same ideas in mind."

> "No did not use ideas from first, this was the opposite of the first task."

> "No, because this was going downhill instead of uphill. I used the total opposite of what I did in the first task set."

These subjects believed they had not used any of their ideas, or any part of their procedure from the first task set when they worked the second. However, as the examples above show, many of them actually described a relation in their

statements. A number of them used the phrase "opposite" in a way which would imply they thought it meant "unrelated".

An additional 8% (33/394) of the subjects said they did not remember what they had done on the first task set of the pair when working the second. That leaves 69% of the subjects who indicated they were relating what they did on the second task set of the pair to what they had done on the first. What forms do these relations take?

The subjects' attempts at consistency can be classified into three categories. The first category contains the subjects who used the appropriate form of consistency. As mentioned above this was about 13% of the subjects in the "reverse" pair study. The second category contains those subjects whose rules showed no recognizable form of consistency. The third category contains those subjects who used some form of "common-sense" relation between the two task sets. Three forms of "common-sense" consistency were identified. These were: (a) to use the same rule, which in some cases meant the same mathematical relation, for both task sets, (b) to focus on one variable and adjust the way they used it in related task sets, and (c) to use a concept or general idea to guide their decisions about how to use the variables. What kinds of explanations do subjects supply to support their approaches?

For those who employ properly related rules on either the toy boats or the toy trucks sets the explanations range from essentially mathematical descriptions to more conceptually oriented statements. For example:

> "Used the formula for the first task set and changed it around."

> "We were figuring opposite values for the two procedures, so I did the opposite function. Division and multiplication."

> "I divided in order to find d in the first set so when the problems were very similar in second set I used this and multiplied D x M in order to find V."

> "Because the spring would be compressed more the higher up a truck was I merely reversed the procedure. The lighter the truck and the further the spring was compressed, the higher up the hill it started from. I reversed this for the calculations."

Most of these subjects talked about "reversing" the formula or going from multiplying to dividing. In other words most of these subjects stated they were thinking about the mathematical relationship involved. Those who did concern themselves with the physical quantities involved were able to connect them effectively with a mathematical relation.

Let's look at the three forms of "common-sense" consistency one at a time. First there were the subjects who used the same mathematical relation for both task sets. Examples of their explanations are:

> "I just divided like I did on the first one because it seemed to work well on the first task."

> "I basically used the same formula, but modified for each set. Both, basically were the same, but with different unknowns."

> "Yes, remembered had divided m by v and from there I reasoned distance divided by m would have greater v."

> "Yes, I remembered the formula I'd used and just modified that to make it work more or less."

> "I multiplied mass and speed to get force."

The subjects who used the same mathematical procedure on the "reversed" sets seldom mentioned the physical quantities in their explanations, and when they did it was as a number to plug in their formula. However, in the other studies subjects used the same mathematical procedure because they could relate it to some physical quantity. The last statement above is an example of this.

Subjects who exhibited variable-centered consistency provided explanations which strongly reflected that fact. Examples are:

> "It was kind of the same procedure except I reversed it-like more weight would compress more versus less weight will go farther."

> "Yes, weighed less push up more, reversed way I used mass."

> "Heavier going down, lighter going up."

> "Weight, more weight-more power, less weight-spring push higher."

> "More mass = more kinetic energy (power of inertia)."

As these examples show, mass was the variable subjects tended to focus on most strongly. Subjects exhibited a definite tendency to "reverse" the way they used the mass, even when they were not entirely focused on the mass. These subjects seemed to be more concerned with the variable of mass than with any attempt to relate procedures mathematically. This form of consistency was the most common. In one group of subjects 50% used greater mass on one of the two task sets in a pair and smaller mass on the other.

The third form of "common-sense" consistency was identifiable from the subjects reasoning statements, rather than from the specific rules they used. Examples of these statements are:

"To me they were opposites. If it was "easier" to get down a slope then it would most likely be harder to get back up and vice versa."

"I used my idea of a boat coasting farther and longer if it was moving faster for both the first and the second task sets."

"I used the general idea about force behind an object. I thought both sets similar in that the force used to compress the spring were similar, or the reverse use by the spring to push the object up the incline."

"Because greater mass objects at greater speed create more force."

"Yes, reasoned that greater momentum goes farther so if it had farther distance it had more momentum, if lower mass it must have had greater speed."

"I believe the heavier arrow would travel farther and therefore be the last to stop." (This is from one of the other task sets, not the "reversed" ones.)

These statements show more variety than the other examples. These statements also tend to use physics concepts such as momentum and force, although the usage is often inappropriate. Subjects working with the task sets related in the other three ways used this type of reasoning in a number of places resulting in the same rule. This was especially true where one task set asked about distance and the other asked about time.

Discussion and Questions

This study is still in progress and was designed to be an initial exploration of subjects' rule usage on related task sets. Since it was designed to be exploratory, implications and inferences should not be drawn. Rather the study raises a number of questions which require subsequent investigations to answer.

This study found that most of the subjects neither used the physically appropriate rules for individual task sets, nor used rules on related task sets which had the proper mathematical consistency. These are two different matters, although they are frequently intertwined. For example, a subjects choice of rule for the first task set in each pair is most likely to made based on his/her physical knowledge of the situation. The subject could then use mathematical reasoning to check that his/her rule is reasonable. However, when the subject comes to the second task set in the pair, now the decision could be made on either mathematical, or physical, or some combination of the two, reasoning. The evidence from this study is that the subjects used physical reasoning with little attention to mathematical considerations.

Based on what they said, the majority of these subjects were attempting to relate what they did on the first task set with what they did on the second. This would indicate that the "common-sense" patterns of consistency found are not just flukes, but have some relation to the subjects knowledge base. All of these patterns are ones which teachers have encountered in their classrooms.

What are some of the questions identified by this study? One would seem to be: Why doesn't a student exhibit the proper consistency on related problems. Based on this study there would seem to be at least three possible reasons. First, the student doesn't see the two situations as related. Second, the student sees the proper relation between the situations but fails to execute properly to produce consistent answers. Third, the student sees the problems as related but applies a "common-sense" form of consistency to working the items. This latter would normally be considered, in a classroom situation, as being in the same category as the first. This study raises the possibility that the first and third reasons above should be approached differently in our instruction.

This study raises some questions about the interface/interaction between subjects' physical knowledge and their mathematical procedural and/or conceptual knowledge. The issue of the relation between procedural and conceptual knowledge in mathematics has received a good bit of attention Hiebert (1986). In this study many subjects used mathematically inappropriate rules such as using the mass to decide, but if the masses are equal then using the other variable. How does the subject's conceptual knowledge of the physical quantities affect his/her consideration of the mathematical character of his/her rule?

Another question is: Do we need to address the consistency issue directly, or is it sufficient to work on helping students change individual conceptions? Phrased this way the question is obviously naive. The relations students perceive are part of their conceptions. So the question is really: do we have to address the students alternative conceptions in several different contexts and let them infer the relation, or do we need to explicitly address the relations between/among the contexts in addition?

Summary

This paper reported on some preliminary investigations of student rule usage for related Rule-Assessment task sets. Task sets were related in one of four ways. First, the same motion was involved, although the objects differed. Second, the object, motion, and one of the two variables were the same; the other variable differed between the two task sets. Third, the object, motion, and variables were the same, but different questions were asked. And fourth, the task sets could have a "reverse" relation. Details of a study on this last relationship were presented.

Few of the subjects used the physically and/or mathematically appropriate and consistent rules. Three "common-sense" patterns of consistency were identified. One was the use of the same rule in contexts requiring different rules. The second was a variable-centered pattern where subjects focused on one variable as fundamentally important. And the third pattern was a conceptual or thematic one where subjects were guided by some idea or concept. Questions raised by these findings were presented.

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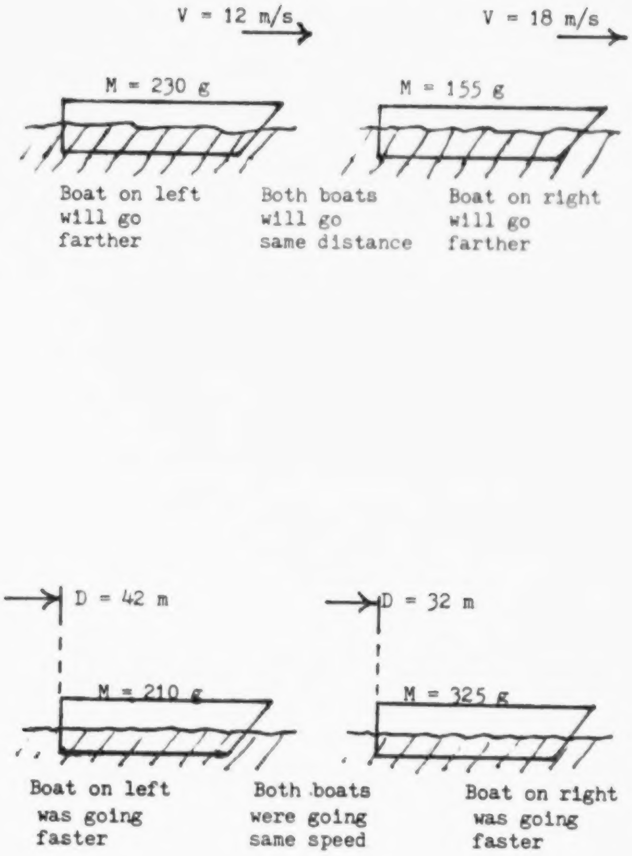
V = 12 m/s
V = 18 m/s
M = 230 g
M = 155 g
Boat on left will go farther
Both boats will go same distance
Boat on right will go farther
D = 42 m
D = 32 m
M = 210 g
M = 325 g
Boat on left was going faster
Both boats were going same speed
Boat on right was going faster

Figure 1
Sample items from Boats Task Sets

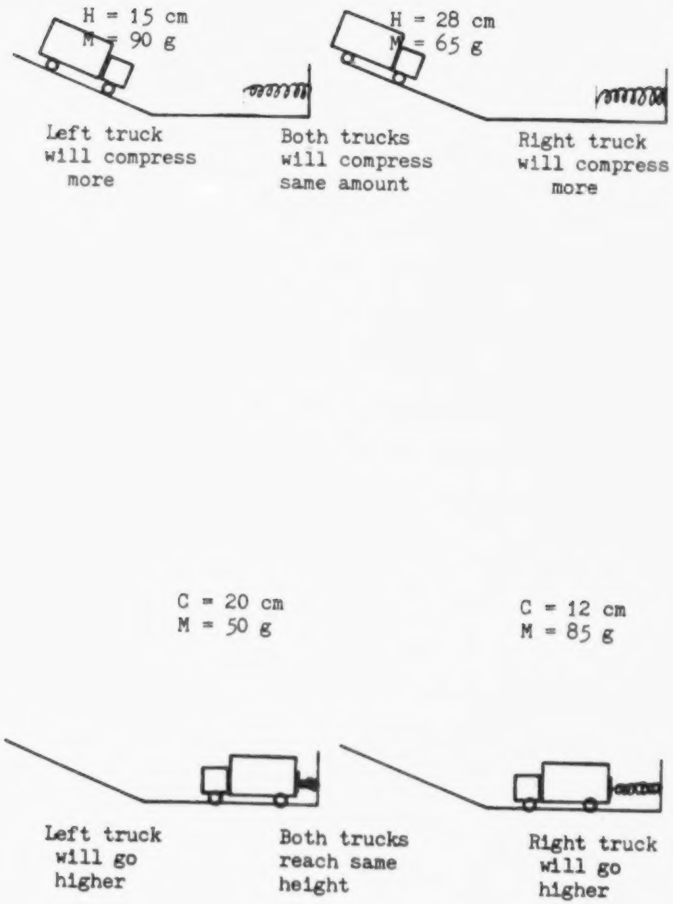
H = 15 cm
M = 90 g
H = 28 cm
M = 65 g
Left truck will compress more
Both trucks will compress same amount
Right truck will compress more
C = 20 cm
M = 50 g
C = 12 cm
M = 85 g
Left truck will go higher
Both trucks reach same height
Right truck will go higher

Figure 2
Sample items from Trucks, Inclines and Springs Task Sets

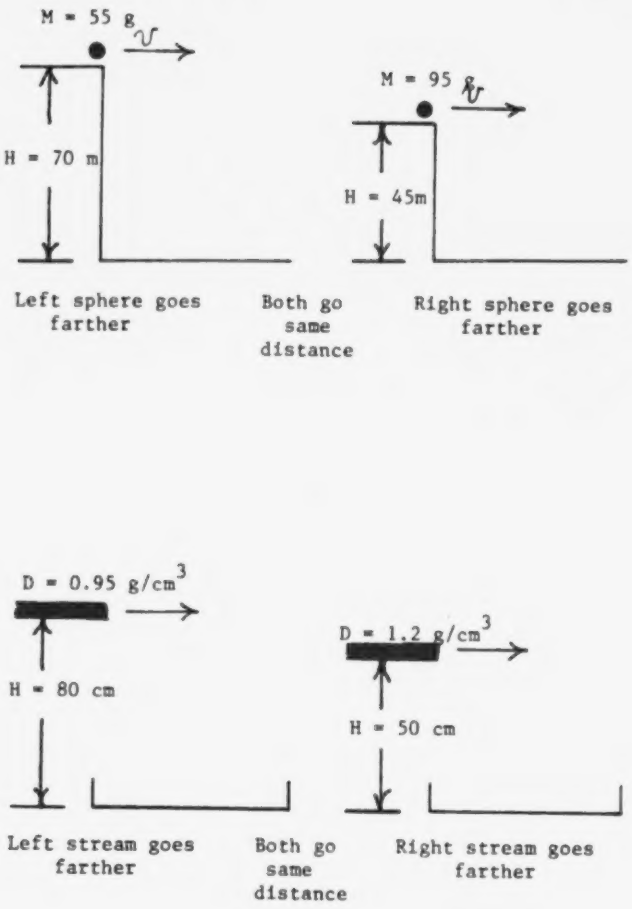

Figure 3
Sample items from Spheres off Cliffs and Falling Streams Task Sets

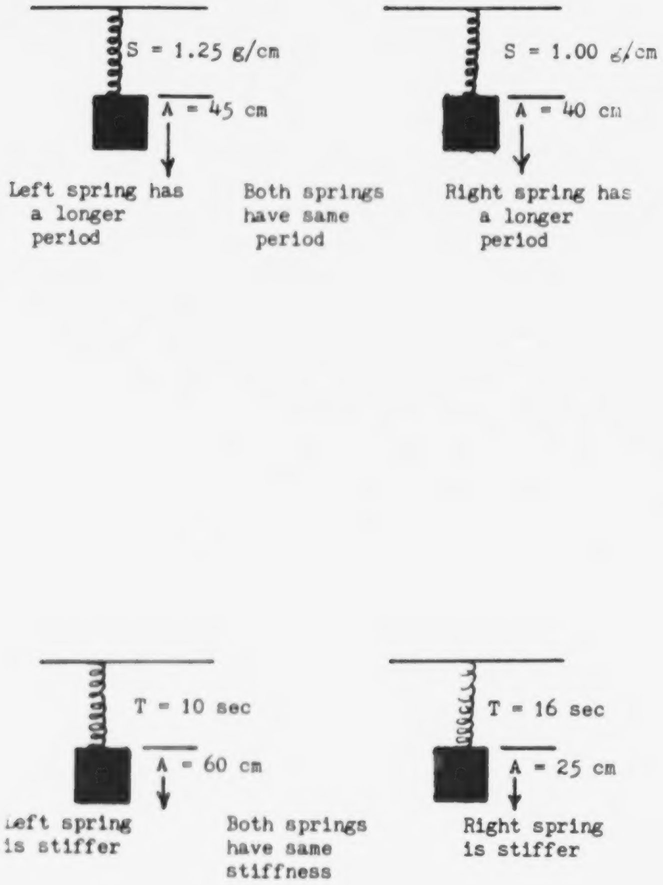

Figure 4
Sample items from Springs Task Sets

CLARIFICATION OF CONSERVATION OF LIQUID QUANTITY AND LIQUID VOLUME

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Abstract

A shift in the method of studying conservation from an examination of the child's responses to an analysis of the scientist's understanding of conservation is proposed. Compensation is the point of departure for the analysis. Existing definitions of compensation indicate disagreements as to whether compensation refers to container shape and liquid level or to an understanding of the transformation on the liquid. This analysis points to the need to describe compensation independent of the container shape, liquid compensation, and also suggests complementary descriptions for compensation.

Four phases in the clarification of conservation of liquid quantity and liquid volume are described. The first phase entails construction of a description of liquid compensation. The second phase postulates complementary descriptions for liquid compensation; liquid compensation described as a transformation on the liquid and as the effect of this transformation. The third phase formulates a theoretical relationship between liquid compensation and conservation and presents the appropriate conservation of liquid quantity and liquid volume experiments. The fourth phase demonstrates that Piaget's conservation of liquid quantity experiment is not in fact a conservation of liquid quantity experiment and demonstrates that the mental structures are the consequence of the scientist's errors in observation.

In the first phase of the clarification a comparison between Piaget's conservation of liquid quantity experiment and a violation of conservation of liquid quantity experiment is made. It is shown that the rise/decrease in water level in Piaget's experiment is not independent of the container's shape whereas in the violation experiment it is. Based on this description, it is demonstrated that the rise/decrease in water level in the violation experiment describes a change in water level whereas in Piaget's experiment a description of a rise/decrease in water level as a change in water level is not possible.

With containers all identical in size and shape in the violation experiment, liquid compensation is described as a change in water level. To satisfy the theoretical requirement that liquid compensation be related to conservation and not a violation of conservation, two displacement of liquid experiments are introduced. Liquid compensation in the first experiment is described as a change in water level and in the second experiment is described as a change in quantity. In each of these experiments liquid compensation is defined as the effect of a transformation on the liquid.

In the second phase of the clarification of conservation of liquid quantity, the descriptions of the transformation on the liquid and the effect of this transformation in each of the displacement experiments are postulated as complementary descriptions. The transformation on the liquid in Piaget's conservation of liquid quantity experiment is described as a transformation in position, whereas the transformation on the liquid in each displacement experiment, liquid quantity and liquid volume, is described as a transformation in shape. Liquid compensation is described as a transformation of the liquid's shape as well as change in water level and a change in quantity.

In the third phase of the clarification, liquid compensation, described as a change in water level and as a change in quantity, is defined as an invariant relationship, a physical measure, which necessarily identifies the type of conservation postulated under a transformation of the liquid's shape. The coordination of the complementary descriptions of liquid compensation in each of the displacement experiments defines the observational structure of conservation of liquid quantity and liquid volume.

In the final phase of the clarification, it is demonstrated that the absence of a physical measure of liquid quantity in Piaget's conservation of liquid quantity experiment leads only to a description of the transformation on the liquid as a transformation in position. Such a description allows the shape of the container to create the appearance of an effect of the transformation - an apparent change in water level. The observational structure of conservation of liquid quantity in Piaget's experiment is, therefore, undefined and this experiment is at best identified as a conservation experiment.

The displacement experiments with physical measures of quantity and volume are identified as the appropriate conservation of liquid quantity and volume experiments; the observational structure of conservation of liquid quantity and liquid volume however, preclude conservation measures of quantity and volume. The need to postulate a non-observable structure for conservation of liquid quantity and volume, whether underlying the observed experiment or the observed subject (mental structures) are shown to result from the scientist's failure to distinguish between liquid compensation and compensation as well as the failure to postulate complementary descriptions for conservation of liquid quantity and liquid volume.

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CONCEPT MAPPING AS A POSSIBLE STRATEGY TO DETECT AND TO DEAL WITH MISCONCEPTIONS IN PHYSICS

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Introduction

An overwhelming number of research studies dealing with misconceptions held by science students has been reported in the literature in recent years. In fact, this is probably the most investigated area of science education nowadays. Promising results have been obtained in this area in the sense of identifying misconceptions commonly held by students - such as the proportionality of force and velocity and the nonconservation of electric current - and of recognizing their stability on the learner's cognitive structure and their relevance for subsequent learning.

Nevertheless, there are some problems in this field of research. For example, so far the best technique for detecting student misconceptions seems to be the clinical interview, which requires expertise and is extremely time consuming . It is true that paper and pencil tests have already been developed using results from clinical interviews, but this type of test has its own pitfalls. Another problem is that most of the studies in this area are still restricted to the detection of misconceptions held by the students. The next step, of course, is to use alternative strategies that take into account the learner's misconceptions, and progressively obliterate them and facilitate the acquisition of contextually accepted meanings. But little has been done in this direction, perhaps due to the lack of appropriate instructional strategies.

Following this trend of research in science education, several studies were carried out with college physics students at the Federal University of Rio Grande do Sul, in Brazil. For example, clinical interviews were conducted to investigate student misconceptions concerning the concepts of electric field, electric potential, and electric current (Moreira and Dominguez, 1986 and 1987) and a paper and pencil test was validated to detect whether or not the student has the Newtonian conception of force and motion (Silveira et al., 1986).

Simultaneously, other research projects were performed to explore the potentialities of concept mapping as a tool for instruction and evaluation in physics education (e.g. Moreira and Gobara, 1985; Gobara and Moreira, 1986).

Comparing the findings of the studies on misconceptions with the results of those on concept mapping it was realized that, in a sense, they are complementary. On one hand, the investigation in misconceptions suggested that the clinical interview was indeed a valuable instrument to detect misconceptions, but it was not appropriate for classroom purposes because it required too much time and practice, and that new instructional strategies were needed to help students overcome their misconceptions. On the other hand, the research on concept mapping suggested that concept maps drawn and explained by students might provide evidence of misconceptions they hold and, perhaps, might also be helpful in overcoming these misconceptions.

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This complementarity of research findings led to the aim of this paper : to propose through examples concept mapping as a potentially useful technique to help teachers and researchers detect and deal with students misconceptions in physics.

The use of concept maps as a strategy to detect and to deal with misconceptions was probably first implied by Novak (1983) when he suggested concept mapping as a metalearning strategy for overcoming misconceptions. Feldsine (1983) in a study similar to the one being reported here confirmed Novak's hypothesis in the area of general college chemistry. Cleare (1983), reporting a study using concept mapping to detect interventions effective on improving pre-service elementary education major's understanding of science topics, referred to "misconceptions that appeared on many of their maps". Possibly, there are other studies showing that concept maps are in fact a useful instructional strategy to deal with misconceptions and to help students to overcome them. The present study intends to be a contribution in this direction in the area of physics.

Concept mapping

In a broad sense, concept maps are just diagrams indicating relationships between concepts, or the words used to represent concepts. Nevertheless, in a more specific view, they can be seen as hierarchical diagrams that attempt to reflect the conceptual organization of a piece of knowledge. Concept mapping is a flexible technique that can be used in variety of situations for different purposes : 1) as a curricular tool; 2) as a teaching strategy; and 3) as a means of evaluation (Moreira, 1979; Stewart et al., 1979).

A concept map may be drawn for a single lesson, for a unit of study, for a course, and even for an entire educational program. The differences are in the degree of generality and inclusiveness of the concepts included in the map. A map involving only general, inclusive, and organizing concepts can be used as a framework for curriculum planning of a whole course of study whereas concept mapping dealing with specific, less inclusive concepts can guide the selection of specific instructional materials. Thus, concept mapping might be useful "to focus the attention of the curriculum designer on the teaching of concepts and on the distinction between curricular and instrumental content - that is, between content that is intended to be learned and that which will serve as a vehicle for learning" (Stewart et al.,1979, p.174).

As a teaching strategy, concept mapping can be used to show the hierarchical relationships among the concepts being taught in a single lecture, in a unit of study, or in an entire course. They are concise representations of the conceptual structures being taught, designed to facilitate the meaningful learning of these structures. However," they are not intended for student use without interpretation by the teacher and, although they may be used to give an overview of the subject to be studied, it is preferable to use them after the students have been already acquainted with the subject, when the maps can be used to integrate and to reconcile relationships between concepts and to provide for concept differentiation" (Moreira, 1979, p.285).

As a means of evaluation, concept maps may be used to get a visualization of the conceptual organization the learner assigns to a given piece of knowledge in his/her cognitive structure . Although criteria might be established to score concept maps (Novak and Gowin, 1984), concept mapping is basically a nontraditional qualitative technique of evaluation.

In both cases, teaching and evaluation, concept maps can be thought as a tool for negotiating meanings. As stated by Novak and Gowin (1984, p.19), "because they are an explicit, overt representation of the concepts and propositions a person holds, they allow teachers and learners to exchange views on why a propositional linkage is good or valid, recognize 'missing' linkages between concepts that suggest the need for some new learning". According to Novak and Gowin, concept maps are intended to represent meaningful relationships between concepts in the form of propositions, that is, they are schematic devices to represent a set of concept meanings embedded in a framework of propositions.

In fact, since propositions are two or more concept labels linked by words in a semantic unit, concept maps can be drawn in such a way that not only concepts would be externalized but propositions as well. That is, if the map maker labels the lines connecting concepts with one or two key words in such a way that the concepts and the linking words form a propositon, her/his map would not only represent her/his own way of organizing a given set of concepts but also propositions that express the cognitive meanings assigned to relationships between concepts. As such, concept maps might be seen as a strategy for externalizing one's conceptual and propositional understanding of a piece of knowledge. Of course, if this is true, when drawing a concept map the learner is likely to externalize his/her misconceptions and misunderstandings as well.

It is under this assumption that concept maps are being proposed here as a strategy to gather evidence and to deal with student misconceptions. Empirical support for this assumption is reported in the next section.

Examples

The following concept maps were drawn by engineering students in an introductory college physics course on electricity and magnetism at the Federal University of Rio Grande do Sul, Brazil, during the first semester of 1986. The course was taught under a self-paced format and the content was divided into 20 units of study, including five laboratory units. Upon completion of the tenth unit (mid course) each student was requested to draw a concept map for the course content studied up to this unit. The following instructions were provided to the students at that time :

Instructions

1. Make a list, for your own use, of the main physical concepts studied up to unit X. (Electricity)

2. Display these concepts in sheet of paper as if it were in a map (a "concept map"), emphasizing, in some way, those that you think are more important.

3. Connect with a line, of arbitrary size and shape, those concepts which you think are related.

4. Write on the lines connecting concepts one or more words that make explicit the kind of relationship you see between the corresponding concepts.

5. Summing up, this "map" must reflect your own way of organizing and relating these concepts. There is no right or wrong answer, what matters is that the map corresponds to your way of seeing the structure of this set of concepts.

6. If necessary, ask the teacher for additional instructions on how to draw this concept map.

Concept maps were not used as instructional materials in this course. This was the first contact students had with concept maps. They had as much time as needed to draw the map and they were free to use any instructional materials they wished.

Immediately after finishing the map, each student was requested to explain it orally to the teacher (of course this was possible because of the self-paced format of instruction).

In order to trigger or to keep the flow of student's explanations,the teacher usually asked questions like the following ones :

- Try to explain your map. How did you start it ? What kind of organization does it have ?

- Which are the most important concepts ? Why ? The least important ? Why ?

- Why did you not include this concept (e.g. electric potential, electric current, electric force) in your map ?

Relevant student's explanations, according to the teacher's criteria, were recorded in a notebook. These "interviews", which lasted about 10 to 15 minutes, were not tape recorded, since no particular need was felt to do so.

Figure 1 shows an example of a concept map drawn by a student at that stage of the course. Below this figure, passages from the student's explanations, judged relevant to understand his map, are transcribed literally.

There is no need of doing a profound and time consuming analysis of the map shown in figure 1 and of the corresponding explanations to conclude that this student has provided evidence of not distinguishing between electrostatic and electrodynamic phenomena and of holding the misconception "no current no field". He also provided evidence of not distinguishing between the concepts of electric potential and electric potential difference and of not having understood Gauss's law and the concept of Gaussian surface. (The connections indicated by dashed lines were added to the map during the "interview".)

Figure 2 shows the map drawn by another student at the same opportunity. Immediately below this figure there are quotes from the student's explanations. These quotes suggest a lot of conceptual misunderstandings in both his previous (i.e. before instruction) and new knowledge. This student is a chemistry major and this fact probably explains why he considered electric energy the most important concept while most of his colleagues thought that electric charge, electric field, and electric current were major concepts of this area. It might also explain why he spoke about the "duality" between electricity and magnetism (although the map was supposed to include only concepts concerning electricity) which was extrapolated from the wave-particle duality of light since in some chemistry courses physical concepts are introduced when needed and quite operationally. That is, chemistry majors are likely to have rote learned many physical concepts well before they are studied in physics, particularly modern physics concepts such as the wave-particle duality.

He has also externalized a misconception concerning forces when he talked about "forces that act on matter and forces that are generated by matter". In addition, at heart, he did not establish any distinction between electric

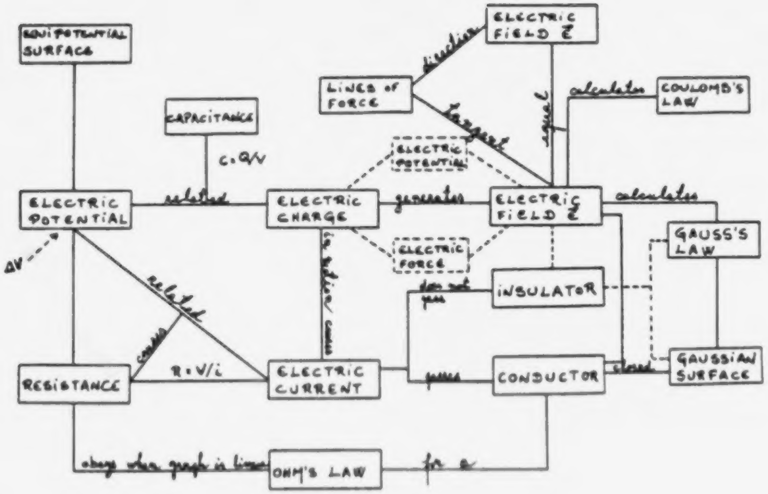

Figure 1 - Concept drawn by student #1 after the tenth unit of study. Dashed lines were added to the map during the oral explanation.

"I put electric charge at the center of the map because it is the foundation of electromagnetism, After that I tried to separate everything in order to see better and I also tried to relate everything that was important."

"Electric force would get in the map through the relationship with potential since potential is related with work and in order to do work a force is needed." (The student was,in fact,talking about the electric potential difference.)

"I don't know where the concept of electric potential would be placed in the map."

"The field of an insulator cannot be calculated through Gauss's law. In an insulator the current does not pass, there is no field inside it."

"The less important concepts would be equipotential surface, direction of $\vec{E}$, Ohm's law; in general, those at the periphery of the map, including the laws because they are used just to calculate the field, they are not important as concepts."

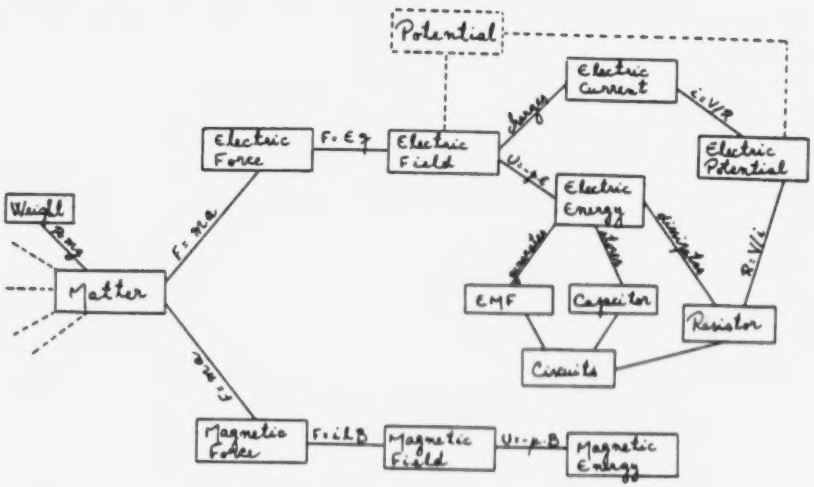

Figure 2 - Concept map drawn by student #3 after the tenth unit of study. The dashed lines were added during the oral explanation.

"There is symmetry between electric force and magnetic force. The map shows a duality between electricity and magnetism extrapolated from the wave-particle duality of light. There as forces that act on matter (e.g. weight) and forces that are generated by matter such as the electric and magnetic forces."

"The map does not include electric charge because when electric force is defined electric charge has been already defined and because electric current and electric charge are at the same level."

"Force changes clothes, is transformed into electric field and when it arrives at the desired point it turns back to force again."

"The diference between potential and electric potential difference is so subtle that they are considered to be equal."

"The most important concept is the concept of electric energy because it is the energy that interacts outside the charge."

"Circuit would be the least important concept because it is just a detail. A little above circuit would be electromotive force."

potential and electric potential difference; however, during the " interview" he realized that there was a basic difference - function x number - and the concept of potential he had used in the map was in fact the concept of electric potential difference and should be replaced by that one, switching electric potential to another position (dashed lines in figure 2).

Figure 3 presents a third example of map drawn by a student, immediately followed by his explanations. These data suggest that this student seems to have had a more meaningful understanding of the subject matter than the other two, and that he clearly was trying to integrate the new knowledge (electricity) with his previous knowledge (mechanics) in physics. But he also showed evidence of not having understood Gauss's law and of not having assigned accepted scientific meanings to the concept of electric potential. (The dashed lines were added to the map during the "interview".)

In that same study, upon completion of the last unit of the course each student was requested again to draw a concept map. The directions were the same given when the first map was requested, except that this time it should include magnetism as well. Furthermore, instead of explaining the map orally, each student was asked to do it in a written form. That is, the map and a written explanation of it should be handed to the teacher.

Figure 4 shows the second map drawn by the same student who drew the mid-course concept map presented in figure 3. Similarly, this concept map, together with the explanations, suggest a meaningful understanding of the subject matter. It also suggests that the concept of electric potential was

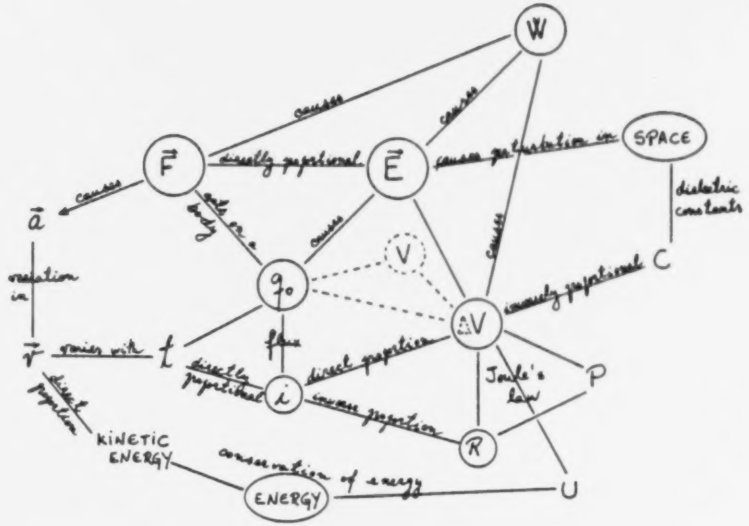

Figure 3 - Concept map drawn by student #16 after the tenth unit of study. Dashed lines were added to the map during the explanation.

"I could have started the map with field, but I have decided to begin with force because force is much related to field and is a very important notion in physics; field is a new notion."

"Force reminds us of acceleration, velocity and kinetic energy; that's why I put these concepts in the map."

"Field is associated to voltage (potential difference) which is associated to potential energy."

"The charge generates the field; a flux of charges is associated to the electric current."

'Gauss's law, in a certain way, is inside the field, it serves to pass from $\vec{E}$ to V (voltage)."

"I didn't even think in including electric potential on the map; we are more used to the potential energy. Potential would be implicit in the work W. There is no potential, what exists is the potential difference. I don't know how to explain potential; it's a definition used to distinguish two quantities with different energies. It would have something to do with entropy; in the same way that there is a trend towards disorder in the universe, the bodies tend to the lower potentials; it's a trend observed in nature such as, for example, when a stone falls "

"Electric field is the most important concept. Space is also important because, depending on the dielectric constant of the space, the perturbation caused by the field will be different."

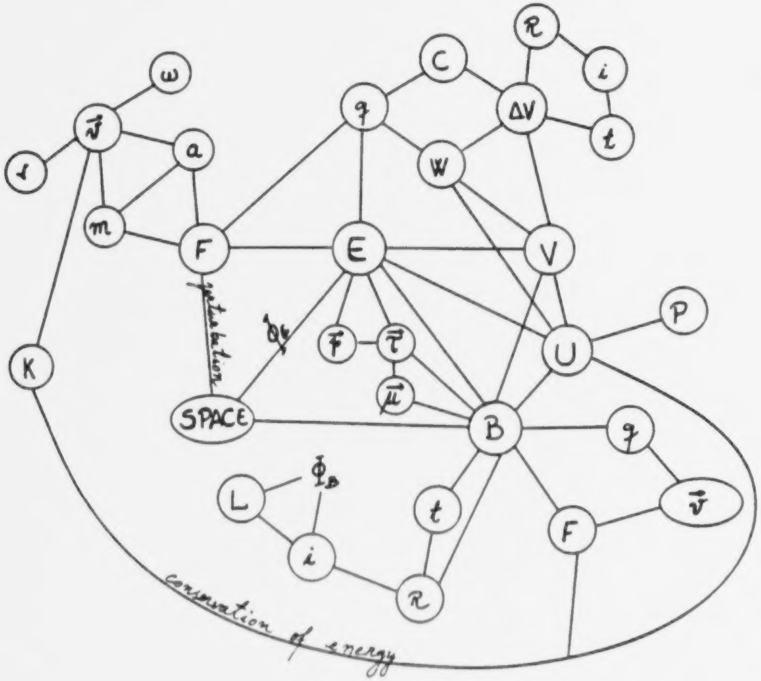

Figure 4- Concept map drawn by student #16 after the last (twentyeth) unit of study.

"In the scheme on the electromagnetism, I started with the word 'space' because I think that, in physics or in any other science that requires abstract reasoning, the concept of space is one of the most important since all the remaining physical entities are intrinsically related to it. For example, connected to space are force, which might be considered an agent causing perturbation in space, and the electric and magnetic fields which together form the electromagnetic field . The connections between space and $\vec{E}$ and between space and $\vec{B}$ are ϕ_E and ϕ_B . I tried to write each entity just once, relating it to the other entities whenever necessary, however, it became increasingly difficult to relate each one with the remaining ones. Thus, some quantities (q, r, i, and t) appear twice, related on one hand to the magnetic field,and on the other to the electric field. Considering the electromagnetic field, we would have next electric potential and potential energy which would be connected to $\vec{E}$ and $\vec{B}$, potential energy being indirectly connected to $\vec{E}$ (through V) or directly (if one wants to say that), but potential energy is necessarily connected directly with magnetic field (I mean, the opposite, magnetic field is directly connected to the concept of potential energy). Afterwards, we find the concepts that come from the main concepts : potential difference, connected to V and bringing the relationships between resistors and capacitors; power, related to potential energy U and the important passage that there is between the kinetic energy and the potential energy. The remaining entities shown in the scheme are secondary."

incorporated to his cognitive structure, although his explanation for the direct connection between this concept and the concept of magnetic field indicates that it was not yet well understood. In addition, a brief examination of the student's justification for the emphasis placed on the concept of space suggests that he might hold the misconception that something must fill the space since he considers force as an agent causing a perturbation in the space.

Conclusion

As it was said before, the purpose of this paper is to suggest concept mapping as a strategy to gather evidence about student misconceptions in physics and, possibly, to lead them to overcome these misconceptions. The examples provided in the previous section are probably enough to make the point that this strategy easily provides evidence of student misconceptions. In addition, these examples also show that concept mapping does not require any procedural expertise and is much less time consuming than the clinical interview. Of course, this does not mean that it is better than the clinical interview; it just seems to be more appropriate for classroom purposes whereas the latter is more adequate for research. As a matter of fact, these two techniques can be combined for research purposes since concept maps can be used in the planning of interviews and in the analysis of transcripts from interviews (Novak and Gowin, 1984, chap.7).

The second point, namely, that concept mapping might be useful in handling student misconceptions in the sense of leading the students to overcome them is more difficult to make, especially because it would require a more extensive use of concept mapping in instruction than it was done in the study referred in this paper.

Misconceptions, also known by different terms such as alternative conceptions, alternative frameworks, intuitive or spontaneous concepts, are meanings of a concept that are at variance with meanings shared by expert concept users in the context in which the concept is embedded, such as a scientific discipline. That is, they are functional meanings for the individuals who hold them but are contextually unacceptable interpretations of the concept.

Concept maps, in turn, might be seen as tools for negotiating meanings because, as put by Novak and Gowin (1984), "they allow teachers and learners to exchange views on why a propositional linkage is good or valid, or to recognize missing linkages between concepts that suggest the neeed of some new learning" (p.19). In practice, this means that teachers and learners must discuss their concept maps

until the achievement of shared meanings, until they agree on meanings of a given concept or set of concepts. When this happens a teaching episode happens (Gowin, 1981, chap.3).

Thus, in order to check whether concept maps are indeed useful in progressively obliterating student misconceptions and in facilitating the acquisition of contextually accepted meanings, they must be used quite extensively. However, even in the case referred in this paper, in which concept maps were used only twice and not fully incorporated in the instructional procedures, clues were obtained suggesting that this hypothesis would hold : in the first map, in almost all of the cases students spontaneously made slight modifications in their maps when explaining and discussing them with the teacher. The concept of electric potential illustrates this point : it is an abstract concept to which most students assign the same meanings of electric potential difference or no meaning at all (Moreira and Dominguez, 1986). This was reflected on the maps since for most students the word electric potential in their maps meant, in fact, the electric potential difference. Nevertheless, after a brief discussion with the teacher, which naturally occurred during the interviews, they realized that electric potential has its own meanings, they tended to modify their maps and, probably, began to acquire these meanings. Isn't this a clue that concept maps could lead students to overcome their misconceptions ?

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PATTERNS OF MATHEMATICAL MISCONCEPTIONS AS REVEALED ON TESTS OF ALGEBRA READINESS

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A standard part of careful test development is carrying out an item analysis. The difficulty level, the reliability, and the distribution of students choosing each option on a multiple-choice item all give significant information about the efficacy of the item. Aside from evaluating the worth of each item, the analysis provides a wealth of information about students' errors. By examining the responses to distracters in multiple-choice items, certain patterns of misconceptions may emerge. This paper is based on such an analysis of items in tests of algebra readiness.

The writing of the algebra readiness tests was done in the context of a doctoral dissertation in Mathematics Education at Teachers College, Columbia University. The research and analysis were done from 1981 through 1984. The first round of testing involved 140 students in four independent schools in New York City, who were randomly assigned one of four subtests (A, B, C, or D). The second round of testing involved 195 students in three independent schools in New York City who were randomly assigned one of three subtests (D, E, or F). The tests were given three weeks into the term in which the students had begun the study of algebra. The students ranged from seventh through tenth grade, but all were new to the study of algebra. The numbers of students taking each test is as follows:

First Round:	Test A; 36	Test B; 37			
	Test C; 36	Test D; 31			
Second Round:	Test E; 66	Test F; 65	Test G; 64		

An item providing insights into students' misconceptions may or may not be considered a good test item. If 70% of the better students answer the item correctly and of the other 30%, if 10% chose each distracter, it is an excellent test item but not revealing for a study of misconceptions. However, if 70% chose the correct answer while 25% chose one certain distracter, it could be good as both a test item and an interesting item for this study. More dramatically, an item may have performed poorly on the test: perhaps only 30% answered correctly while 50% agreed on a certain wrong answer; that will be of great interest to us for this paper. Hence the items discussed below will often be weak in terms of the original test purpose; indeed several were not included in the second round of testing for that reason.

Of the 135 original test items, 31 are discussed below as revealing some general misconceptions. These 31 items involved the following types of confusion:

7	the use of variables
7	properties of fractions with numerical components
10	properties of fractions with variables as components
7	others, including percent, divisibility, terminology

A discussion of the problems containing variables follows. Note that percents given do not quite add up to 100% due to rounding. If the total falls significantly below 100%, the remaining students omitted the item. The first two questions which were also consecutive on both tests, should be considered together:

(1) For any number "N", the value of $N + 2 - 2$ is:

	(a) N+4	(b) N-4	(c) N *	(d) not determined
Test F	0	1	57	7
Test A	0	1	24	11
Total	0	2	81	18
(N=101)	0%	2%	80% *	18%

* will be used to indicate the correct answer

(2) For any number "N", the value of N x 3 ÷ 3 is:

	(a) Nx9	(b) N÷9	(c) N *	(d) not determined
Test F	2	0	45	18
Test A	0	1	24	10
Total	2	1	69	28
(N=101)	2%	1%	68% *	28%

It is interesting not only that so many students cannot answer these questions without knowing the value of N, but also that the question becomes significantly harder when the operations are x and ÷ rather than + and -. These next two questions should also be considered together. They were also consecutive on both tests A and F:

(3) If you start with a certain number called N, then add 5, you would do which of the following to get the answer "N":
(a) increase by 5 (b) take 1/5 of it (c) divide by 5
(d) subtract 5

	(a)	(b)	(c)	(d) *
Test F	18	3	3	41
(N=65)	28%	5%	5%	63%

The presummed attraction of choice (a) is word association; "increase" seems to go with "add" which distracts one from selecting the inverse operation. The corresponding item on Test A on the first round, was a fill-in. It was answered correctly by 75% of the students.

(4) If you start with a certain number called "N", then multiply it by 12, you would do which of the following to get the answer "N": (a) subtract 12 (b) divide by 12
(c) increase by 12 (d) divide by 1/12

	(a)	(b)*	(c)	(d)
Test F	5	46	5	9
(N=65)	8%	71% *	8%	14%

This seems easier than item 3 above, but it had no choice similar to (a) of 3. The interesting error here is (d) divide by 1/12. It seems to be a case of choosing the inverse operation as well as the numerical inverse without realizing that these together become the original process; i.e. to divide by 1/12 is to multiply by 12. On Test A of the first round, this was a fill-in item which 67% of the students answered correctly.

(5) If N is a certain number, then (+N) + (-N) =

	(a) 0 *	(b) N	(c) -N	(d) it depends on the value of N
Test F	44	6	5	9
(N=65)	68% *	9%	8%	14%

The error of interest here, as in questions 1 and 2 above, is the belief that it depends on the value of N. Such students do not seem ready to accept algebraic axioms stated in general terms. On Test A, this was a fill-in item which 56% of the students answered correctly.

The next two questions should be considered together. One asks about the difference of A and B, one about the product of A and B. In each case the error of note is the claim that the values for A and B need to be known.

(6) In order to decide whether A - B = 0 is a true statement:
(a) it is enough to know that A is bigger than B.
(b) it is necessary to know that A and B are both 0.
(c) it is enough to know that A and B are equal.
(d) it is necessary to know the values of A and B.

	(a)	(b)	(c)*	(d)
Test E	1	3	27	32
Test C	1	1	14	17
Total	2	4	41	49
(N=102)	2%	4%	40% *	48%

(7) Two whole numbers, each bigger than one, are called A and B. Here are two statements about their product, AxB: STATEMENT 1: AxB is larger than A. STATEMENT 2: AxB is larger than B. What do you know about these two statements?
(a) Only statement 1 is true. (b) Only statement 2 is true.
(c) Statements 1 and 2 are both true. (d) It depends on the value of A and B.

	(a)	(b)	(c)*	(d)
Test F	0	0	36	29
Test A	0	0	21	14
Total	0	0	57	43
(N=101)	0%	0%	57% *	43%

One difficulty with the previous two questions may be their complex directions; the items may involve a large component of reading comprehension. Still, the reluctance of many students to accept rules stated in general terms (with variables) should be considered in curriculum planning. One wonders how meaningful, much less how intuitively obvious, the algebraic axioms are to beginning algebra students.

The next group of problems all involve the concepts of fractions. Problems 8 through 14 illustrate some common misconceptions about arithmetic fractions, having only numerical components.

(8) The mixed number $2\frac{3}{4}$ equals which of these:

	(a) $2+\frac{3}{4}$ *	(b) $2-\frac{3}{4}$	(c) $2\times\frac{3}{4}$	(d) $2 \quad \frac{3}{4}$
Test E	46	1	16	2
Test C	26	0	10	0
Total	72	1	26	2
(N=102)	71% *	1%	26%	2%

The curious error here is that students who have barely started the study of algebra would interpret the two parts of a mixed number as being multiplied, as if reading it as an algebraic expression (as if it were 2w), despite years of experience with mixed numbers in arithmetic. Perhaps many students in upper elementary grades do not think of $2\frac{3}{4}$ as representing $2+\frac{3}{4}$.

The next two items should be considered together. Each involves writing a fraction in lowest terms.

(9) The fraction 36/56 has the same value as:

	(a) 3/5	(b) 9/14 *	(c) 306/506	(d) 38/58
Test E	13	42	5	6
Test C	4	22	6	1
Total	17	64	11	7
(N=102)	17%	63% *	11%	7%

The error of choosing 3/5 is curious; it is as though students were "cancelling the 6's" or selecting out of the fraction the distinctive 3 and 5. If students choosing (a) really believe this "rule", then choice (d) is also correct, since it would equal 3/5, hence it would also equal 36/56 - but perhaps in this case equality is not transitive!

(10) The ratio 60:80 can be written in lowest terms as 3:4. What is the ratio 120:150 in lowest terms?

	(a) 4:5 *	(b) 2:5	(c) 12:15	(d) 3:5
Test G	32	13	6	8
(N=64)	50%	20%	9%	13%

This item was a fill-in on Test D; 55% answered correctly.

The interesting error here is (b) 2:5. As in the previous item, this suggests "cancelling" all digits common to both numerator and denominator and leaving only the digits which are distinctive. These two items reflect a type of "visual thinking" which is fallacious. This way of thinking is surely related to advice commonly given by teachers of upper elementary grades: to reduce a fraction such as 50/80, "cross off the zeros." One wonders how many students who are expert at crossing off zeros realize that they are dividing out the common factor of ten.

The next two items both involve a confusion about the meaning of the multiplication of fractions.

(11) The expression $\frac{5}{8} \times \frac{15}{16}$ is equivalent to which of these:

	(a) $\frac{5x15}{8x16}$ *	(b) $\frac{5x16}{8x15}$	(c) $\frac{5+15}{8+16}$	(d) $\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{8}}} \times \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{16}}}$
Test D	11	2	2	14
(N=31)	35% *	6%	6%	45%

This item was not used in the second round of testing.

The surprise here was that the item was so difficult. Since no "reducing" or "cancelling" can be done, the simple rule of multiplying the two numerators and multiplying the two denominators applies. In order to choose (d), the most popular option, a student has to approve of "cancelling" the 5 into the 15, although they are both numerators, and the 8 into the 16, although they are both denominators. One curious result is that this produces the answer 3/2, which is greater than one. The students who chose option (d) were not stopped by the impossibility of multiplying two proper fractions and obtaining an improper fraction.

(12) "One-third of 10" can be written as any of these except one. Which choice does NOT mean "1/3 of 10"?

	(a) $10 \div 1/3$ *	(b) 10/3	(c) 1/3 x 10	(d) $10 \div 3$
Test E	21	17	11	17
Test C	14	9	4	8
Total	35	26	15	25
(N=102)	34% *	25%	15%	25%

One possible confusion here is the negative wording; it is always harder to select the one false answer than the one true answer. However, the negative idea was repeated and emplasized; students merely looking for a correct answer would be most likely to choose (c), which the fewest did. There are two related errors here: one is that many students believe that 1/3 of 10 really is $10 \div 1/3$. The other error is in not perceiving the equivalence of all three wrong answers. In any multiple choice question, if all but one choice are recognized as the same, it is easy to identify the odd one. It becomes essential in algebra to realize the equivalence of "x thirds", "one-third of x", "x divided by 3", and "one-third times x".

The next item involves what may be to some students confusing terminology.

(13) The Least Common Denominator of $\frac{1}{2x2}$ and $\frac{1}{2x2x2}$ is:

	(a) 2x2x2 *	(b) 2	(c) 2x2x2x2x2	(d) 2x2
Test G	13	25	11	10
Test D	10	11	7	2
Total	23	36	18	12
(N=95)	24% *	38%	19%	13%

The item is interesting on several counts. Firstly, if the question had been asking what denominator to use in order to add 1/4 and 1/8, it would have probably been an easy item. This is the identical question, but its form, with the denominators factored into primes, forces direct confrontation with the concepts of multiples and factors. The student is being asked to find something which contains all factors of each denominator. A student who does not grasp the essence of the concept but who uses a rote method will be lost. Secondly, the words "Least Common" are inherently confusing. The natural tendency is to look for something small - the "least" number which is "common" to both denominators. Hence the popularity of the answer "2". Students are notorious for confusing "factcr" and "multiple", and in particular "Least Common Multiple" and "Greatest Common Factor". It doesn't help that the Least Common Multiple is invariably larger than the Greatest Common Factor. This terminology should perhaps wait for algebra.

The last item about numerical fractions involves size comparisons.

(14) Which of these statements is FALSE ?

	(a) $\frac{8}{11} < \frac{8}{10}$	(b) $\frac{7}{18} < \frac{8}{18}$	(c) $\frac{3}{4} < \frac{5}{8}$ *	(d) $\frac{17}{101} < \frac{17}{100}$
Test G	10	7	41	3
(N=64)	16%	9%	66% *	5%

The corresponding item on the first round of testing had a typing error; there were no correct answers.

As in the previous item, the wording adds confusion to the item; however, students trying to choose a true statement would probably select (b), which few did. The interesting error is (a). That $\frac{8}{11} < \frac{8}{10}$ is false would indicate a belief that $\frac{8}{11} > \frac{8}{10}$. Indeed, understanding that a larger denominator means a smaller fraction is a subtle and difficult concept in elementary grades and may not be truly believed even after being "learned". Applying this same reasoning to choice (d) makes that also correct, hence the test-wise student rejects both answers. On the other hand, many students' strategy is to go with the first correct answer without checking further. It may also be true that many students don't know an algorithm for comparing sizes of fractions. In this case, however, 3/4 is easily changed to 6/8 for an easy comparison.

The next group of problems involved both types of difficulty: fractions and variables. Perhaps it is not surprising that so many of them were difficult, given the trouble students have had in each of these areas.

The first two problems both ask about multiplying the fraction 2/3 by the variable N, but the questions posed are quite different, and produced quite different results.

(15) 2/3 of some number N is equal to which of these:

	(a) $\frac{2N}{3N}$	(b) $N \div \frac{2}{3}$	(c) $\frac{2}{3} \div N$	(d) $\frac{2N}{3}$ *
Test C	7	14	7	6
(N=36)	19%	39%	19%	17% *

This item was not used on the second round of testing because of its great difficulty. The surprise here is not just how difficult the item was, which might have produced a more random distribution of answers, but in the great preference for option (b). Just as in item (12) above, where only 34% of the students realized that "1/3 of 10" is not "10 $\div$ 1/3", here 39% have identified 2/3 of N as N $\div$ 2/3. The notion that a fractional part of a number is obtained by dividing by the fraction, rather than multiplying, is a persistent and curious belief held by many students. How it comes about and how to avoid it is an important issue. The next item addresses the same question as this one, but is posed differently.

(16) If N is any positive number, then multiplying it by 2/3 makes the value: (a) increase (b) decrease (c) stay the same (d) it depends on the value of N

	(a)	(b) *	(c)	(d)
Test F	15	28	0	22
(N=65)	23%	43% *	0%	34%

This item was on Test A with only the first three options:

	(a)	(b) *	(c)
Test A	5	28	1
(N=36)	14%	78% *	3%

The contrast is revealing. Students seem to be clear that the value can't stay the same; option (c) is virtually ignored. Having narrowed the choice to increase or decrease, on Test A, most seem to realize that "decrease" is the better choice. However, given the extra option on Test F, "it depends on the value of N", many students select that as a safer choice. Presumably, many who chose the correct answer on Test A were not sure it was correct. Presented with a variable, many students feel that no size comparisons can be made. This tendency to withhold judgement if possible is shown on two of the next three questions. All three involve the concept of multiplying fractions.

(17) For any fraction $\frac{A}{B}$, if you multiply $\frac{A}{B} \times \frac{B}{A}$, you get:

	(a) $\frac{A}{B}$	(b) $\frac{B}{A}$	(c) 1 *	(d) $\frac{A^2}{B^2}$
Test F	3	4	44	14
(N=65)	5%	6%	68% *	22%

This item on Test A was a fill-in which 42% answered correctly. The results on Test F were not bad, and it is reasonable that choice (d), as a more complex answer, had some appeal. The item makes an interesting contrast to the following:

(18) If N is any number except zero, than $N \times \frac{1}{N} =$

(a) N (b) 0 (c) 1 * (d) it depends on the value of N

Test F	9	4	26	26
(N=65)	14%	6%	40% *	40%

This item was a fill-in on Test A; 22% answered it correctly and 36% answered "N". The two previous items offer some interesting contrasts. In each case, the fill-in was more difficult than the multiple-choice format. For either format, however, $N \times \frac{1}{N}$ is more difficult than $\frac{A}{B} \times \frac{B}{A}$ even though they both exemplify the same rule: any non-zero number times its reciprocal equals one. The question here is whether $N \times \frac{1}{N}$ is really more difficult, or is it that the option "it depends on the value of N" is so attractive that it becomes popular whenever available. It would be worthwhile to ask item (17) with the option "it depends on the values of A and B". In any event, algebra teachers who present such statements as "intuitively obvious" need to be aware that to many students they are far from obvious.

The next item also concerns multiplying fractions.

(19) $\frac{A}{C}$ and $\frac{B}{D}$ are each fractions between 0 and 1. Their product, $\frac{A}{C} \times \frac{B}{D}$, (a) is bigger than one of the fractions and smaller than the other one, but you don't know which. (b) is bigger than either fraction. (c) is smaller than either fraction. (d) it depends on the exact value of the fractions.

	(a)	(b)	(c) *	(d)
Test F	8	16	20	18
(N=65)	12%	25%	31% *	28%

On Test A, the choice was somewhat different. Instead of option (d) the choice was: "is bigger than A/C, smaller than B/D." The responses to that item were:

	same (a)	same (b)	same (c)*	different (d)
Test A	14	8	10	2
(N=36)	39%	22%	28% *	6%

The results are comparable; roughly 30% of students realized that the product of any two proper fractions is smaller than either of them. It is curious that so many on Test A felt that the product was between the two fractions in value (choice a) as if it were an average of sorts, although this was the least popular choice on test F, where the distribution of responses is almost random. Again on Test F we see the appeal of not committing to the relative size of an expression with a variable.

The next question looks like a proportion but is essentially a question about equivalent fractions.

(20) If $\frac{X}{Y} = \frac{3}{4}$ then one possible set of values for X and Y is:
(a) X=8, Y=6 (b) X=15, Y=20 * (c) X=3+5, Y=4+5
(d) Y=3, X=4 The responses are as follows:

	(a)	(b) *	(c)	(d)
Test E	4	43	2	15
Test C	1	25	3	7
Total	5	68	5	22
(N=102)	5%	67% *	5%	22%

The error of interest is (d), Y=3, X=4. It is not clear how much error is caused by the fact that Y came before X in this distracter and how much is just the attractiveness of the numbers 3 and 4 as precisely what is called for. The problem of dealing with variables did not seem overwhelming here. This question seems to reward the careful reader; one interesting question is how much students' success in algebra is a result of such care.

The next two questions both relate to comparing quantities expressed in terms of variables.

(21) If you know that C $>$ D, which of these must be true:

	(a) $\frac{1}{C} > \frac{1}{D}$	(b) $\frac{C}{E} > \frac{D}{F}$	(c) $\frac{E}{D} > \frac{E}{C}$ *	(d) $\frac{1}{D} < \frac{1}{C}$
Test B	15	4	9	4
(N=37)	41%	11%	24% *	11%

This item illustrates the double difficulty of fraction concepts and using variables. The responses, however, are far from random. The popularity of the first choice seems based on a belief that the larger denominator produces a larger fraction, other things being equal, which is the exact opposite of the true property of fractions. Also, students who chose (a) were probably unaware that it is equivalent to (d), which rules out both options for any multiple-choice question. This item was not used in the second round of testing, nor was the following:

(22) If C $>$ D and also E $>$ F, which of these must be true:

	(a) $\frac{C}{E} > \frac{D}{F}$	(b) $C-E > D-F$	(c) $\frac{C}{D} > \frac{E}{F}$	(d)* $C+E > D+F$
Test B	17	1	4	12
(N=37)	46%	3%	11%	32%*

Again, the surprising result is not merely that the item was difficult, but the strong preference for the first choice. One possible explanation is that students use an invalid test-taking strategy: starting with the first choice, they search for a confirming example rather than a counter-example. If they find one (or a few) they conclude that they have found the correct option.

Another possibility is that they believe that if one fraction has both a larger numerator and a larger denominator than another fraction, that it must be the larger one. This view agrees with research presented at the NCTM conference in April, 1987 by Carol Novallis Larson[1]. She found that students would correctly identify fractions such as 1/2 and 4/8 as "equivalent fractions", but when asked to select which fraction was larger, these same students stated that 4/8 was larger. This result deserves further study.

The next question elicited interesting responses.

(23) What must be multiplied by A to get B ?

	(a) A-B	(b) B-A	(c) $\frac{A}{B}$	(d) $\frac{B}{A}$ *
Test G	10	4	23	23
Test B	2	6	10	15
Total	12	10	33	38
(N=101)	12%	10%	33%	38% *

It is not surprising that this was difficult; it is a novel question which involves both fractions and variables. It is encouraging that students realized that the expressions with subtraction were not relevant to a question about multiplying. It would be interesting to know what strategies were used. The next item is somewhat related to the previous one but also makes an interesting contrast to a purely numerical question.

(24) How many E's are there in one K ?

	(a) K-E	(b) E-K	(c) $\frac{E}{K}$	(d) $\frac{K}{E}$ *
Test D	4	3	7	11
(N=31)	13%	10%	23%	35% *

This item was not used on the second round of testing. Again, it is interesting that most students rejected any expression having subtraction as a poor choice, although not too many chose well between E/K and K/E. When the same question is posed with numbers, it becomes very easy; 84% of students answered correctly an earlier item, "How many 1/5's are there in 3 ?" It is clear that the difficulty in item (24) is the use of variables.

The next group of seven questions relate to three different topics: two items involve percent, two involve divisibility, and three involve terminology.

(25) The number 3 has the same value as:

	(a) 3%	(b) $33\frac{1}{3}\%$	(c) 300% *	(d) 3.00%
Test E	22	6	10	27
Test C	7	2	13	13
Total	29	8	23	40
(N=102)	28%	8%	23% *	39%

There are several surprises here. First, that an item which is based on a standard topic in the curriculum should be so difficult; second, the attractiveness of choice (a); third, the failure to recognize that the favorite, choice (d), is the same as (a). Percent is a topic which confuses many students; its placement and pedagogy as well as what students believe about it need to be looked at carefully.

(26) The number $4\frac{1}{2}\%$ equals which of these:

	(a) 4.5	(b) 4500	(c) .45	(d) .045 *
-Test C	25	1	4	6
(N=36)	69%	3%	11%	17% *

This item was not used on the second round. The overwhelming choice, that $4\frac{1}{2}\%$ equals 4.5 is the same error as in the previous question, where 3 was said to equal 3%. The willingness of students to ignore the percent sign, or to feel that it doesn't change the value of the number it follows, is disconcerting. Whether the symbol itself is difficult, the word "percent" is difficult, or the entire concept is difficult is not clear. Percents do not seem to be well integrated into the number system for many students.

The next two questions concern divisibility.

(27) If the units digit of a number is 4, which statement is true: (a) both 2 and 4 divide into the number evenly.
(b) 2, 4, and 8 all divide into the number evenly.
(c) 2 divides into it evenly; 4 may or may not.
(d) nothing is certain about what divides into the number.

	(a)	(b)	(c) *	(d)
Test D	20	3	6	0
(N=31)	65%	10%	19% *	0%

It is surprising that so many students chose (a); it could well be a case of looking for confirming instances rather than seeking a counter-example. This item was not used in the second round.

(28) If the units digit of a number is 3, 6, or 9 then which of the following statements must be true?
(a) 3, 6, and 9 all divide into the number evenly.
(b) 3 and 6 both divide into the number; 9 may or may not.
(c) 3 divides into it evenly; 6 or 9 may or may not.
(d) nothing is certain about what divides into the number.

	(a)	(b)	(c)	(d) *
Test D	6	5	12	6
(N=31)	19%	16%	39%	19% *

This item was not used in the second round because of its great difficulty. The reason for the popularity of choice (c) is not clear. Perhaps students were thinking of the numbers 3, 6, and 9 themselves as all being evenly divisible by 3. In this case, confirming examples are not as easy to find as counter-examples.

The last three items all contain the word "exceeds", which many students find troublesome, and which many algebra textbook authors employ.

(29) Which of these represent the amount by which 10 exceeds 7 :

	(a) 10+7	(b) $10 > 7$	(c) 7-10	(d) 10-7 *
Test F	5	30	0	30
Test A	5	11	1	18
Total	10	41	1	48
(N=101)	10%	41%	1%	48% *

The interesting error is (b), mainly because $10 > 7$ does not represent an amount, but makes a statement that 10 does indeed exceed 7. Reading symbols correctly seems to be the problem here. This question is re-cast using variables in the next item.

(30) Which of these represents the amount by which X exceeds Y ?

	(a) X+Y	(b) X > Y	(c) X-Y *	(d) X ÷ Y
Test F	5	29	31	0
Test A	4	12	18	1
Total	9	41	49	1
(N=101)	9%	41%	49% *	1%

The choices, except for (d), parallel those of the previous item. The essential difference in the two items is that this one uses variables whereas the previous one used numbers. The results are almost identical, with 41% choosing (b)although it is the only option which does not represent an amount. The issue here seems to be one of the reading of mathematics, a subject which needs work.

The next item asks the same question as the previous one but gives different options.

(31) By how much does G exceed H ?

	(a) G-H *	(b) H-G	(c) $\frac{G}{H}$	(d) $\frac{H}{G}$
Test G	36	9	10	3
Test D	18	3	6	2
Total	54	12	16	5
(N=95)	57% *	13%	17%	5%

The question is easier without the tempting option G > H. The choice of (c) is strange, and may indicate an interpretation of the question as "G is how many times as large as H ?" Exploring students' interpretations of the word "exceeds" should prove interesting.

Students' misconceptions have been looked at in this paper mainly in the areas of fraction concepts and the use of variables. Issues discussed include reducing fractions, the relationship between the size of a denominator and the size of the fraction, the multiplication of fractions and of reciprocals, and fractions with variables as components. Other issues concern the comparison of expressions containing variables, the meaning of percent, and the use of confusing terms such as "exceeds" and "least common denominator". The kinds of questions which need to be explored are as follows:

1) What do students believe about these mathematical entities?
2) Where in the curriculum should these topics be taught, in terms of readiness and of sequence?
3) What materials and methods should be used in teaching these concepts to avoid misconception?
4) Can terminology be found that would make the learning of certain concepts easier, while still being correct mathematically?
5) What is the role of reading mathematics, and how can it be taught?
6) What present teacher practices lead to these misconceptions, and how can they be modified?

Note: The test items quoted in this paper are copyrighted material. All rights are reserved by the author.

Footnote:

1 Carol Novaillis Larson, University of Arizona, "Associating Fractions with Regions, Number Lines, and Rulers: Implications of Research", presented at the April 1987 annual meeting of the National Council of Teachers of Mathematics, held at Anaheim, California

COMPREHENSION MONITORING IN LEARNING FROM SCIENTIFIC TEXT

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Students in the upper secondary and tertiary levels of education learn the conceptual content of science, to a great extent, in a receptive mode. In studying receptive learning and, specifically, receptive science learning, special attention has been paid to "what the learner already knows", according to the well known dictum of Ausubel. Educational psychologists use constructs such as "subsumer", "frame", or "schema", all of them describing in several ways the information which the learner already has. Moreover, research on students preconceptions has generated a great amount of literature and interest, as this seminar attests. However, the prior conceptual knowledge of the student is not the only variable which influences the meaningful learning of potentially meaningful information. Educational and developmental psychologists have been paying attention during the last years to variables of a higher level: metacognitive variables.

Metacognition, a concept introduced by John Flavell and others in the 1970s, "refers to one's knowledge concerning one's own cognitive processes and products or anything related to them " (Flavell, 76, p. 232). When reading a text, for example, active meaning makers differ from passive memorizers of information in the metacognitive skills which they bring to the task.

One of the necessary metacognitive skills for a meaningful acquisition of information is an adequate control of one's own understanding. This is the ability to know when one is comprehending or not comprehending something. A considerable number of studies have been done on comprehension monitoring when reading or when receiving instructions, both in children (for example, Markman, 77, 79; Markman & Gorin, 81) and adults (for example, Baker, 79, 85a; Baker & Anderson, 82; Glenberg, Wilkinson & Epstein, 87; Ryan, 84; Schommer & Surber, 86.) This paper deals with the comprehension monitoring abilities of secondary school students when reading scientific texts. The work which is presented in the following pages has several purposes. In the first place, it is an attempt to study the extent to which comprehension is controlled by students when reading scientific texts. Studies in comprehension monitoring have been carried out predominantly using materials of a general nature. The second question it attempts to answer is whether this control is dependent on the context in which learning takes place. The inflexible behavior which occasionally show many science teachers (Russell, 84), together with the authoritarian presentation of scientific knowledge in traditional teaching materials (Otero, 85), suggest a possible negative influence on the learner's ability to control ccomprehension. According to this, differences in comprehension monitoring were sought when the texts to be read by the students were placed in academic and extra-academic contexts. In the third place, we were interested in identifying specific comprehension monitoring strategies which students could use when reading scientific texts.

Procedure

This study has been carried out within the so-called "contradiction paradigm". Contradictory or inconsistent statements are introduced in a text. A failure to identify the inconsistencies is taken as evidence of a comprehension monitoring failure. Besides, the method employed follows closely that used by Baker (79) in a work on comprehension monitoring in college students.

Two classes of 18 year old students (n_1 = 31, n_2 = 38) in a public school in Madrid were chosen for the study. These students were in their last year in secondary school before entering the university. All of them were taking physics after having studied physics and chemistry the two previous years.

The materials for the study consisted of booklets divided in two main sections. In the first section there were six texts ranging from 86 to 94 words each. Contradictory statements were located at fixed positions (second and last sentences) in four of the texts (#2, #3, #5, and #6.) The physical proximity of the sentences in a text is a factor known to have an influence on the integration of information (Walker & Meyer, 80) and, because of this, in the detection of inconsistencies. Following is a translation of one of the four texts containing inconsistencies:

> Crystallization is a method to purify products used in scientific research. It consists in solving the product in water or in any other solvent. After that, the substance is allowed to crystallize. This process is repeated several times until products of a great purity are obtained. The process can take several days. Crystallization can only be carried out using water as a solvent. (Text #2)

There were written instructions in the booklet asking the students to rate the comprehensibility of the text using a scale ranging from 1 to 4 (1= difficult to understand...2= easy to understand). In case that the response were 1 or 2 there were additional instructions to: a) underline the sentence or sentences where the difficulties were found, and b) explain the reason for these difficulties in the space provided below.

The second section of the booklet contained information on the inconsistencies which were in the texts. The students were asked whether they had noticed the inconsistencies. In case thet they had detected the inconsistencies but did not underline the sentences or explain the confusion, they had to explain the reasons for this omission. The subjects had been instructed to follow the instructions in a strict sequential order without skipping pages or retroceding.

In order to create a different context for the reading of the texts in each group, experimenters were introduced differently. In one classroom the experimenter was presented by the litereture teacher. The students were informed about the purpose of the test: to identify difficulties which a learner experiences when reading a text. Their cooperation was requested to evaluate the comprehensibility of some short passages (supposedly) taken from newspapers. No mention was ever made to science texts or textbooks. In the other classroom the experimenter was introduced by the physics and chemistry teacher. The subjects in this group were told that the purpose of the experiment was to identify difficulties which students have when reading scientific texts like those found in their science textbooks.

In addition to different introductions there were also different headings for the texts in each group. In the "literature" group the headings identified the newspapers from were the paragraphs were (supposedly) taken. In the "science" group titles of science textbooks were given as headings. Otherwise, the texts were exactly the same in both groups.

Two criteria may be used to decide when an inconsistency has been detected. According to the first one, a conservative criterion, an inconsistency is detected when the student underlines the contradictory statements while reading, and explains the cause of the confusion. According to the second criterion, more liberal, an inconsistency is detected when it is identified while reading, as in the first criterion, or when the student declares, in the second section of the booklet, to have noticed the contradiction without having underlined the problematic sentences. The liberal criterion has been used to obtain the results given in the following section, except when it is otherwise indicated.

The subjects were given 50 minutes to complete the test.

Results and discussion

The overall number of inconsistencies detected (from a total number of 69 booklets x 4 problematic texts = 276 inconsistencies to be detected) is shown in Table I.

Place Table I about here

An analysis of variance revealed that there were no significant differences in the number of inconsistencies detected by students in each group. This could be interpreted at least in two different ways. It is possible that the context factor did not have an influence in the detection of inconsistencies in science texts. That would indicate that the comprehension monitoring abilities necessary for scientific text processing are strongly attached to the individual with little variations due to context. On the other hand, a different interpretation would be reached questioning an auxiliary hypothesis: perhaps a real difference in context was not created during the test. The same school setting was kept for both groups, the writing style of the texts did not conform to that of a newspaper, and the content of the passsages evoked that of science textbooks. All this suggests that more careful research is needed on the influence of this factor. Because of these reasons the following results are given conjointly for the total number of students in both groups.

There were found significant differences ($p < .01$) due to the effect of passage or text. It is well known that inconsistency detection depends on text variables: whether the inconsistency involves a main point or a detail, for example (Baker, 79.) More main point inconsistencies are detected than those about details. Although these are not variables in which we were specifically interested, the obtained data support previous findings. The only inconsistency clearly involving a main point is that in text #3, which corresponds to the highest number of students noticing the confusion (Table II.)

Place Table II about here

The number of students who notice zero, one , two, three, or all four inconsistencies is given in Table III. There are 21 students, about 30 % which detect at most one of the four contradictions. Only 13 students detect all four inconsistencies. This points to a sensible percentage of science students unable to control appropriately their comprehension processes. It indicates, at least, an absence of a disposition to process and/or integrate the individual propositions existing in the text--an essential ability to understand scientific text or any text whatsoever.

Place Table III about here

There are 18 students, about 26 % of the total number, which having noticed some of the confusions do not underline the contradictory sentences or explain the nature of the inconsistency. The explanations given by these subjects in the second section provide some interesting cues on the processes used by science students to control their own understanding. These students can be classified according to the stated reasons as shown in Table IV. They do not sum up to 18 because some students provide more than one type of reasons.

Two main groups can be distinguished according to the stated reasons: those subjects who tolerate the existence of inconsistencies, and those that do not and take some remedial action. The students in the first group find the existence of an isolated comprehension problem in a text unimportant, even after being instructed to ascertain its comprehensibility. Typical responses in this category are the following:

"It didn't seem to be important" (28.P)
"I thought that it was not important" (15.C)

Place Table IV about here

These students are taking the existence of comprehension gaps as something to be expected naturally when reading a text. It is interesting to examine some additional reasons provided by the students in this group:

> [I did not underline the sentences, although I noticed a problem, because]
>
> " I thought that contradictory sentences didn't have to be underlined. Only those that I could not understand" (2.P)
> " The sentences were easily understandable and there were no problems to interpret them" (22.P)
> " I understood the content well, it was clearly written and the opposition of the two concepts is not a difficulty to understand the text as a whole" (11.P)
> "I was more concerned with them as independent ideas" (22.C)

These reasons point toward a limited usage of comprehension criteria. These students are perhaps employing lexical, syntactic or external consistency standards of evaluation, but not internal consistency criteria (Baker, 85b.)

The student in the second group take a more active position when they realize the problem. They use "fix-up procedures" (Baker, 79) to obtain an explanation acceptable to themselves. Some subgroups could be identified. Two of the subjects resolve the contradiction by imputing it to some kind of printing error--a resource without much interest.

There are two students which explain the confusion in terms of a presumed ambiguity in scientific language:

> [I did not underline the sentences, although I noticed a problem, because]
> "Although they were contradictory it was clear to me that the solvent was water" (3.P)
> "It was not clear enough so I thought that it meant that water was the most common solvent" (31.C)

The word "only" in the last sentence of the text on crystallization reproduced in a previous page is interpreted as "preferably". These students seem to believe that it is possible to employ scientific language with such a high degree of ambiguity. It is a belief perhaps grounded on the way language is used in everyday matters.

Finally, there is one subject who resolves two contradictions making use of a remarkable argument. For this student, science seems to have enough authority to support contradictory statements. Referring to text #3 ("The speed of light is the greatest that can be achieved. No object can travel at a speed exceeding that of light because its mass would become infinite...There exist detailed observations of the behavior of some objects when they surpass the speed of light") the student explains:

> [I did not underline the sentences, although I noticed the problem, because]
> " I thought that it was something scientifically established and that there were no errors" (6.P)

Being one only individual generalizations are problematic. But this answer illustrates the negative effects of a possible history of authoritarian science teaching. The learning set of the student has apparently being distorted to such an extent as to be prepared to relinquish the principle of contradiction in blind obedience to the authority of scientific text.

Conclusions

The study has revealed a sensible fraction of secondary school students having inadequate control of their own comprehension. This will impair their ability to learn from conventional science textbooks. Some of the reasons behind this failure lie in the usage of limited or inadequate criteria to evaluate comprehension. There is an important number of students who fail to adequately process and/or integrate propositions when reading a scientific text. There are others who find it difficult to apply internal consistency standards,using instead lexical, syntactic or perhaps external consistency standards to evaluate comprehension. Other students seem to have a lax conception of the standards regulating the use of scientific language. That is an obstacle to acquire the precise meanings which a scientific text intends to convey. Finally, it is possible for science students to develop an inappropriate submission to the authority of scientific texts. As a consequence their capacity to control their own comprehension is severely diminished.

The preceding results suggest new teacher activities to address the metacognitive problems of science students. They also exemplify some of the roles which metacognitive variables play in science learning. This is an area where, undoubtedly, much promising work remains to be done.

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Criterion	Inconsistencies detected	
	n	%
Conservative	127	46.0
Liberal	155	56.2

Table I. Total number of inconsistencies detected (N= 69 x 4= 276

		Text number			
		2	3	5	6
Students detecting the inconsistency	n	39	55	34	27
	%	57	80	49	39

Table II. Students detecting each of the inconsistencies

Criterion		Students noticing				
		0	1	2	3	4
		inconsistencies				
Conservative	n	12	18	18	11	10
	%	17	26	26	16	15
Liberal	n	3	18	20	15	13
	%	4	26	29	22	19

Table III. Students noticing 0, 1, 2, 3, or 4 inconsistencies

Category	Number of students
1. Students tolerating the existence of inconsistencies in some of the texts	12
2. Students not tolerating the existence of inconsistencies in some of the texts	8
2.1. Typographical error	2
2.2. Ambiguity in language	2
2.3. Authority of science	1
2.4. Other reasons	3

Table IV. Distribution of students according to the reasons which they provide for not having underlined contradictory sentences

CHILDREN DERIVE MEANING FROM SOLVING PROBLEMS ABOUT PHYSICAL MATERIALS

Donald M. Peck, University of Utah

A heterogeneous class of fifth grade children derived their understandings and meanings about fractions by solving problems posed about physical materials. A different accelerated class of fifth graders, selected because of their apparent high abilities in mathematics, were taught via a conventional textbook approach to fractions. The test performance of the two groups as measured by right answers on addition and subtraction problems was almost identical. However, interviews and the posing of problems which required insight and meaning revealed a wide gap in interpretive and problem solving abilities in favor of the heterogeneous class.

The accelerated group for the most part directed their attention to mechanical manipulations of symbols according to perceived rules. Many of the procedures they used led to inconsistent results. When that occurred, they were unable to detect that they had been led astray.

The heterogeneous group, on the other hand, centered their attention upon meanings they had derived from solving problems posed about physical objects. They were more prone to utilize the meanings they had acquired to solve problems and were better able to tell for themselves whether their attempts were sensible.

The two groups came from essentially the same middle class socio-economic background and attended schools in close proximity to each other. The school attended by the thirty-three students belonging to the accelerated group utilized a comprehensive testing program and grouped for homogeneity on the basis of those test results. The average ranking for this group of students, as measured by their performance on the Iowa Tests of Basic Skills, was the eighth stanine.

The school attended by the heterogeneous group of twenty-four students was led by an administration who subscribed to a philosophy of mixing students according to their abilities as measured by test results. The Iowa Tests of Basic Skills placed this group of children at the fifth stanine as an average.

The discussion which follows describes the observed differences in approaches to problem situations utilized by children within the two groups, compare the specific instructional goals for each group, and review the assumptions and specific instructional procedures which undergirded the instructional program for the heterogeneous group and which led to the observed increases in their capabilities.

DIFFERENCES IN APPROACHES TO PROBLEMS AS EXHIBITED BY THE TWO EXPERIMENTAL GROUPS

The differences in approach to fraction problems will be discussed in terms of the nature of the test exercise given to the two experimental groups, an analysis of students responses, a comparison of strategies employed by representative members of each group, and a summary of the characteristic differences in perception and strategy employed by each group.

Analysis of Student Responses

Both the accelerated and heterogeneous groups had received instruction on fractions during November and December of 1986. Near the latter part of March 1987 they were given a set of five fraction examples to do. The first four examples were closely related to specifically what the students had been taught. The fifth example was unusual and required the students to fall back on their own resources to find a solution. The test exercise is shown below:

Problems common to the instructional programs of both groups

1. $\frac{2}{3} + \frac{1}{4} =$
2. $\frac{3}{7} - \frac{2}{5} =$
3. Which is larger? $\frac{3}{7}$ or $\frac{4}{9}$
4. Fill in the circle to make the statement true. $\frac{\bigcirc}{8} = \frac{3}{6}$

An uncommon problem ⟶

5. Fill in the circle to make the statement true. $\frac{\bigcirc}{5} = \frac{3}{4}$

The results of the exercise are set out in the table below:

Proportion and percentage of correct responses

Problem # ⟶	1	2	3	4	5
Accelerated group(N = 31)	$\frac{22}{31}$ – 71%	$\frac{25}{31}$ – 81%	$\frac{13}{31}$ – 42%	$\frac{20}{31}$ – 65%	$\frac{0}{31}$ – 0%
Heterogeneous group(N = 24)	$\frac{20}{24}$ – 83%	$\frac{17}{24}$ – 71%	$\frac{18}{24}$ – 75%	$\frac{16}{24}$ – 67%	$\frac{14}{24}$ – 58%

Table 1.

Analysis of Student Responses

As can be seen from Table 1, the addition problem (problem 1) was completed correctly by 71% of the accelerated group and by 83% of the heterogeneous group. The subtraction exercise (problem 2) yielded similar results with 81% of the accelerated group and 71% of the heterogeneous group performing it correctly. It appears the two classes were performing equally well as far as addition and subtraction of simple common fractions was concerned.

Problem three (Which is larger?) posed significant difficulty for many of the accelerated students. Only 42% of the students were able make the correct decision, while 75% of the heterogeneous group (essentially the same students who had successfully completed the addition and subtraction problems) performed this exercise accurately.

Problem four didn't seem to be quite so difficult for the accelerated group and 65% them were able to do the example.

The first four problems were taken from the usual fare given to students in their study of fractions. Even though the results were somewhat similar as far as getting right answers was concerned, interviews revealed some fundamental differences about how the students viewed fractions and made decisions about them. The accelerated students seemed to center their attention on a ritualization of the mechanical processes associated with the usual algorithms for fractions. They looked for relationships between the symbols themselves without regard for what the symbols meant. For example, consider one students' explanation of why he chose 4/9 as being larger than 3/7 (The students had not been advised as to whether their responses on the test exercise were correct or not, nor were they told until all the interviews were completed.):

Student: *If the top number is more for the one(fraction) than the other(fraction), it would be larger.*

Interviewer: *I'm not sure I follow. Could you show me what you mean?*

Student: *Like in this* (referring to problem three in the test exercise), *nine is five bigger than this four* (in 4/9) *and the seven is only four bigger than the three* (in 3/7), *so 4/9 is bigger than 3/7.*

Even though this student's response was included among those making right answers. Although he verbally expressed the belief that the fraction symbol meant to cut something up into equal portions, he looked at the relationship between the symbols 4 and 9 and tried to make his decision based on the numerator-denominator relationship of the numbers. (Could it be that his rule had worked often enough that he had adopted it as being reliable?) He was not making random judgements about comparing fractions, but was very consistent in his procedure. He thought, like the students noted by Rumelhart and Norman (1981, pp. 355-356), that his procedure made sense. When asked to compare 5/8 and 4/9 he was happy with the result and could not "see" that there was something wrong with 4/9 being larger than 5/8). This student, as Confrey (1984) has noted about others, perceived mathematics as being "out there" and unrelated to any personal meaning.

As the interviews with the accelerated students continued, it became apparent, even though they had produced right answers, that the large majority of them had misconceptions similar those possessed by the student just described even though they had produced right answers. If the accelerated students had been expected to provide some rational for their decisions as a part of the testing program, the picture would indeed be gloomy. It turned out that only three of these accelerated students (10%) perceived meanings in the symbols and operations they were performing. The ritual of moving and operating the symbols was performed without concern that the movements should somehow make sense in the sphere of their experience . The learnings this group had been exposed to apparently gave little opportunity for reflection on how the learnings might be related to common experience, and, even more importantly, how those learnings might help them quantify and solve problems about their world in general. Schoenfeld (1982, p. 29)was perhaps referring to similar observations as those noted here when he stated that

> *All too often we focus on a narrow collection of well-defined tasks and train students to execute these tasks in a routine, if not algorithmic fashion. Then we test student on tasks that are very close to the ones they have been taught. If they succeed on those problems, we and they congratulate each other . . .(but) . . . to allow them, and ourselves, to believe that they "understand" the mathematics is deceptive and fraudulent.*

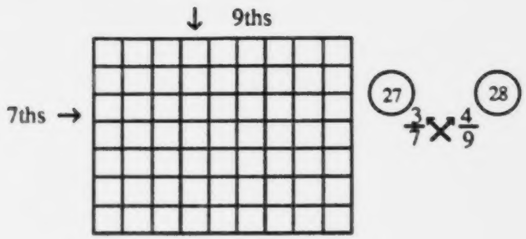
The students in the accelerated group viewed their teacher as the source of mathematical rules. They also believed that they must know the rules before they could proceed. They also exhibited dependent behavior by constantly trying to include the instructor, or the interviewer in this case, as an element of their problem solving strategy.The students constantly attempted to get the interviewer to tell them what to do or verify their efforts. They could not decide for themselves if they were making sense.

The heterogeneous group, on the other hand, had developed ways of viewing the symbols and operations on fractions that related them to common objects and elements of their own experience. These students expected to be able to solve the problems and did not seem inclined for the most part to try and include the interviewer in the discussion as the ultimate determiner of right or wrong. They understood that it was their job to figure out the problems and explain their solutions to each other and to the teacher. They did not seem tied to rules, but rather used their own logic to figure out what needed to be done in particular situations. For instance, consider the following excerpt from an interview with one of the students who had responded correctly to which is larger 3/7 or 4/9:

Student: *I decided by just multiplying 7 x 4 and and 9 x 3. Seven times four is larger than nine times three, so 4/9 is larger.*

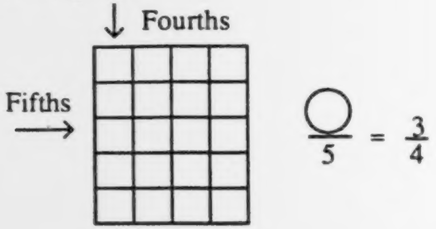
Interviewer: *How does that help you decide?*

The student drew the sketch below:



He argued: *If I cut the rectangle into 7ths one way and 9ths the other, then each 7th has nine pieces, so three of them have 27 pieces. Each 9th has seven pieces, so four of them have 28 pieces. Four ninths has 28 so it's more.*

This student could also show that her method of deciding worked using other physical materials as well. She had acquired some referents for the fraction symbols that served as a foundation for the method she used to decide which was larger. Her possession of a way of viewing fractions that was related to her experience gave meanings that she was able to extend to the unfamiliar example represented by problem five in the test exercise. She had completed it correctly. Her approach was representative of those used by other students in the heterogeneous group who had also successfully solved problem five. When asked how she knew her solution was correct, she sketched the following figure:



She explained: *Each fourth has five pieces so 3/4 must have a total of 15 pieces. If they* (the two fractions) *are equal then there must be 15 pieces for the other fraction too. Each fifth has four pieces so three fifths has 12 pieces. So I need three more pieces from the next 5th. That is 3/4 of a fifth, so the number that goes in the circle is 3 3/4.*

None of the accelerated group managed problem five or even came close. Their efforts focussed on trying to figure out some relationships between the symbols, but none of them possessed referents scheme that would allow them to reason their way to a solution. However 14/24 (58%) of the homogeneous group did the problem correctly and could explain how they knew their solution was correct. There was an additional two students who conceptually knew what to do, but didn't know how to write down the results of their thinking in numerical form. If these two students are given credit for understanding what was required, that would make a total of 67% of these students being able to address and solve problem five successfully. The difference between the two groups seems to be in the fact that one group held some referents based on experience with physical objects combined with an expectation that it was their job to figure out what was going on and explain their decisions. The other group did not possess adequate references for the symbols and held an expectation that understanding was to be provided them from external sources.

The interviews with the students of both groups pointed to some fundamental differences in the way the students were thinking and what they perceived their role to be in the mathematics classroom. The following statements summarize those differences:

Accelerated Group	Heterogeneous Group
1. Students directed their attention to symbol manipulations to symbolic relationships	1. Students directed their attention to referents for the symbols drawn from experience with real objects.
2. Students expected the teacher to decide whether answers were right or wrong.	2. Students expected to figure "it out" for themselves and make decisions about right and wrong themselves.
3. Students expected the teacher to show them how to do the problems.	3. Students expected to show the teacher how they worked out the problems and how they knew their work was reasonable.

COMPARISON OF INSTRUCTIONAL PROGRAMS

The instructional programs for the two groups will be discussed in terms of the differences in instructional goals, the underlying assumptions for the instructional program of the experimental heterogeneous group, the problem of evaluation , the role of physical materials, and a classroom example of how the physical materials were actually used.

Differences in Instructional Goals

The goals for the accelerated group as perceived by the teachers and expressed in the curriculum guide provided by the school district was aimed at mastery of basic skills at a high level of proficiency. Achievement of the goals was accomplished by the instructor demonstrating the algorithm and then providing considerable practice from textbook pages. Textbook practice was supplemented by drill sheets containing selections of problems aimed at establishing the procedure in memory as an automatic response . Each topic was fragmented into a series of subskills illustrating a step by step approach to a general concept. Attainment of the goals were measured by a performance of the algorithms on a timed paper and pencil test. The decision as to the correctness of responses was determined by an answer keys administered by the teacher. All in all, the program was typical of the usual textbook and management approaches to the mastery of arithmetic skills.

The goals for the heterogeneous group differed in that a particular algorithm or procedure which could be used to produce answers was not the aim of instruction. The most basic and most important objective was to put the children in a position to answer the question, "How can you tell for yourself?" This was accomplished by assisting the students to acquire a base upon which to build meanings, reason out answers, verify and prove to themselves the adequacy of their own decisions and become confident and sure about their own thinking. Any particular computational scheme was to arise from the student's own decisions about how things work as opposed to being presented authoritatively by the instructor. The realization of such goals would be impossible if the students were taught by drill, by lecture, by telling them how to do problems, or by other popular methods which insulate the student from complete immersions in conceptualizing for himself or which preclude the necessity for him to solve problems daily and determine for himself whether he has solved them sensibly.

A major feature of the instructional program for the heterogeneous group was the reversal of the traditional student-teacher role. Students solved problems and explained their solutions and showed the correctness of their work. The teacher asked questions , listened to explanations and doubted. He was never an expositor or authority about right or wrong. The authors working definition, "A child doesn't understand if he has to use a rule or pattern" served to guide the instructor in determining whether understanding had been achieved.

The Problem of Evaluation

The usual tests of mastery involving paper and pencil tests posed a unique barrier to progress in terms of problem solving and thinking. Tests used indiscriminately for evaluation reward children for procedural accuracy but are generally unable to detect that the children may have little or no understanding of what the answers mean or the conceptual considerations involved. This fact has been amply noted in the preceding sections of this paper as well as by others such as Whitney (1985).

The presenters have noted that children can be roughly categorized into four groups on the basis of right and wrong answers produced on paper and pencil tests (Peck, Jencks & Connell 1987). There are those that have a procedure that may produce right answers, but the students lack or have fuzzy conceptual understandings. A second group has sound conceptual understandings and procedures for expressing their understandings. A third group has correct understandings but

have made inadvertent errors or possess an inefficient computational process. The final group has neither process or concept available to help them perform well in the test situation . To avoid the dangers of causing the children center their attention on answer production or the memorization of rules at the expense of conceptual understandings, the instructor used brief interviews to supplement written work and test results. The interviews attempted to see if the students had acquired meanings and conceptual understandings of the operations or problem situations. A student may have written a correct response, but if he lacked the proper conceptual understandings associated with the situation, he was placed with those needing further instruction. The students soon sensed that it was more important to search for meanings than it was to simply acquire a skill at producing answers.

The Role of Physical Materials

The instructor believed that children construct and reconstruct their own knowledge in order to make sense of it. This assumption suggested that the children required experiences that permitted them to construct their own understanding. To accomplish this end, it seemed necessary for the children to find personal meaning in the mathematics they studied by being able to relate it to their own experience with how things work in the real world and how the symbols of mathematics can be used to describe this experience. Physical materials were thus considered the logical ground upon which to build the children's experience background.

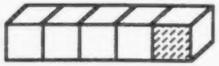
As a consequence of experience, children adopt belief systems that either enhance or obstruct their progress in adjusting to new or novel situations particularly problem situations (Cobb 1985). Carpenter, Linquist, & Silver, (1983) when they reported on the results of the Third National Assessment noted that students believed that mathematics was rule governed, superficial, and only subject to expert external authority. These negative beliefs in the judgement of the authors had to altered to perceptions of mathematics as being reason governed, substantial and personalized. The instructor set about helping the students form this belief system by exchanging the usual teacher role of explainer and judge for that of question asker, listener and doubter(Peck, Jencks & Chatterley, 1980). He encouraged the students to explain to each other and bounce ideas off anyone who would listen as they struggled to define and conquer a problem situation. The instructor insisted their conclusions and answers be in terms of showing how they were organizing the materials and how they knew for themselves they were making sense. Reward was withheld until the students could show clearly from the materials that their reasoning was valid.

There is nothing new about using physical materials to develop mathematical ideas, nor does their use automatically lead to desirable outcomes (Holt ,1982). The presenters have often noted that when physical materials are used to merely illustrate a rule or to demonstrate a principle, the desired understandings which permit the children to use the ideas acquired as a basis for reasoning and decision making do not necessarily follow. The use of physical materials as tools of decision making combined with the change in teacher role helped the children view mathematics as subject to their own reasoning.

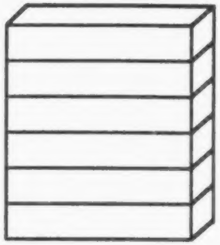
An Example of the Use of Materials with the Heterogeneous Fifth Graders

The students came to the fifth grade experience with various counterproductive perceptions and strategies already in place (Peck 1984). It was necessary to modify these views. Experience has taught that entering an instructional activity with an already familiar topic (even though the students may be having considerable difficulty with it) is not productive and brings about little change in perception. For this reason the heterogeneous groups' experience was begun in an unfamiliar setting. It was determined to begin the heterogeneous grouped children's experience with the concepts of area and volume since they had had little or no experience with the concepts involved.

The children were introduced to volume and area by using some base five multibase blocks. A unit cube was used to represent a rubber stamp and the back of the instructor's hand was used to represent a stamp pad full of black ink. The instructor began by pressing the unit cube to the back of his hand (the stamp pad) and placing it on one end of a five rod as shown here:



He asked, "*How many stamps would be necessary to completely cover the rod with ink?*" Using the material supplied the students soon decided that twenty two stamps would be necessary to cover the rod. The instructor then posed a problem by stacking the rods as shown below:



Instructor: *Can you find a way to determine the number of stamps required to completely cover the stack with ink no matter how high it gets?*

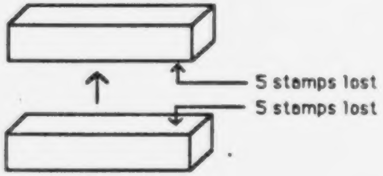
The students came up with a number of ways of doing the problem which they explained to each other, the instructor, principal and anyone else who happened to enter the room. One strategy developed by a number of students was exemplified by Amy's argument:

Suppose there are 31 rods stacked up, then each stick on the front would require five stamps, so there are 5 x 31 stamps on the front side, and the same number on the back side. Each end gets 31 stamps, so that's two times 31. Then the top and bottom each get five.

She then added the subtotals (155 + 155 + 31 +31 + 5 + 5) to get the final number of stamps required.

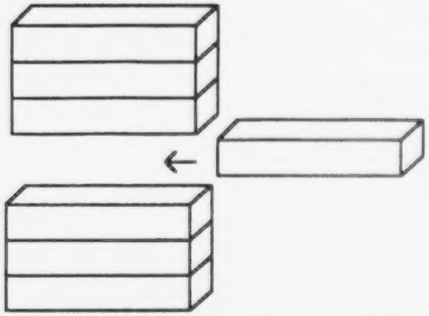
Michael on the other hand had noticed a pattern that suggested the addition of a rod increased the number stamps required by twelve. His method was to start with the twenty-two stamps required to completely cover the first rod and then add twelve each time a rod was added. He could not explain how it happened at first. The instructor simply indicated he could not accept his results unless he was able to explain how the increase of 12 occurred and that it would always continue in that manner. He and several of his classmates tackled the problem. They offered the argument below on the following day:

Student: *If you start with just one rod, it takes 22 stamps. Now as you bring up another rod to glue on, it has 22 stamps too, but when is attached to the first rod there are five stamps that are lost on each side of the crack.*



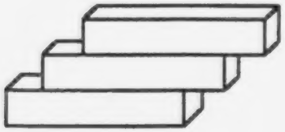
That's ten lost altogether so you have only added 12. That happens each time you add a rod, so that's 12 each time. If there are 27 rods stacked together then you start with 22 and add 12 for each of the 26 rods added, so that is 22 + (26 x 12) stamps.

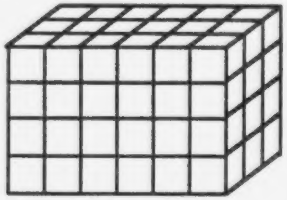
Another variation on this argument was offered by Cassandra. She started by gluing two rods together to start with. It took 34 stamps to cover the two rods. She then commenced building the stack by inserting rods in between the end rods.



Cassandra argued the end rods retained the same number of stamps but that each rod inserted could only be stamped on the front and back and the two ends making a total of 12 stamps added each time.

The instructor posed variations on the problem the problem and provided for repetitive practice of the conceptual ideas underlying finding the surface areas of various configurations . (Practice time was never devoted to mechanical procedures, but only to conceptual ideas and problem situations.) Two examples are given below:





By asking the students to find the number of cubes and the numbers of stamps needed to cover various figures built of cubes, the students learned to deal effectively with both surface area and volumes in a great variety of situations and circumstances. They also systematized some effective methods of doing so for themselves. Once the children were effectively in charge, the number of little squares (stamps) needed to cover a region or object was defined as area and the number of cubes required to construct the object as it's volume.

There was a gradual alteration in the perceptions the children held of the instructor and their own role in engaging mathematics. It took almost three months of activities similar to those just described, however, before the children began to lose their tendency toward establishing dependency networks and expectations that the instructor would eventually bail them out by providing a procedure they could memorize. They began to explore and solve more significant problems and even began posing themselves questions to explore. By mid year, the students were deeply involved and had developed the positive perceptions and strategies described earlier in this paper.

SOME CLOSING COMMENTS AND SUMMARY

It would be nice to be able to say the program worked perfectly and that all the children in the heterogeneous class were positively affected. Six of the children in the heterogeneous experimental class were unable to break out of their dependency patterns and use the materials as a basis for thinking, reasoning and building mental roadmaps to guide them in their problem solving efforts. Two of these children had dyslexia. The remaining four attempted to standardize the manipulations of the objects and commit these manipulations to memory in much the same way they attempted to mimic the movements of symbols in an algorithm. Yet, from the observations presented here, it becomes evident that the majority of these children (about 75%) of them were in a superior position to the accelerated group in that they could independently solve problems, think clearly and justify their efforts at finding solutions.

In summary, a mathematics program for an accelerated class of fifth graders based on teacher explanations and extensive practice of the algorithms for operating on fractions led students to center their attention on fraction symbols without regard for underlying meanings. Although many of these students produced right answers, for the most part they lacked meanings that would help them to interpret results and decide whether they were making sense. Interviews suggested the level of success accorded the students by placement in an accelerated program was not warranted

A heterogeneous class of fifth graders performed as well the accelerated group on addition and subtraction of fractions as regards right answers. They had acquired a set of referents and meanings for fractions based upon instructional activities with physical materials used as tools for decision making and thinking that helped them solve unfamiliar problems and interpret the results of their own thinking. Although ranking far below the accelerated group in terms of placement via test performance, the heterogeneous group proved themselves to be better able to address the cognitive aspects of mathematics and think for themselves.

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PRECONCEIVED KNOWLEDGE OF CERTAIN NEWTONIAN CONCEPTS AMONG GIFTED AND NON-GIFTED ELEVENTH GRADE PHYSICS STUDENTS

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The purpose of this study was to see whether non-gifted physics students have more difficulty with certain mechanical concepts than their gifted counterparts. The instrument used for this study was a Preconceived Knowledge Test which consisted of nine simple drawings and five tasks involving the motion of ordinary objects encountered in everyday life. The newly compiled test was administered to 25 gifted and 24 non-gifted eleventh grade students. Both sample groups were completing the same high school physics course. The gifted students were found to consistently earn higher grades in physics than did their non-gifted counterparts. However this was not the case with the results on the Preconceived Knowledge Test. On several of the fourteen items the non-grifted sample did appreciably better than the gifted sample and were nearly even on the other seven items. This suggests that even gifted students encounter difficulities in grasping certain basic mechanical concepts. Possible reasons for this discrepancy should be explored including implications for physics teachers. This study was derived from previous studies done by McDermott, Clement, Osborne, Minstrell and others.

In order to compare the gifted and non-gifted students with regard to their levels of misconceptions the problem was considered as the null hypothesis:

> H_0: There is no significant difference in the levels of misconceptions concerning certain mechanical concepts between gifted and non-gifted students currently taking the same physics course.

Forty-nine students taking an honors physics course with the same teacher and text book were chosen for this preliminary study. Twenty-five students were identified by the school as gifted with an average IQ of 146 and twenty-four students were non-gifted with an average IQ of 116.

These students were not identified as gifted or non-gifted for purposes of being placed in the honors physics course and were mixed fairly even between both physics sections. The physics teacher was also unaware of those students identified as gifted, which incidentally in Pennsylvania requires an IQ of 130 or higher.

An interesting but not surprising relationship between IQ and grades was immediately evident when comparing the gifted and non-gifted samples. The grade for the first marking period which covered Newtonian Mechanics, the relevant concepts to this study, were appreciably higher for the gifted students. An average grade of 91 for this marking period for the gifted group and 80 for the non-gifted group. The physics grade for the entire year showed a similar and appreciable difference. An average final grade of 93 in physics for the gifted students and an average final grade of 81 for the non-gifted students. There is sufficient data available to show that the correlation between IQ and grades are very high. However, it is not certain whether the correlation between IQ and creatively and imagination is equally as strong.

This study attempted to compare the level of misconceptions between the gifted and non-gifted students. Fourteen items dealing with ordinary and everyday motion were chosen as test items. Nine wre drawings involving Newtonian motion found in most physics textbooks. These include the following: (1) picture of a pendulum whose string is broken and the student is asked to indicate its path; (2) a picture of a rocket with its engines firing and the student is asked to indicate its path; (3) a picture of a cannon firing a ball from a horizonital cliff and the student is asked to indicate its path; (4) a picture of a ball moving in a horizontal circle at the end of a string and the student is asked to indicate its path when the string breaks; (5) and (6); a ball is bounced off a table, thrown horizontially and is shown in two positions once as the ball is falling toward the table and in a second position as it bounces up from the table. The student is asked to draw the direction of the force or forces on the ball in each position; (7,8, and 9); a ball is thrown straight up and falls back to the ground. The ball is shown in three positions first on its way up, a second position at its very peak and a third position as it falls downward. The student was asked to draw the direction of the force or forces as the ball in each of these positions.

Included as part of this test were also five tasks. These included the following: (10) The student was given a lead and aluminum ball of equal size and after having determined that one was heavier than the other was asked to predict the manner of their decent if allowed to fall together. (11) a ball was released in an elevated end of a plastic tube-placed horizontally on the table of about 3/4 of a complete circle. The student was asked to predict the motion of the ball as it emerged from the distal end. (12) a ball was released in an elevated end of a plastic tube also place horizontally on the table but this time the tube was coiled around to complete three circles and the student was asked to predict the motion of the ball as it emerged from the distal end. (13) The student was instructed to demonstrate how he/she would release a puck in order that it would complete a quarter circle track without leaving the track. (14) The student was asked to walk briskly with a tennis ball held at his/her side and instructed to release the ball so that it would fall in a coffee can placed on the floor.

For the sake of space economy a complete description of these activities has been deleted however, one can get a better idea of the activity by refering to the pictures at the end of the paper.

For the purposes of this study, responses to the nine questions and five tasks of the Preconceived Knowledge Test were considered either correct or incorrect according to protocol established by researchers presently studying preconceived knowledge of mechanical concepts (Clement, 1982; McCloskey, 1983; McDermott, 1984; Viennot, 1979). All judgments regarding the correctness of the 14 responses were made by this researcher in order to minimize variations in bias and interpretations. For a response to be designated as correct, for purposes of analysis, only the answer had to be correct, regardless of the validity of the reasoning used in arriving at the answer. No effort was made to account for valid explanations given for incorrect responses or invalid explanations given for correct responses. The face validity of the Preconceived Knowledge Test, assumed as part of this study, was derived from those recent studies, previously cited, dealing with preconceived knowledge (Isaac & Michael, 1982). The reliability statistics concerning the items on the Preconceived Knowledge Test produced a Kuder-Richardson coefficient of .60584.

The newly compiled but unvalidated Preconceived Knowledge Test was administered to the physics students using the Piagetial interview method. The test consisted of nine written questions and five task questions dealing with motion of common objects involving certain mechanical concepts and was administered individually to all subjects. The average time required for each subject to complete the written and task portions of the test was 30 minutes. The student responses on the Preconceived Knowledge Test were tested as the dependent variable and giftedness as the independent variable and was compared using the chi-square analysis. The questions represented different concepts of motion, therefore, it was decided to separately analyze the relationship for each of the individual questions of the Preconceived Knowledge Test.

The data were presented to fit 2x2 contingency tables with 1° of freedom. Consequently, the Yates correction was utilized and the test for significance was chosen at the 1%-level (Ferguson, 1966). The responses on each question of the Preconceived Knowledge Test given by each subject tested were scored as either correct (+) or incorrect (0). The chi-square method was chosen because it relates only to frequency data and required that each individual event is independent of each other. Since the hypothesis under investigation in this study was stated as a null hypothesis and assumed to be true, a calculated value of x^2, which was greater than the critical value required a rejection of the null hypotheses. In order to test the hypothesis, the gifted sample was compared with the non-gifted sample on their responses to questions on the Preconceived Knowledge Test.

The table shown at the end of this paper includes the results on the fourteen items and the calculated chi-square values and probabilities. The chi-square values do not indicate any significant difference at the 1% level for any of the test items. Consequently the hypothesis can be accepted under these conditions.

Research on the nature of preconceived knowledge in the physical sciences and on its implications to science learning has provided an exciting new perspective for viewing giftedness, particularly as it relates to the learning of science. The present exploratory investigation has attempted to expand the model of giftedness by examining preconceived knowledge dealing with mechanical concepts in gifted and non-gifted students of similar academic training.

Conventional testing in the sciences has traditionally emphasized isolated facts and empirical relationships. It has generally been assumed that gifted students demonstrate more advanced cognitive abilities, such as problem-solving, than the average student of the same chronological age (Stanley, George, & Solano, 1977). The results of the present study may help determine whether some cognitive abilities, such as those associated with the structuring and internalizing of preconceptions dealing with mechanics, may not develop spontaneously, simple as a concomitant of the advanced intellectual development associated with higher I.Q. scores. Also, the present study may indicate the perceptions of the physical world held by most students are based on more than the level of intellectual development, cognitive skills, or the amount of formal science instruction received.

Renzulli (1978) had cautioned educators not to determine a priori giftedness upon IQ alone. For to do so, according to Renzulli, would ignore students who possessed intellectual and non-intellectual skills not measured exclusively by tests of intelligence or creativity. Ludow and Woodrum (1982), examining the problem solving strategies of gifted and average learners (determined by IQ scores), showed that gifted learners at the elementary level do not necessarily develop problem solving abilities as a natural consequence of higher IQ's. Delisle and Renzulli (1982), using various behavior models to determine giftedness rather than personality and intellectual traits, have shown that a student's concept of self is more likely to result in academic success than the IQ or rank in class. Their study showed that 56% of their study population, consisting of middle and junior-high school students, who do not meet the IQ requirements for inclusion in a figted program demonstrated gifted behavior on tasks and projects of interest to the student. Delisle and Renzulli (1982) suggest that students in a traditional gifted program (based on IQ tests) lack both direction and knowledge of their roles as "gifted" students. Consequently, Delisle and Renzulli (1982) warn that the exclusive and stringent use of IQ scores or achievement tests for identifying the gifted students is indefensible on the basis of current research findings.

A recent study by Davidson and Sternberg (1984) examining the role of "insight" in intellectual giftedness revealed that a student's "insightful" grasp of a particular question or problem is not a direct function of either IQ or grades. In some cases the so-called average student seemed to demonstrate a greater "feeling" for the question or problem than did the gifted student. This would certainly support the findings fo the present study that understanding of physical concept derived from preconceived notions are not necessarily facilitated by intelligence based on IQ scores.

Apparently one cannot consider the student to be neutral or innocent in his/her appreciation of the mechanical world. The concepts taught in the classroom find resistance and sometimes total rejection resulting from stable preconceptions that students have constructed over a number of years of interacting with their environment. The present research, was intended to increase awareness for a possible expanded or new model of giftedness which may include preconceived knowledge as one of the criteria to be included in the determination of giftedness. It can also aid in the development of instructional strategies which enhance a correct understanding of science concepts on the part of the student, rather than their rejection which the literature indicates is currently the norm.

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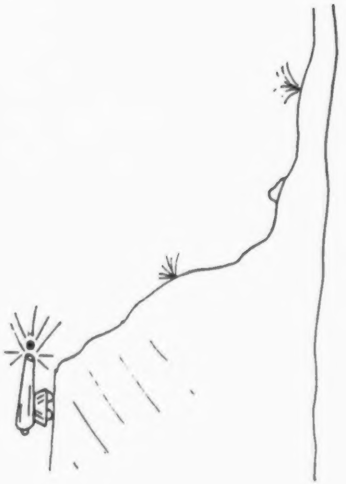
TABLE OF RESULTS

Correct Responses on Preconceived Knowledge Test for Eleventh Grade Sample:
Chi-Square Analysis for Giftedness vs. Responses to Question

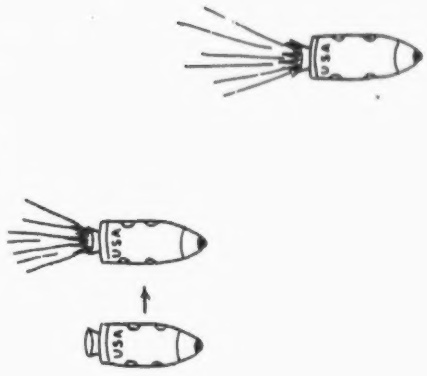
d.f. = 1

Question #	Gifted N=25 n	 %	Non-Gifted N=24 n	 %	.Yates x^2	p.
1	15	60	11	46	.500	.4796
2	06	24	08	33	.165	.6843
3	21	84	21	87	.000	1.000
4	16	64	11	46	.982	.3218
5	04	16	08	33	1.163	.2809
6	02	08	07	29	2.383	.1226
7	02	08	04	17	.239	.6247
8	11	44	12	50	.018	.8931
9	24	96	21	87	.319	.5724
10	14	56	05	21	4.983	.0256
11	20	80	17	71	.171	.6791
12	18	72	13	54	.996	.3182
13	14	56	14	58	.000	1.000
14	23	92	19	79	.766	.3816

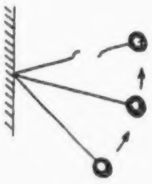
Question #3



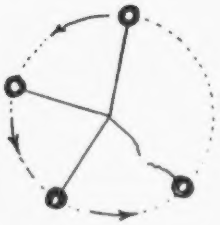
Question #2



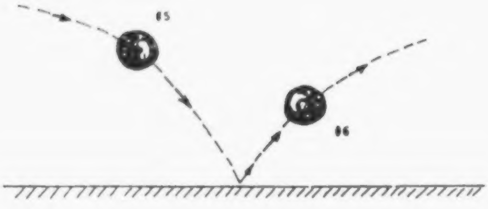
Question #1



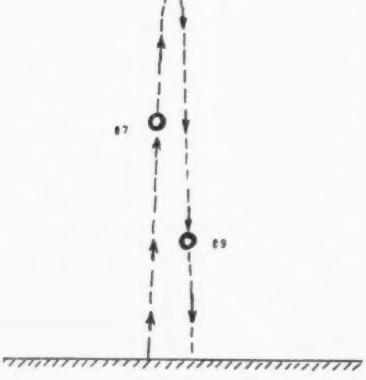
Question #4



Questions #5 and #6

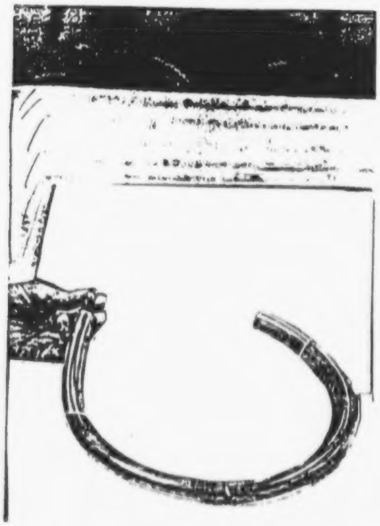


Questions #7, 8, and 9

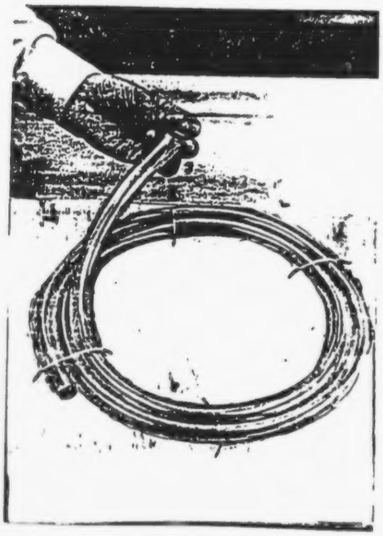


Question #10

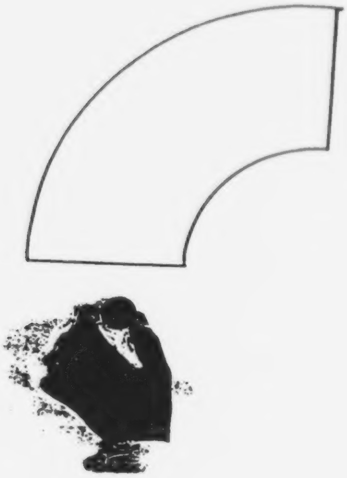
Question #11



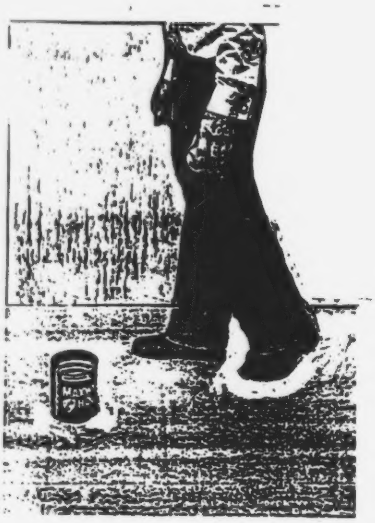
Question #12



Question #13



Question #14



ALTERNATIVE FRAMEWORKS ABOUT THE LEARNING OF CHANGES OF STATE OF AGGREGATION OF MATTER:SORTING OF ANSWERS INTO MODELS

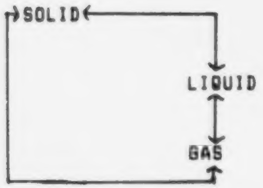
JOSEP RAFEL and CLAUDI MANS (Institute "Carles Riba" and University of Barcelona.Catalonia.Spain)

1.- Introduction.

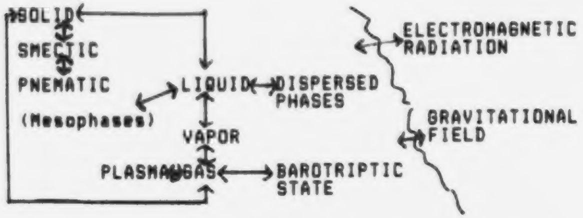
Secondary school pupils usually leave primary school considering only three states of aggregation of matter:solid,liquid and gas. Living matter,colloids,and in general all the dispersions do not fit under that scheme.And states like plasma,barotriptic,smectic, pnemàtic,the gravitational field,light and other electromagnetic radiations are not taught in schools.So,pupils arriving at secondary school in the best cases have heard something about only these six changes of state:

-Solidification	-Fusion	-Sublimation
-De-subliamation	-Condensation	-Boiling

arising from the following outline:



Instead of, at least , these 24 based upon the following outline:



Any research on the improvement in teaching of equilibria between phases in secondary school(including those allowing the learning of more changes of state than the above mentioned ones)could look at the ideas that pupils bring at arrival.

The main purpose of this work is to obtain a small set of data of pupils aged 14-16 concerning their alternative frameworks and misconceptions about the changes of state of matter,in a wide constructivist interpretation of scientific concepts building as indicated by DRIVER(1983) and by OSBORNE and WITROCK(1985).

The existence of these misconceptions lead pupils to great difficulties in the understanding of the scientific point of view of changes of state. The goals of this research include finding and identifying the main difficulties that pupils have in reaching adequate comprehension. So operational and conceptual influences and their linkages have been studied.

Mass media interactions have also not been neglected. The most usual techniques employed in this kind of research includes questionaires,interviews and class-room observation.

In this research the first data were obtained from questionaires on these topics,as indicated in other works.See RAFEL, PORTA & MANS (1985). and RAFEL & MANS (1987).A copy of the questions included in the distributed test is shown in Appendix I.The questionaire consists of fifteen questions (Q1 to Q15) including sets of multiple choice questions (MCQ) and open-ended questions.

The analysis of the MCQ has been carried out mostly through the employement of exloratory data analysis and the open-ended questions have been studied using network analysis techniques,and a brief summary of the results is presented here.

2.The Investigation.Sample and Questionaire.

After several phases of revision of pilot tests a questionaire was finally distributed to 103 pupils aged 14-16, (49 boys and 54 girls);33 of them who attended one secondary school (18 boys and 15 girls) and 70 who attended another one (31 boys and 39 girls) in the town of Barcelona in the school year 1985-86.Both were public schools. The teachers came from a common selective system and had more or less the same educational background.

Pupils attending both schools had similar home backgrounds since people living in the town districts around these schools belong to the lower middle class.They were selected for baccalaureate at 13-14 , but not by specific abilities.

The test basically consisted of 11 Multiple Choice Questions(MCQ) ,where pupils were also asked for their confidence on the answers they gave; and two open-ended questions,plus a third one not studied yet. MCQ results are analysed in Section 3 and the Open-Ended Questions,(OEQ), in Section 4.

3.- Study of the MCQ responses.

3.1.-Quantitative Results.

The results of the MCQ are summarized in the Table No.1, where facility should be understood as the quotient obtained dividing the number of pupils scoring in each question,by the total number of pupils.

TABLE No.1:

I=ITEM PC=PUPILS CORRECTLY ANSWERING F=FACILITY

PBC=POINT BISERIAL CORELATION ("r"coefficient)

I	PC	F	PBC	\	I	PC	F	PBC
2	66	64	.69	\	9	63	61	.70
3	40	39	.70	\	10	21	20	.57
4	72	70	.65	\	11	78	76	.61
6	61	59	.70	\	13	66	64	.69
7	59	57	.71	\	14	43	42	.71
8	31	30	.65	\	Average :	54.5	53	

(s^2=17,63 & st=4.20)

NOTE:P.B.C.as a measure of consistency was calculated as follows:

r=((XM(p)-XM(q))/st)*SQR(pxq)

where p=proportion those succeding each item; XM(p)=mean score in the whole test; q=proportion which fail, and XM(q)=1-XM(p).

3.2.-Qualitative discussion of MCQ results.

Eight out of the eleven MCQ included changes of state as a principal or secondary target. Two main sets :

a) including questions Q8, Q10, Q13 and Q11 and

B) including questions Q11, Q9 and Q14 are discussed and shown as follows:

-Q8)Given two figures I and II,the variation of temperature is represented against the required time for heating with a heater: a piece of ice at -15°C until it reaches 120°C in a well closed jar.Which is the correct one? (Figure I did show a continuous curve line and figure II did show several straight lines growing up from -15°C until 120°C, with horizontal fragments at 0°C and 100°C)

A) Figure I	B)Figure II
C)There will be horizontal lines in figure II,only by turning-off the heater	D)Both are all right.

-Q10)You have two pans, A and B, in two identical heaters. They have the same ammount of potatoes,water and salt ;and they are covered by water during the whole experiment.Selected heater positions are the same.When "A" and "B" start boiling,the "B" heater is switched to a "minimum" position and "A" isn't.

Please,indicate which potatoes are cooked at a greater temperature:

A)The ones from pan "A"

B)The ones from pan "B"

C)Both are the same

D)The ones from "B" will need a greater time.

-Q13)In winter on a very cold and wet day you wish to dry some clothes. Where would they dry better: In the open-air or in the bath-room?

A)It's the same.Temperature doesn't matter!

B)In the open-air

C)In the bath-room

D)There is nothing to do.You must wait for a better day.

-Q11)Fog is formed by:

A)Boiling water in rivers

B)Vaporized water in rivers

C)Condensation of humid air,after strong cooling

D)Vaporized water from mountain snow

-Q9)A small jar is filled with ice. The lid is screwed on tightly and the outside wall of the glass is dried with a tea towel. Some minutes later the outside wall of the jar is all wet. Where has the water come from?

A)From melting ice coming through the glass

B)From cold which has converted into water

C)Water already existing in the air sticking to the cold glass

D)The towel did not dry the water enough

-Q14)Clouds are formed by water from sea, rivers and lakes. How does water form these clouds?

A)Through a boiling process thanks to the heat of the Sun.

B)Through boiling processes but only in very hot seas like the Caribbean.

C)Through boiling and vaporization processes.

D)There is not any cloud formation through natural boiling processes.

Pupils got the highest scores (76 and 64%) in Q11 and Q13,and the lowest scores (20 and 30 %) in Q10 and Q8,all of them belonging to set α).

A good reason for the low score in Q8 could be the requirement of handling of tables or graphics.

Questions Q8,Q10 and Q13 were all related either to vaporization or boiling . The great score difference observed between questions (Q8 and Q10) and Q13 has shown a surprising lack of homogeneity;specially between Q10 (with the lowest score value) and Q13, both related to everyday life events.

Considering set β), lack of homogeneity appears again: with only a peak value (76%) for Q11, while similar score for questions Q9, Q11 and Q14 was expected since they were writen to test pupils knowledge on condensation under typical parallel criteria belonging to science-world ideas schemes.

But lower scores were achieved for Q9 (61%) and Q14 (42 %), which suggest alternative schemes in pupils-world conceptions.

And finally a remarkable association between scoring and confidence was also found ,which indicated us that confidence is not always related to the adequate way of responding.

4. Study of the OEQ responses.

4.1.-Networks and assigned codes to answer categories.

The present networks were created under the scheme of BLISS et al.(1983) and they are shown below. The title of each studied question is followed by an outline for each network,for both questions No.1 and No.5(open-ended ones):

FIRST QUESTION WAS:"Sometimes,when camping your tent appears wet in the morning.Where do you think the water came from?".

Principal headings of the conceptual network included:

TITLE	CODE	No.PUPILS USING IT
-WAY OF ANSWERING:		
-NO ANSWER	1.1	9
-RATHER WELL	1.2	13
-ACCEPTABLY	1.3	27
-POORLY	1.4	54

-SOURCES OF HUMIDITY:		
-FROM THE SOIL	1.5	5
-FROM THE AIR	1.6	37
-FOG AND CLOUDS	1.7	6
-FROM PLANTS	1.8	7
-NONE OF THOSE	1.9	48
-TIME ALLOCATION:		
-EARLY MORNING	1.10	19
-BY NIGHT	1.11	42
-NOT AVAILABLE	1.12	42
-KIND OF PROCESS:		
CONCRETE (With no sequences) :		
-NAMING (Only)THE DEW-POINT	1.13	0
-FALLING (Like Raining)	1.14	1
-COOLING	1.15	7
CONCRETE (With some sequences) :		
-USING THE IDEA OF DEW-PONT	1.16	27
-INCORRECT CONDENSATION EXPLANATION	1.17	16
FORMAL :		
-CHANGES OF STATE	1.18	44
-VAPORIZATION	1.19	7
-CONDENSATION	1.20	33
IRRELEVANT:		
-NOT INTERESTING AT ALL	1.21	3
-FALSE IDEAS ABOUT CHANGES OF STATE	1.22	3
-TERMINOLOGY:		
-VERBAL	1.23	95
-MATHEMATICAL	1.24	8

AND FIFTH QUESTION WAS:"Describe your sensation when you put some cologne or alcohol on your hands.Then,explain what you think happens?".

Principal headings of the conceptual network included:

TITLE	CODE	No.PUPILS USING IT
-TEMPORAL LENGTH		
-IN A VERY SHORT TIME	5.1	12
-NOT INDICATED	5.2	91
-SENSATION (Feeling)		
-COOLING RELATED	5.3	78
-HEAT(Biology Answ.)	5.4	12
-WITHOUT SENSATION	5.5	13
-PROCESS(Including reactions) :		
-CHEMICAL REACTION	5.6	3
-OTHER KINDS OF REACTION	5.7	2
-TEMPERAT. DIFFERENCES	5.8	16
-HEAT PRDTN.or INTERCHG.	5.9	1
-ALCOHOLIC DEGREE RELATED	5.10	50
-WITHOUT INTEREST	5.11	4
-CHANGE OF STATE IDEAS	5.12	27

4.2 Sorting of pupil answers and fitting them into different models.

Up to this point the research about the open-ended questions has been carried out following the network analysis techniques as described in the literature (BLISS et al.(1983) and SOLOMON et al.(1985)).From now it will be necessary to compact the available information.To do so,the following has been proposed. Considering the available information on the use of different kinds of answers,after the coding operation made as a result of the network analysis the expected answer pattern for outstanding responses should include the use of certain codes: 1.6,1.10 or 1.11 and 1.18 or 1.19 or 1.20 in Question No.1 and 5.1,5.3,5.9 or 5.12 in Question No.5.This grade of excellence was not achieved by any pupil.So,new sorting of answers into models was prepared as shown in the Table No.2, where "T"=Total No.of pupils per Model,and "S"= No.of pupils scoring at least 7 in the MCQ.

Note that quotient S/T , except for fifth model, has been calculated.

From all these groups, the group formed by pupils with unexpected answers it is especially interesting for the aims of this research.

Table No.2:

MODEL NUMBER	COMBINATION OF CODES USED	T= TOTAL PUPILS by GROUP	S=SCORING 7 or MORE IN MCQ	QUOTIENT S/T (%)
1	1.6 and(1.10 or 1.11)and 5.3,and 5.9 and 5.12	9	6	67
2	1.2	9	5	56
3	1.3	25	10	40
4	NONE OF THE ABOVE	49	8	17
5	(1.5 or 1.8)and(5.8 or 5.10)	11	5	--

NAME OF EACH MODEL: 1=ADVANCED, 2=RATHER GOOD

3=ACCEPTABLE, 4=POOR ANSWERS, 5=UNEXPECTED COMB.

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5.Relations MCQ to OEQ.

-Pupils have been classified upon answers fitting to five models. In the first two models a good association between the use of the typical codes of each model and the variable Q (which takes the value Q=1 if a pupil scores 7 or more for the 11 MCQ,and Q=0 for scores under 7)was found,after a crosstabulation studying its Chi-square values.In the third model the association is much smaller,and in the fourth it's high again, but negative.

-As indicated in Table No.1 similar "r" values have been obtained in the PCB calculations for each question. Nine out of the eleven values are ranging between .65 and .71 which are acceptable considering the nature of the test (a research one instead of an achievement one).

-A strong association exists between assigned model and obtained total score in MCQ.

And, as a general trend, homogeneity is obtained between MCQ score and sorting into models according to the use of codes in the open-ended questions.

-Conceptual networks are not exempt from ambiguity. The chosen version of the proposed network of each studied question can be constantly modified and improved, but they summarize for us the information obtained in the open-ended questions and allow us to continue our search for a hidden knowledge structure.

-Quotient S/T ,shown in Table No.2, appears to be a good indicator of the association between model assignment and the achieved score. Their values decrease from 67% for model No.1, to 17%. for model No.4.

-Quotients S/M (where S=No.of pupils scoring at least seven in the MCQ, and M=No.of pupils in each model)were calculated for every question and for the four first models: As a general trend coeficients related to the same question decrease from first model to fourth.Although some irregularity was found which reinforces the idea that pupils' ideas are not so easy to foresee as they seem from the science teacher point of view .

-The sorting presented in this work is an immediate tool which helps easily to obtain a quantification of qualitative answers ; which comparison was really difficult.

6.Reflections on the questionaire design.

Some methodological issues related to the valuation of the applied techniques for data collection and their analytical study have been appeared as an interesting complementary result of this work. A main question arises from this work. Are both kinds of questions (MCQ and OEQ) really worthy?

Effectively both have been widely useful:

-Thanks to MCQ qualitative analysis it has been possible to confirm that the concepts,relationships and hierarchies on changes of state for childrens are not directly related to those for science teachers.

785

786

-With the OEQ analysis an impressive wealthy world of some basic alternative concepts related to changes of state of matter that pupils held has been discovered.

It is quite useful to start a comprehension of the learning difficulties for concepts which are not so obvious for the science teacher who usually ignores these alternative hierarchies and relationships before studying them.

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APPENDIX I.

ENGLISH VERSION OF THE QUESTIONS OF THE DISTRIBUTED TEST

1)Sometimes,when camping your tent appears wet in the morning.Where do you think the water came from ?

2)You have a jar with a tightly screwed on lid,with a small volume of liquid inside. You heat it and the liquid changes completely to vapor.What will be the vapor volume?

A)The same as the liquid B)It will occupy the whole jar

C)The same as the liquid but only in the higher part of the jar. D)Double volume than the initial liquid volume

3)A bus leaves a garage,and begins to run on a sunny day. What do you think will happen to its tires?

A)The tires are heated and the mass will grow.

B)The tires are heated,but the mass will be constant.

C)The air mass in the tires are greater than when they are cold

D)The tires are not heated,and the mass will be constant.

4) A ping-pong ball has just been flattened.Which of the following proposals do you think will be the most adequate to unflatten it?

A)Put it quickly in dry ice.

B)Put it in boiling water.

C)Put it in the freezer

D)Leave it as an useless ball.

5)Describe your sensation when you put some cologne or alcohol on your hands.Then,explain what you think happens?"

6)Imagine you've just arrived at your winter vacation resort in a little mountain village.The weather forecast indicates that the night temperature will reach 20 degrees below zero. Your friends car will need some protection for the radiator. Which of the following products do you recomend to add to the radiator water to avoid congelation ?

A) Oxigeneated water B)Kitchen salt

C)Vinegar D)Sugar

7)Where do you think that the ice on the freezer walls comes from?

A)From the food humidity.

B)The water that remains,after preparing ice cubes, on the floor of the freezer.

C)Water coming from the pipes inside the freezer

D)From humidity in the air which enters the freezer when you open the door.

8)Given two figures I and II,the variation of temperature is represented against the required time for heating with a heater: a piece of ice at -15°C until it reaches 120°C in a well closed jar.Which is the correct one? (Figure I did show a continuous curve line and figure II did show several straight lines growing up from -15°C until 120°C, with horizontal fragments at 0°C and 100°C)

A) Figure I B)Figure II

C)There will be horizontal lines in figure II,only by turning-off the heater D)Both are all right.

9)A small jar is filled with ice.The lid is screwed on tightly and the outside wall of the glass is dried with a tea towel. Some minutes later the outside wall of the jar is all wet. Where has the water come from?

A)From melting ice coming through the glass

B)From cold which has converted into water

C)Water already existing in the air sticking to the cold glass

D)The towel did not dry the water enough

10)You have two pans, A and B,in two identical heaters. They have the same ammount of potatoes,water and salt ;and they are covered by water during the whole experiment.Selected heater positions are the same.When "A" and "B" start boiling,the "B" heater is switched to a "minimum" position and "A" isn't.

Please,indicate which potatoes are cooked at a greater temperature:

A)The ones from pan "A"

B)The ones from pan "B"

C)Both are the same

D)The ones from "B" will need a greater time.

11)Fog is formed by:

A)Boiling water in rivers

B)Vaporized water in rivers

C)Condensation of humid air,after strong cooling

D)Vaporized water from mountain snow

12)

12.1 In which of the following months,does the sea water temperature at the Barcelona beaches reach its highest values?

A)June B)October C)January D)October and June

12.2 Please indicate the atmospheric preasure in MEXICO,D.F on a sunny day.

12.3 Please indicate the distance from Barcelona to New York.

12.4 Please indicate the mass of an official soccer ball.

13)In winter on a very cold and wet day you wish to dry some clothes.Where would they dry better: In the open-air or in the bath-room?

A)It's the same.Temperature doesn't matter!

B)In the open-air

C)In the bath-room

D)There is nothing to do.You must wait for a better day.

14)Clouds are formed by water from sea,rivers and lakes. How does water form these clouds?

A)Through a boiling process thanks to the heat of the Sun.

B)Through boiling processes but only in very hot seas like the Caribbean.

C)Through boiling and vaporization processes.

D)There are not any cloud formation through natural boiling processes.

15)Please, compare what will happen if you puncture two balloons: one filled with water and the other one with air. Please indicate also the reasons of their different behaviour.

Note: Question No.12 was prepared only to see the pupils' estimation capacity

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Pupils' Conception of Matter.
A phenomenographic approach.

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My pupils and I were talking about what would happen if we slightly heated a piece of wood on a stove. One pupil said: *I know that wood can boil.* My reaction to that was: You mean boil like boiling potatoes? He replied: *No, like water.* Another pupil said: *There will be more atoms in it when it is warm.*

I am a teacher with a constructive view of learning and this led me to carry out some research in the EKNA -group with Dr Björn Andersson on pupils' perception of boiling (Andersson, Renström, 1979) and heating (Andersson, Renström, 1981). This background did however not help me in the described situation. The more I found out about pupils' conceptions concerning a special area, the harder it was to teach. In the teaching situations I recognized conceptions in what the pupils said, but this recognition alone did not help me organize better teaching contexts or devise strategies for teaching. I needed something more.

Even if the pupils are taught a great deal about atoms and molecules, they do not use these concepts to solve chemistry problems. If they do, they do it in a «wrong« way, as the pupil mentioned above *There will be more atoms in it when it is warm.* Here is another example. One of my pupils, an average boy in the eighth grade, who has been taught chemistry for more than a year and a half, asked me *What is an atom actually?* What can you do for a pupil who obviously wants to know about atoms when you already have tried all the variations and strategies you are familiar with?

To be an efficient chemistry teacher, I needed to know what relationship, if any, exists between different conceptions. It was not enough to know about existing conceptions: I needed some kind of structure or hierarchical model showing the relationship between the conceptions. As the literature on the subject was limited, it was necessary to construct a new model.

Chemistry deals with what various substances are made of and what happens when they are mixed together. Therefore I had to find out what conceptions the pupils had about various substances, «matter«, and in what ways the conceptions might be related.

Two things made me certain that there must be a structural relationship among different conceptions. First, I come to an understanding concerning chemistry and hopefully some of my pupils, who now have these strange conceptions, will reach the same understanding some day. Second, some definitions of learning, which have been introduced by a research group at my department in Göteborg and will soon lead us to the subtitle of this paper: A phenomenographic approach, have become a part of my thinking about conceptions. Characteristic for phenomenography is that when the reseacher is «pinning down« a certain way of understanding a phenomenon, they are looking for the most distinctive characteristics of the conceptualization and is aiming at a **structural description** (Marton, 1986). In phenomenography the objects of study are of a relational character; i.e. a **statement is seen as a relationship between the individual and aspects of the world around him/her.** A phenomenographer is trying to describe **aspects of the world as they appear to the pupils and not how they are in fact.** This important distinction is made by Marton (1978, 1981) and means that the phenomenographers have adopted a **second-order perspective**.

We will soon go deeper into the phenomenographic definitions. First I would like to make some comments about chemistry teaching as I have known it. Once I was the one, who was looking at my teachers, trying to understand what they were saying and not being able to ask questions. Now, I was the teacher and I was exposing my own pupils to situations similar to the ones I had often found uncomfortable. Even if I was a teacher of my time and had the pupils' own thinking as a point of departure for teaching, I could see myself in my pupils. Furthermore, even if I had come to an understanding with the subject and become a chemistry teacher, I once more found myself to be the one, who did not understand. I did not understand the pupil who said: *I know that wood can boil.*

Today I believe I know how to ask the next question and today I have ideas about the relationship between conceptions, including the conceptions behind the two answers *I know that wood can boil* and *There will be more atoms in it when it is warm.*

The research problem

My research concerned the following question:

How do the pupils conceptualize matter?

This question was essential because I had to be sure that it would cover as many aspects of thinking about matter as possible. Otherwise I could not search for the relationship between the conceptions. This question was divided into three parts.

1. The first concerns how the pupils conceptualize various substances such as a grain of salt, water in a cup, a piece of iron, oil in a cup etc. and the present phase of the substances.

How do the pupils conceptualize what various substances are made of ?

After the pupil has had an opportunity to recognize the substance X, I asked questions about the substance in the following way:

- X yes, what is X?
- What do think X is made of?
- Can you draw it?
- Suppose we have a machine that enlarged X so that we could see how X is built up, what would we see?

2. To reach a deeper understanding of how the pupils conceptualize these questions, the second part of the question is

How do the pupils conceptualize the substance X, when X is

a/ divided,

b/ heated (first a little and then very much),

c/ cooled, and

d/ interacting with other substances.

Of special interest here is of course, if the pupils need the atom concept or not, when they talk about the substances. If they do, in what way do they use the concept?

3. The third part of the question is

How do the pupils express the atom concept?

The interviews

Information about conceptions is usually obtained through interviews. The interview method I have used is Piaget's revised clinical interview method (Piaget, Szeminska, 1952). Regarding the concrete material on which the conversations were based, I had the following requests:

1/ that the pupils had confronted the substances of which the concrete material consisted, and

2/ that these substances had been partly studied previously.

The pupils were as far as possible interviewed about at least two, but mostly three substances in the solid, liquid and gas phases.

One substance at a time was placed in front of the pupil. The first question put to the pupil was a very ordinary one: "Do you know what this is?"

Only one interview with the pupil. I look upon every pupil as being an owner of a certain number of concepts about matter. Through such concepts the pupil is able to express his or her view of various substances. Because individuals are always active minded, is the relationship individual - world also repeatedly exposed to development concerning the concrete materials chosen. My general idea was, that if the pupil received several opportunities to show as much as possible of his/her concepts during one interview, I would have more information from which to construct an analysis of their view of matter. To reach several contexts I changed the substance discussed just a little. In these situations I challenged the pupil's ideas, the conceptions, and therefore I call them **the challenges**.

I wish to describe what the pupils express. For this reason I have not mentioned or asked about a concept before the pupil has mentioned it. For example I have not asked questions like: -What is «matter«? -What is a solid substance? or -What is a liquid? If the pupil used a concept such as liquid, I asked. -You said «liquid«. What do you mean by liquid?

It should be mentioned here that all the pupils knew that I was a teacher of physics and chemistry, and that the interviews were to find out their particular ideas and points of view. They therefore knew that every pupil's ideas were of the greatest importance to me.

Carrying out the interviews. Twenty pupils were interviewed at the end of the semester in May, 1983. Seventh-graders had one year of study chemistry, the eighth-graders had two years and the ninth-graders three years. The pupils were chosen randomly. The interviewed pupils were distributed in different grades as follows:

Grade	Number pupils	Boy = B Girl = G
9	10	3 B + 7 G
8	6	5 B + 1 G
7	4	4 B

The concrete substances were:

Solid sustances	Liquids	Gases
Salt	Water [2]	Air (in the room)
Iron [1]	Oil [2]	Oxygen [3]
Aluminium [1]		Carbon dioxide[3]
Wood [1]		

1 Thin pieces 5x10x0,3 cm.
2 In plastic cups ca 20 ml.
3 In tubes.

The actual interview. At each interview I started by spreading some grains of salt on the table in front of the pupil, after which I asked: Do you know what this is? Together we drew an enlarged picture of a grain and I posed questions about the picture.

- What is salt made of?
- How do you imagine salt?
- How do you think it is built up?
- Imagine, we had a piece of machinery that made it possible to enlarge the salt still more ... what would we see?
- What is it you have drawn here?
- What is in here (inside and around)?

Then, with my nail, I crushed the grain of salt and asked the following:

- Can you see?
- What happened to the salt?
- You made a drawing like that? What is really happening?
- Could you describe it?
- If I choose one of the small pieces here and crush/break it and then I choose one of those small pieces ... how long can I keep on going?
- Is there any piece I cannot crush?

Another grain was chosen as an object of our study. Sometimes the pupil described that one as well. This description has been valuable to the analysis. The pupil was asked:

- What do you think will happen if we put this on a stove and heat it up just a little ... to about sixty degrees (60° C)?
- You said warmer .. what does that mean?
- What is the difference between the salt that has been heated and the salt that has not?
- You draw a picture like this ... does anything happen to that?
- What happens if it is heated even more?
- Can you draw a picture?

Depending on how the pupil answered the questions above, the following were determinded:

1/ nothing happens,
2/ the salt melts, or
3/ it burns.

Usually I repeated what the pupil had said. Sometimes I said it in other words, such as:

- You said melt ...
- Could you draw what you told me?
- How do you imagine .... melting?
- Burning ... what really happens?
- Flammable substance .. what do you mean?

After salt I continued with water. Generally, I put the same questions as I did when salt was discussed. I started from the beginning as if I had forgotten what the pupil said earlier. They were also asked to describe what they thought would happen if water was cooled down and then moved to a freezer.

When the pupils had discussed both salt and water separately, the natural consequence was to put them together. We did that in the following way:

- You said that salt looks like this (the drawing, the pupil has now the opportunity to change his mind about salt - a challenge.)
- What happens to the salt?
- You said water was like that ..(a challenge)
- What happens to the water?
- Salt water? ... What is that like?

Afterwards we continued with the other substances iron, aluminium, air, wood, oil, oxygen and carbon dioxide. Similar questions were asked about all the substances.

All the interviews were done at the pupil's own school. Each interview took around 40 minutes. Every interview consisted of roughly 30 problems. The interviews were all tape recorded and transcribed completely. Attached to the interview protocols are also the drawings and my notes.

From qualitative to contextual analysis

According to phenomenography, there are only a certain number of ways of expressing one phenomenon in a society; i.e. "**each phenomenon, concept or principle can be understood in a limited number of qualitatively different ways**" (Marton, 1986). This implies that every phenomenon has a limited **outcome space**, within which the researcher can find possible thoughts about the phenomenon.

A **qualitative analysis** of how an individual has conceptualized a certain phenomenon implies that the researcher works on interpreting the meaning, the conception which is manifested in a statement. According to Svensson and Theman (1983), a qualitative analysis should result in the researcher identifying conceptions and categories of conception, which have the attribute that they represent a selection as well as a summary and an organization of the content of the statements.

In a conversation individuals have to make a large number of quick assumptions about the meaning in order to reach understanding during the conversation. An orientation towards understanding is therefore continuous. We strive for the meaning and therefore we tend to smooth over all shades of meaning in what is expressed. This can constitute a «cradle« for misunderstanding if we freeze statements and analyse them detached from their context (Theman,1985). In scientific analysis, the small shades of meaning are those, which can help us to reach more differential interpretations of the meanings. For this reason, Svensson and Theman (op cit) consider that the researcher, when making his analysis, should keep the statement in a complex context in order to be able to support interpretations and conclusions more convincingly. This will provide us with an interpretation on a deeper level. This interpretation of a phenomenon in a second-order perspective is **contextual** (Svensson, 1979 and 1985).

When you are looking for the most distinctive, structural aspects of the relation between the pupil and the phenomenon in a contextual way, you will end up with **categories of description**. Marton says "categories of description which, though originating from a contextual understanding (interpretation), are decontextualized and hence possible to use in other contexts than in one in which they have been arrived at" (1986, op cit p 9).

The amount of information concerning a given phenomenon, which is «pinned down«, through the contextual analysis in the categories of description, is also a tool, a part of the contextual analysis. With that as a point of departure, it is possible to interpret statements, even those with a poor vocabulary.

What is a result? My four distinctions

Before my description of the result of this investigation I would like to make clear what a result, categories of description, means for my study and how it can best be presented.

I would like to describe my four analysis stages through my own relation to the content of the four stages and how I have formed my «concepts«; i e. the result of the investigation. My relation to my problem is varying between what Svensson describes as an atomistic and a holistic approach to learning.

With these concepts Svensson (1976) expresses differences in **skill** concerning **how** his students understand a text that they have read. He is expressing this aspect in terms of a relation between the students' understanding of the text and the author's intention with the text.

The first distinction: Identification of conceptions and categories of conception

According to phenomenography, there is **one** conception manifested in **one** statement. What I had to do first was to search for and identify possible conceptions and list them in categories of conception.

The statements which form the basis of my very first analysis, a qualitative analysis, are the pupils' descriptions of various substances, a fourth of all statements. A qualitative analysis is not at the same level as a contextual analysis. This implies that not even the results will reach the same depth as the results of the contextual analysis. However, I see this introductory qualitative analysis and the distinction that it led to pave the way for the coming contextual analysis.

The focus of the analysis has been the content of the statements; i.e. in phenomenographic terms the **what**. In order to penetrate and explicate the pupils' thoughts reflected in their statements, I have used a method common in phenomenological psychology and the group around Amadeo Giorgi (1975), (Alexandersson, 1981), namely to write what the pupils have expressed in other words a couple of times. This procedure takes time, but during this writing exercises understanding of typical features of different conceptions will successively be built up.

My ideas about possible conceptions became more and more definite. My awareness of the possible conceptions expressed by statements finally took over. The identified conception was now related to the content of the statement. My work had changed character to what I want to describe as a holistic approach, but it was still at the same level since I was still forming the same «concept«.

This is called the first distinction.

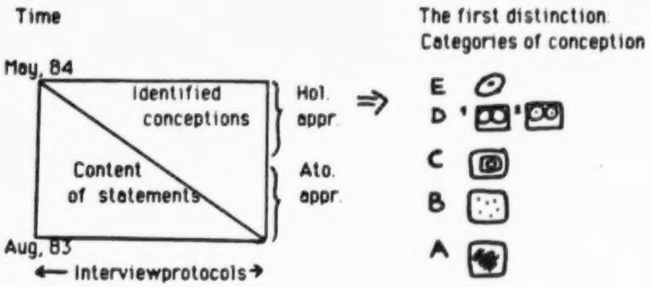


Figure 1: The conceptions have become a «concept«

The categories of conception could now be arranged according to my understanding of what conception was most or less advanced.

To this analysis period I would like to add the qualitative analysis of the pupils' answers to the question **What is an atom?** Before I describe the categories of conception and the categories of atom concept, I want to present the whole research process so that this first distinction can be seen in a broader prespective.

The second distinction: Qualitatively different «whats« are delimited in categories of description

The second distinction was derived from the phenomenographic definition of a conception: **a conception is an entirety consisting of parts** (Svensson, 1984). Now I had to «pin down« and delimit these qualitatively different entireties and their parts. In phenomenography a conception is seen as a result of **learning,** and a sign of **learning** is a changed conception. Therefore a phenomenographer is not only looking for a relation between the entireties but also for a relation between the parts (Johansson, Marton, Svensson, 1984).

All statements were now involved in the analysis which had a contextual character. Statements expressing one conception were compared with statements expressing close conceptions. Similarities and differences concerning the «what« of the conceptions appeared more and more. The identified conceptions could be delimited step by step and be «pinned down« as entireties consisting of parts.

My relation to the demand, the delimiting of the entireties and their parts, can be described in the same way as the work with the first distinction. In the diagrams below a model of categories of description is also included.

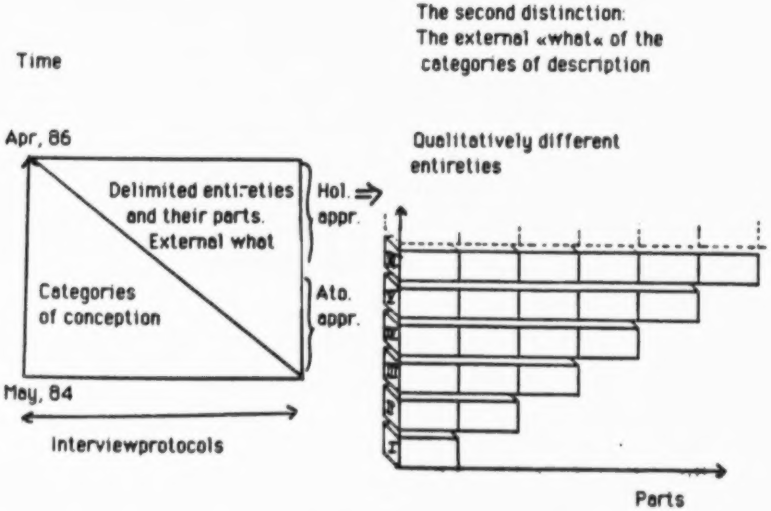

The second distinction:
The external «what« of the categories of description
Time
Apr, 86
Delimited entireties and their parts. External what
Categories of conception
May, 84
Interviewprotocols
Hol. appr.
Ato. appr.
Qualitatively different entireties
Parts

Figure 2: The delimited entireties and their parts are «pinned down« and can now be described as the external «what« of the categories of description.

The distinction this analysis led to is called the "matter-staircase-house" according to the form of the description and how it was vanquished, step by step. The outcome space is seen as a «house«, something the pupils are in.

Of particular interest here is the new part in each category of description. The structure of that part affects all the other parts and therefore characterizes the whole entirety. The description of an entirety and its parts can be seen as the underlying structure of the conception, a structure which directs what the pupil expresses in a statement in one way or another

The third distinction: «How« the pupils think about the «what«

Different individuals' statements are not alike even if they manifest the same conception and are an expression of the same underlying structure. The **how**-aspect which illuminates the way someone conceptualizes a phenomenon is a variable in each category of description. The way someone has conceptualized a «what« is seen indirectly in **how** the **what** is conceptualized. The variations in «what« which are described in the matter-staircase-house can therefore further be differentiated and refined through a new contextual analysis. I could now describe the **internal what** in each category of description through the how-aspect.

This time the reading was directed towards **how** the pupil thinks about the «what«. My research process can be described as follows: My relation to the demand, the how-aspect or the skill with which the pupil thinks about the «internal what«.

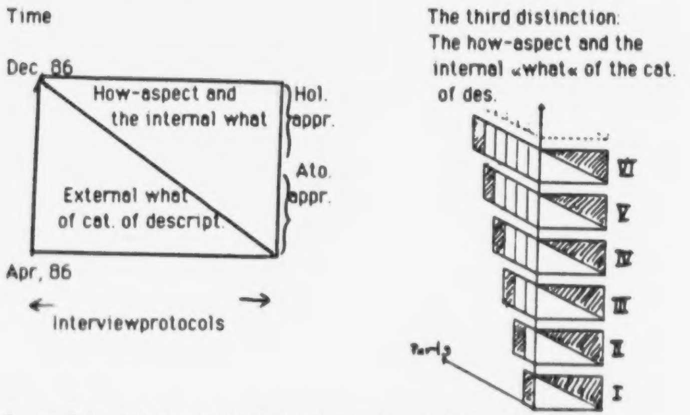

Time
Dec. 86
How-aspect and the internal what
Hol. appr.
Ato. appr.
External what of cat. of descript.
Apr, 86
interviewprotocols
The third distinction: The how-aspect and the internal «what« of the cat. of des.
VI
V
IV
III
II
I

Figure 3. My relation to the demand and the model of distinction

I was now describing the pupils' skill in thinking of a particular underlying structure. The similarities and differences which appeared after reading about a hundred statements of the same structure can be described best with the learning approach the pupil expresses. I could relate «how« the pupil expresses the «what« to my knowledge of the entirety, i.e. the category of description. The described entirety corresponded to the author's intention in Svensson's study. I saw each entirety as they are described in the matter-staircase-house as the potential of each conception.

If the pupil expresses an atomistic approach he/she will start off focusing on and stressing a special «what« and then switch over to talk about another «what«. For a conversation partner or an interpreter the last «what« can appear as if the pupil has a lack of concentration or as if the pupil just invented something for the sake of having something to say. If the pupil is asked about that «what« he/she will answer *I do not know*. According to Svensson it is characteristic of the atomistic approach that one cannot relate the first «what« to the second. For this reason I have analyzed **when, where and how** a pupil says *I do not know*, and this has led to important information.

A pupil who expresses a holistic approach does not have to say *I do not know* because he/she can relate their first «what« to the second, which enables them to vary their two «what«. The interesting thing here is that the relationship between the «what« focussed on and stressed by a pupil who expresses an atomistic approach and his second «what« is reversed by a pupil with a holistic approach. If the concept **figure** is used for the «what« that was focussed on and stressed first and **ground** for the second «what«, we can see that the «what« which is figure for an atomist is ground for a holist and vice versa. Both figure and ground are in a way present in the learner's consciousness (Marton, 1986).

The wide range of statements which expresses the same underlying structure makes it possible for me to arrange the statements according to the emphasis laid on the «figures«. Let me take an example. Suppose that we have four statements which all express the same conception. Statements 1 and 2 express an atomistic approach of learning and statements 3 and 4 holistic approach. In statement 1 the pupil emphasizes a figure (fig-at) and he just mentions the ground (gro-at). In statement 2 the pupil starts off with the same figure (fig-at) but he talks more about the ground (gro-at), prompting the interviewer to ask about it. In statement 3 we can see how the figure (fig-ho) is related to the ground (gro-ho), while in statement 4 we can see such an emphasized figure (fig-ho) that the ground is hardly mentioned. A guide to those four statements and how they can be arranged is proposed in the figure below.

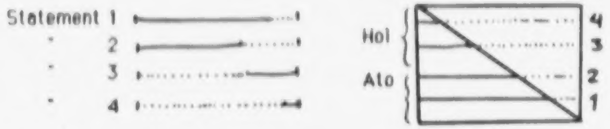


Figure 4. Four statements and how they can be arranged.

With this detailed description I want to emphasize how I have constructed this simple diagram from an analysis of similarities and differences in what can be stressed, in what way it is stressed and how it is related to the unstressed. The diagram is like the diagram I have chosen for my description of the development of my research process. The difference is that the last diagram does not describe **one individual's** development. The diagram does not describe development over time, but as I see it today it would have been pure luck if this had been the case.

One important observation here is as I see it that pupils who express an atomistic approach to learning, within all categories of description, are not able to express a phase aspect or chemical interactions. Changes are described through two different pictures of the substance, static pictures, one picture before the change and the other after. Questions about the «change« are always answered with *I do not know.* If the pupil has adopted a holistic approach, the pupil can express changes, phase aspects and chemical reactions through choosing a suitable variation of the relation between the figure and the ground.

The fourth distinction: Relation between parallel conceptions of the categories of description

It was possible to make one more distinction in a contextual analysis if we focus on the similarities and differences in the existing «figures« of the categories of description. According to the terms of phenomenography, we are now looking at the «how« and the «internal what« in an external perspective (Marton, 1986), which is the demand in my last analysis. My research process can be described as in the figure below.

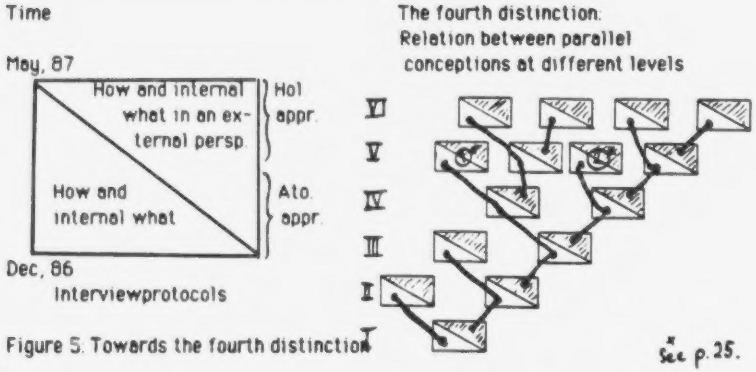

Time
May, 87
How and internal what in an ex- ternal persp.
Hol appr.
How and internal what
Ato. appr.
Dec, 86
Interviewprotocols
The fourth distinction: Relation between parallel conceptions at different levels
VI
V
IV
III
II
I
I

Figure 5. Towards the fourth distinction

*See p. 25.

If you look back at the third distinction you will see that an atomistic figure is quite different from a holistic one in each category of description. The conclusion I can draw from the similarities in the stressed «what«, the figures, which appear in an analysis of statements from closely related categories of description is that it is possible for a learner to leave a conception and constitute a new emphasizing either the atomistic figure or the holistic figure. Even already at level II I can track variations of meanings. These variations within a category of description are called **parallel conceptions** because they express the same underlying structure.

Of the four distinctions the fourth is the one that corresponds to the knowledge which I asked for from the beginning. It is also the distinction whose description has to be further refined. I am only describing the most obvious similarities. In order to describe the relationship in a satisfactory way, it will be necessary for the interviewer to be familiar with the second and third distinctions.

In this paper I want to present the first distinction and the second distinction.

The identified conceptions of matter

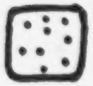
The first distinction

The substance is conceptualized as

A. a continuum

The substance **exists**. The pupil cannot say anything about the substance or how it is possible to recognize it. The substance is formed in one way as a homogeneous mass. One has to learn what substances look like and learn their names.

B. a continuum with atoms



There will be atoms in the homogeneous mass. The atoms could be made of the substance (water atoms in water), or they could be atoms that are the same in all substances.

C. a dishomogeneous unit

The pupil creates a delimited **unit**, which can have a shell, film, peel or skin and some kind of nucleus. In this unit there can be «taste«, atoms etc. The units can be of various sizes. Sometimes they are formed as layers.

D. consisting of particles which are

1/ of a continuum

The substance is now **in the particles**. There could be another substance between the particles.

2/ like atoms but made of a continuum

The particles of all substances are often drawn in the same way. Names and attributes belonging to the atomconcept are used.

E. consisting of atoms

The pupil knows what atoms and molecules are, what elements and chemical compounds are.

Despite repetitive analysis of many different statements concerning descriptions of substances, which express the same conception, I could not gain any information about how pupils conceptualize the present phase of the substances. Regardless of the form, solid, liquid or gas, the substance is conceptualized in the same way. To put together statements to match the actual phase of substances and look for phase-specific things, I made an analysis based on my own thinking pattern about the three phases of substances.

With surprise I realized that even pupils who expressed the conception E - The substance consists of atoms - could not say what «phase« was. One pupil drew a picture of water and steam (see below) with the comment *"but I have never understood what makes that water and that steam"*

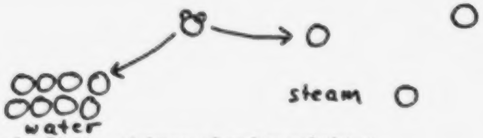


Figure 6: «Static« pictures of water and steam

Joseph Nussbaum (1985), who has also presented a category system within the same area, and Helga Pfundt (1981) stress that matter often is conceptualized as a continuum and that the matter is **«static«**. That matter is conceptualized as being static is very true. Not being able to find anything specific for phases became a real problem to me. In order not to get stuck, I started to tabulate

1/ the pupil' conceptions of the various substances

2/ the number of expressions of each conception, and

3/ the number of expressions per substance and category.

The tables did not help me further. They are interesting and I am glad I have them, because they provide a good picture of **the pupil**.

Finally I analyzed the pupils' answers to the question **What is an atom?**

The four conceptions of the atom concept

A. No conception

The pupil does not know what an atom is. After three years of school chemistry one pupil said: *I don't know how to draw that. I don't remember anything about it.*

B. Atoms are small «things«

Atoms are very small «things« that you can find in matter. It is the smallest particle **in** a substance and it is very, very small.

C. Atoms consist of atoms

An atom is the smallest **existing** particle and an atom is not **indivisible**. Everything consists of atoms, even an atom. If you divide an atom, the small pieces will be the smallest existing particles and they will now be the atoms.

D. Atom model

In this category I cannot distinguish answers learnt by heart from those of real conviction. The pupils draw the same kind of pictures, a nucleus with electrons in orbit.

Now I had two systems of categories, which I was handling in an "atomistic" way. The first step towards relating the systems was in an analysis, which had a controlling function. I wanted to know if a pupil had expressed a more advanced atom concept in their statements than in their answers to the question What is an atom? That analysis resulted in that two interpretations of 142 were reinterpreted.

In the second analysis I focused on the **content**, the «what«, and which pupil had said this «what«, was disregarded. The first similarities which came into sight were very obvious. Let us look at the symbols of the two category systems.

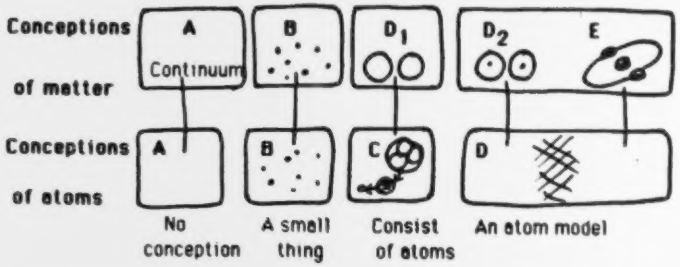


Figure 7. Similarities between the two systems of categories

Category C - The substance is a dishomogeneous unit - has no place in the figure above. Why? The next analysis was therefore focused on all statements without an atom concept; consequently all statements which express the *continuum* conception and the conception *the substance is a dishomogeneous unit* were of greatest interest. I had to find out if the *dishomogeneous unit* was a sign of an atom concept, since the pupils sometimes gave the same name to the parts of the unit as to the parts of the atom.

The analysis was now contextully and the first step towards the second distinction had been taken. Suddenly, the tenth of June, 1984, I was able to delimit the three first **categories of description**. The «unit« was not a sign of an atom concept, but of an embryonic **phase concept**.

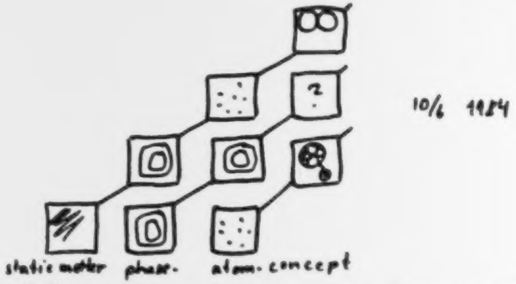
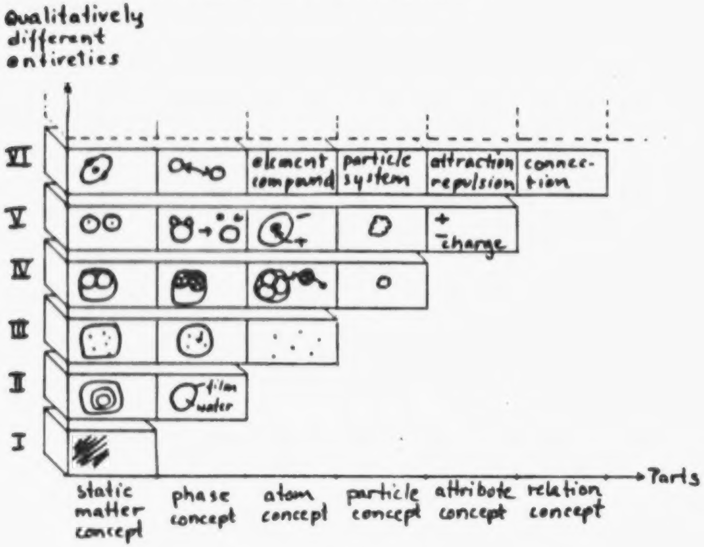


Figure 8: The conception *a dishomogeneous unit* has got a place and we can for the first time see a conception as an entirety consisting of parts.

The most difficult step had been taken. From then and on I «left« the pupils and only focussed on **«what«** they had expressed and **«how«** they had expressed it. For the first time I fully realized what the phenomenographers meant by their refinement of the constructivistic view of learning emphasis on a **constitutional view of learning**. The pupils are caught in a collective way of thinking, in **the existing conceptions**. The pupils are not free to construct their own conceptions, even if the conceptions are sometimes very unusual and seem to be individual (Marton, 1984).

Figure 10. The **six categories of description**



The categories of description

The second distinction: The matter-staircase-house

This presentation of the six categories of description will distinguish each category of description as an entirety consisting of parts. This means that I am now describing the underlying structure in each conception and not the various meanings each structure can give rise to. The categories of description found are all at different levels. Let me start to describe the matter-staircase-house step by step, level by level.

Level I: The substance is conceptualized as a continuum

The pupils are aware that various substances **exist or do not exist**. Each substance has "something" which is "typical" of especially that substance, but the pupils are unable to describe this in words. All thinking activity seems to be used to recognize the substance and keep that substance in mind. For that reason a substance always has to be **in the same way** and its homogeneous mass is therefore not changeable. If a substance were changeable it would be impossible to recognize it.

The only thing I can see that the pupils can say about the substances is that they are divisible and can be put together. This is something a human being can do with the substances. This is a sign of a special man-centered relation to the substances that you can find only at this level. "The self is not separated from the not-self" as Reiser expressed it in 1939 (cited in Marton, 1986).

Because of this man-centered attitude to the substances the pupil is not able to ascribe attributes to a substance, which implies that the pupil cannot conserve matter. It is **we**, human beings, who make decisions about the matter. What substance and how much we have depends on our wish concerning the substance. If we want water **we** are pour a certain amount, and if we want hot water **we** heat it. Every change is caused by **us**, because we are the ones who measure, divide, heat, clean and destroy a substance at a specific moment.

A description of a substance depends on **how** the pupils are able to think about one or two existing substances. In order to describe how wide a category of description can be, I would like to mention here, what is described in detail in the next distinction. If they are able to focus on one substance at a time (atomistic approach), they will accept, for instance, that oil is a substance among others. But if they are able to think of two substances at the same time (holistic approach), we can see that the two substances **water** and **air** can exist as homogeneous masses in all other substances, like a ghost in a wall. That means that oil can be conceptualized as being water and fat in suitable amounts. What a chemist regards as a gas is seen by students as air with a little homogeneous mass of a suitable substance.

If the pupils are able to think of two substances at the same time, they are also able to explain changes. The relation between the two in each other existing substances is change. In hot water there will be water and hot air, more of water than of air. If the water is boiling there will be too much hot air in it and you will see bubbles. Humid air is air with water in it. If there is too much water in the air, it will rain.

It is interesting to note that according to the pupils there are three types of air: **air, hot (warm) air** and **cold air.** Hot air could be the same as **heat** and cold air as **cold**. In this conception it is not possible to ascribe attributes to a substance and air is no exception. Let me present a couple of examples. If you are out on a cold winter day and you breath with your mouth open, your warm air will meet the cold air. When cold and warm air are existing in each other, there will be **water**. One pupil said to me *You can see that every evening on TV when you are watching the weather. When warm air is meeting cold air it will rain.* One girl, Charlotta, explains hot water and ice as in the pictures, as follows:

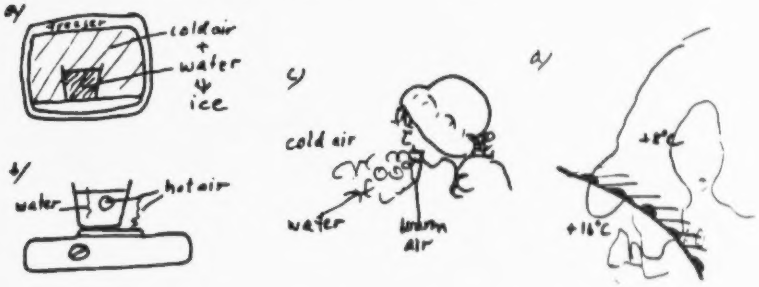


Figure 11. Water and «cold air« in a freezer a/, water and «hot air« on a stove b/, «warm air« and «cold air« meet and form water c/ and d/.

By considering that two suitable substances exist in each other at the same time, the pupil has a remarkably great area of explanation. The pupil can explain phases and phase aspects, and chemical interactions.

Level II: The substance is conceptualized as a dishomogeneous or delimited unit

The outstanding feature of this category of description is that the pupils delimited a substance by creating an appropriate **unit** for the substance. The **unit** is a sign of distinguishing the substance from other substances and the **limit** is the **embryo of the new concept, the phase concept** or the new step in the matter-staircase-house.

A drop of water is often seen as **a unit** which consists of a thin film of water in one «form« and this film contains water in another «form«. Water exists in two «forms«. A grain of salt is a common **unit**. It can have a shell and a nucleus. In the nucleus we can find the «taste« or the saltwater. Salt exists in two «forms«.

I should point out here that the **unit** is not a unit in a physical or a chemical sense. The **unit** is created in a suitable size and it is characteristic of each substance. By creating a **unit** the pupil makes it possible to ascribe attributes to the substance, and furthermore the pupil is now able to **conserve mass.**

Here follows some examples of how pupils can describe substances.

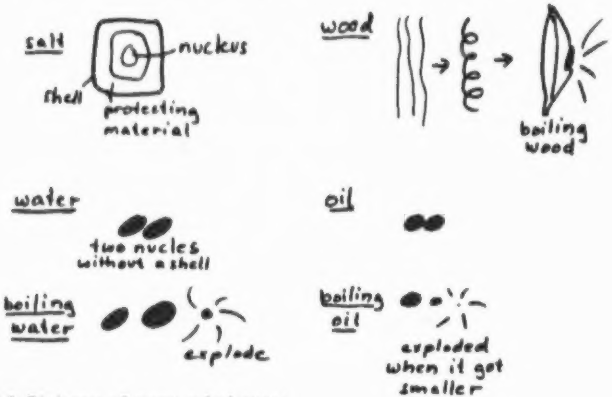


Figure 12: Pictures of some substances.

In spite of the fact that the pupil has constituted a conception which may be regarded as an embryonic phase concept, we can see that pupils think about the two «forms« within the substance in a way which shows they cannot express phases. The phase aspect depends on **how** the pupils think of the two «forms«. I will come back to that in my description of the third distinction.

Level III: The substance is conceptualized as a unit of the substance with "small atoms"

This conception - A **unit** of the substance with "small atoms" - is familiar to many researchers (for example, Nussbaum, Pfundt and others). It is described as **the cake with raisins** or **the plums in a plum-pudding**.

But how the "cake" and the "raisins" are conceptualized separately or related to each other is not well known.

Before I go into this, I would once again stress the phenomenographic definitions that a conception is a sign of the relation between the individual and the world and that a conception is an entirety consisting of parts. The new part here is the **embryo of the atom concept** and this new part **belongs to** the entirety. When the pupils locate atoms in the **unit** of the substance it is a sign of learning. They have left one conception and formed a conception that is new to them, but one that already exists. Other individuals have expressed it before and others will do it after them. The new conception that they have constituted is an important **step** in the development of the matter concept.

In the contextual analysis I can see that the "cake" is not conceptualized as a continuum described as at level I but as at level II. For this reason this conception must also include the aspect **«the unit of the substance«**.

This entirty consists of three parts. This means that when the structure of the new concept is a reality, all the other parts in the entirty will be changed or coloured by this structure. Let me start with the change in the phase concept.

In spite of the fact that the **unit** into which the pupil puts the «small atoms« is of «holistic character«, which means that if the same unit has been expressed at level II, the pupil has been able to explain a phase aspect, we now cannot see anything of that phase aspect attached to the **unit**. Instead, the pupil shows that he/she does not know how to express the phase aspect. The pupil has to learn **how** to express the phase aspect with the constituted conception. I will go further into this in the third distinction, since the phase aspect depends on whether you have an atomistic or a holistic approach.

Here follows some examples of how pupils can conceptualize substances:

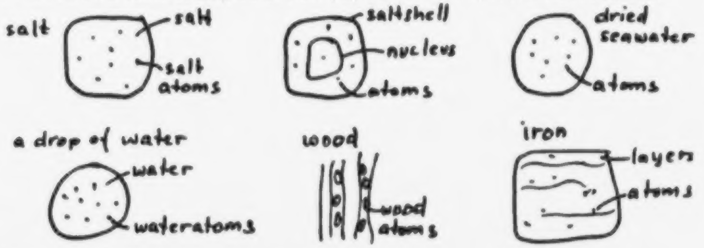


Figure 13: Pictures of some substances.

It is important to mention here that my interpretation of this conception has made me realize the function of the two parts of the **unit**, the shell and its content, to a greater extent. The shell, film, peel, etc, will more and more have the function of **keeping the substance together**. This will still be more obvious at the next level, level IV. A category of description which is soon probably will be split into two categories of descriptions, because of the contextual analysis of the how-aspect. We can namely interpret that the pupils express an **embryo of a binding concept**.

814

Level IV: The substance is conceptualized as a unit of the substance which consists of particles

The new concept in this entirty is a **particle concept**, and its structure colours the meaning of all the other parts in the entirty. It is true that «small atoms« and other small things are called particles, but they are not particles in the sense used here. This particle concept leads not only to a new conception but also to a new type of understanding of matter, namely that **« everything consists of particles «**, i.e. a particulate understanding of matter. As with the earlier levels, this particle concept is also an embryo.

A particle can be named atom or only particle. A particle consists of particles or of atoms, and an atom consists of particles or of atoms.

A substance is still seen as a **unit**, but what the substance is made of is now **in the particles**. Let us look at level III and what the pupils could stress there: the **unit** of the substance or the **small atoms**. Now we can see that the pupils can create particles of either the **unit** of the substance or the **small atoms**. Neither care, the structure of the particle concept is the same.

The border of the **unit** is not involved in the creation of particles. Instead it has the function of keeping the particles together. This is the last category of description in which we see the **unit** and its parts, especially the border.

All thinking activity seems to be used for creating particles and keeping the created particles in mind. The way the pupils think about the particles is close to how they think about a substance at level I. For instance, they cannot ascribe attributes to the particles and the particles have no particular size. The «only« thing they seem to know is that «everything consists of particles« and that is what they express even when they explain phases or interaction. Sometimes the border, the shell, bursts and all the particles fall out. If one of these particles is focused on the pupil seems to conceptualize that particle as a new **unit**, and therefore it is possible to create particles in it. Even the knowledge **an atom is the smallest existing particle** is interpreted from this structure, as shown in the following picture.

815

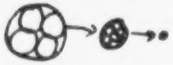


Figure 14. Atoms consists of atoms - otherwise the «smallest« existing particle cannot be an atom.

The underlying structure of this category of description is not very useful. Since it is repetitive in nature, there is no end. There is a constant regeneration of particles. We see that the pupils who conceptualize particles in a similar way are able to put another substance between the particles. This is a sign of a binding concept.

Level V: The substance is conceptualized on the basis of the attributes of the particle (les)

The particle concept has developed in such a way that the pupils are now able to ascribe attributes to the particle, implying that the particle has some kind of structure to reckon with.

The atom concept. Now we can see that the pupil has a special interest in the atom and its subparticles. There is no order of precedence or relation between the different particles. An electron has the same «value« as the atom, which it is a part of.

The «static« matter concept. Here I have found four more prominent ways of expressing a substance, namely based on the attribute of

1/ one particle, which can be named atom, but whose content is like a continuum,

2/ one atom or molecule,

3/ and 4/ two or more particles conceptualized as 1/ or 2/. The pupils can here even ascribe attributes to the binding between the particles.

The phase concept. If the pupils are able to adopt a holistic approach, we can see that the pupils can express phase aspects with the four variations described.

Let me present some examples.

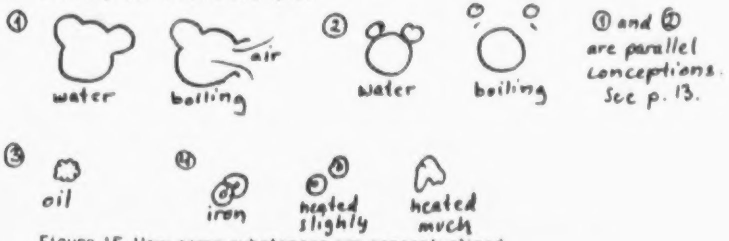


Figure 15. How some substances are conceptualized.

Level VI: The substance is conceptualized as a particle system

In this category of description the new structural aspect is the **relation concept**. The pupils can realize that there are connections, relations between different particles in a system.

The relation concept implies for the

attribute concept: that the pupils are now able to reason about relations between charges. Only now are they able to understand what attraction, repulsion, etc, means, or that the colour of a solution depends on the relation between the particles.

particle concept: that the pupils realize that there are relations between the subparticles in an atom. For the first time we can see what a chemist calls an **element concept**. Likewise, we can see that they realize that there are relations between the subparticles in a molecule, which implies that the pupils have a concept of the **chemical compound.**

«static« matter concept: that they consider that the particles of a substance constitute a system. Between the particles unchangeable «relations« prevail. The pupils' interest in the **space, the interspace or the gap** between the particles is very obvious, which is **a sign of the relation concept**. The interspace is a source of variations, because there can be «nothing«, «emptiness«, «energy«, a «cloud« or a «binding« there.

phase concept: pupils believe that the particles are invariable and that the «nothing«, the «emptiness« is what is changeable. No one of the twenty pupils interviewed was able to change the «nothing« and express a phase aspect.

Implications for teaching

The development of the concept of matter, which I described in the matter-staircase-house, can be compared with the stages that a mealworm goes through in developing to a beetle, or the three forms a substance can exist in.

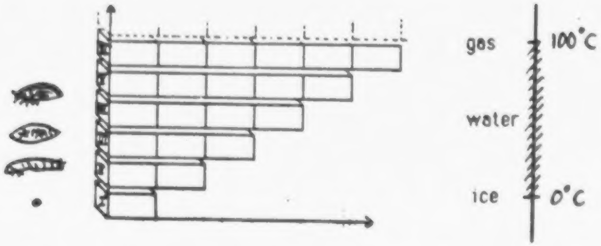


Figure 16. Analogies between the development of the mealworm and the matter concept and between the three forms of a substance and the matter concept.

Even if we talk about the development of concepts, we teach only the «ready-made« concept. In a way we neglect the «development« when we say «not good enough« or «misconceptions« about the result of our pupils' work. If we see the development of a concept as the development of a mealworm, we ought to be happy about every «step« the pupils take.

Suppose we have a hundred mealworms in the second stage, the larval stage. If we look at them closer, we will find that the small creatures are not alike. The differences we see indicate that there is a development within the larval stage. I have around hundred statements at each level, more at level I and fewer at level VI, and I find an internal «what« depending on differences in **how** the «what« can be emphasized in the approach adopted. What I see I interpret as a development within each level.

The other analogy I want to make is a comparison between the three phases of water and the categories of description. Suppose that we have two glasses of water and the water is in liquid form. If we look closer at the water, we will find that there is cold water in one cup and hot water in the other. In a temperature diagram we could point out that the cold water is closer to the ice form than the hot water, and the hot water is closer to the gas form. The internal «what« is similar. There is a variation which we can interpret to mean that some expressions of the same underlying structure are «new« and others are «old«, or that some are closer to one level than another level and so on.

The different signs of development of the matter concept, which I see in the matter-staircase-house, is already a base for my teaching. In a way I am prepared for answers as "*I know that wood can boil*", *There will be more atoms in it when it is warm*", and others.

I see each statement as a "photo" of the knowledge and the skill the pupil possesses the moment the statement arose and something which the pupil can develop according to the described matter-staircase-house. Thus, with an phenomenographic approach I describe knowledge as the "what" and the skill as the "how" in forms of conceptions which I "pin down" in categories of description.

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Misconceptions in Environmental Chemistry Among Norwegian Students

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Introduction

Ideas related to chemicals being synthetic and poisonous substances are widespread am ong the general public. In many European countries the words "chemical" and "chemistry" are almost synonymous with poison and danger (PAOLONI, 1981). Is there any disparity between the student's interpretation of the chemical world in which he lives and that accepted by the chemists? Which common misconceptions on the concepts chemical substances, poisonous compounds, acid rain and air do the students harbour? Do these erroneous beliefs in environmental chemistry persist formal instruction in chemistry at school? Is there any link between identifiable misconceptions and attitudes to science?

Research on students' misconceptions in science constitutes a growing body of recent research in science education. Different methods have been employed from surveys, concept mappings, classroom observations to clinical interviews. In Norway Sjøberg and Lie have analysed students' answers to questions with forced choices and free responses. Their research was on students' understanding of the concepts force and work (LIE & SJØBERG, 1981). Harnæs has performed clinical interviews to identify the misconceptions held by students on temperature and heat (HARNÆS, 1985). The study reported in this paper is part of an international survey with a series of multiple choice questions. The Second International Science Study (SISS) was directed by The International Association for the Evaluation of Educational Achievement (IEA) under the auspices of the Australian Council for Educational Research (KEEVES & ROSIER). The SISS-instruments consisted of a core of international items together with optional national ones, the latter being of spesific interest in this paper. The investigation was carried out in 25 countries in 1983-84.

Purpose of study

The aim of the international SISS-study was to develop a map of science education worldwide and thus expose the teaching and learning practices and the outputs of instruction. Our national items were to a large extent designed to map misconceptions in different areas of science. This paper will deal with some on environmental chemistry.

Method

The 7 multiple choice questions which I shall refer to here constitute only a small part of the complete SISS-study. 6 of the items are national items designed by our group headed by Sjøberg. Each item has 5 possible alternatives. One or more of the distractors were written in order to monitor hypotheses posessed by the research-group as to what might be common misconceptions of the concepts in question. Six of the items belong to a group of anchor items which were administred to more than one student sample. 6500 Norwegian students participated in the SISS-study among those some 1400 students from the 4th grade and the same number from the 9th grade. The third sample consisted of pre-university students in the 12th grade comprising 2400 science students (12S) and 1200 non-science students (12N). The samples were drawn independently, and were representative for their target populations.

I might include some words on science teaching in Norwegian schools. We have compulsaroy schooling from the age of 7 to 16 (grade 1-9) and the students read combined science for a total of 9 periods a week in primary school and a total of 9 in junior high school (grade 7-9). Roughly half of the student-population continues to senior high school (grade 10-12) and some 40 % of these students take the science line.

The 12S-students read separate science subjects. They have studied chemistry, biology and/or physics for 3-10 weekhours per subject the last 2 years at highschool.

Table 1 shows the number of students per population and the accumulated number of periods in science that each population has had of formal instruction from grade 1-12.

Table 1 NUMBER OF STUDENTS IN EACH SAMPLE AND NUMBER OF ACCUMULATED PERIODS IN SCIENCE AT SCHOOL

sample (grade)	notation	no. of students	accum. no periods in science
4th	4	1386	4
9th	9	1490	18
12th non-science	12N	1212	23
12th science	12S	2405	23 + option (8-18)

The test battery in the SISS-study consisted of three questionnaires for each student including 60 scientific items and questions on his attitudes to school science and to science and technology. As for the scientific items the frequencies of the keys and the different distractors are recorded.

Results

The responses to the 7 multiple choice questions on environmental chemistry are given i Table 2 as percentages of students supporting the alternative choices.

Table 2 MULTIPLE-CHOICE ITEMS ON ENVIRONMENTAL CHEMISTRY. STUDENTS IN GRADE 4, 9 AND 12. % OF RESPONDENTS. N=non-science students, S=science students, @=international item

1. WHICH ONE OF THE FOLLOWING SENTENCES EXPRESSES A CORRECT STATEMENT?

4	9	12N	12S		
31 0	20.9	20.2	11.3	A	Chemical substances do not belong in nature, man has made them
13.4	9.3	4.6	1.2	B	Natural substances cannot be harmful
23.4	18.3	11.7	4.3	C	Chemical substances are usually harmful
10.0	27.7	43.7	73.5	D*	Natural substances are also chemical ones
13.4	21.8	18.3	9.1	E	Natural substances are composed of atoms while chemical substances are composed of molecules
8.7	2.0	1.5	0.6.	()	

2. CARROTS ARE SPREAD WITH POISON AND THEN BOILED. MAY THEY BE EATEN?

4	9	12N	12S		
9.4	3.0	2.7	1.8	A	No, the poison is more easily absorbed when the carrots are boiled
10.5	3.8	1.4	0.6	B	No, the vitamins disappear
20.0	41.9	67.8	83.4	C*	No, the stuff may be equally dangerous after boiling
39.1	39.5	25.4	12.2	D	Yes, all poisonous compounds are destroyed when boiled
15.4	10.3	1.7	1.6	E	Yes, but the boiling period must be as long as for boiling instruments in a hospital
5.6	1.5	1.0	0.5	()	

3.● TWO SUBSTANCES ARE COMBINED TO FORM A POISONOUS COMPOUND. WHICH SENTENCE IS CORRECT?

9	12N	12S		
5.7	1.0	0.3	A	Both reactants were poisonous
17.5	14.8	7.1	B	At least one substance was poisonous
8.1	3.2	0.5	C	One substance was poisonous, the other was not
3.7	2.9	0.8	D	Neither of the substances were poisonous
63.2	77.5	90.7	E*	Neither of the substances had to be poisonous
1.7	0.6	0.6	()	

4. WHAT IS AIR?

4	9	12N	12S		
7.0	0.9	0.2	0.1	A	Nothing, vacuum
3.0	0.7	0.2	0.0	B	The same as nitrogen
48.9	38.5	39.8	5.2	C	The same as oxygen
14.2	46.4	50.2	92.4	D*	A mixture of different gases
20.3	12.4	8.6	2.3	E	Invisible water vapour, mixed with oxygen
6.6	0.9	1.0	0.1	()	

5. WHAT IS THE FORMULA OF SULPHURIC ACID?

9	12N	12S		
3.4	4.1	2.8	A	HCl
61.6	46.4	8.4	B	SO_2
2.5	2.0	1.3	C	HNO_3
26.1	42.9	86.8	D*	H_2SO_4
4.2	3.2	0.6	E	$NaCl$
2.0	1.4	0.2	()	

6. WHAT IS CORRECT ABOUT ACID RAIN?

9	12N	12S		
27.2	6.9	6.1	A	There would be no acid rain if all factory smoke was cleaned
19.9	45.3	61.3	B*	Combustion of sulphur-containing substances like charcoal and oil will eventually give acid rain
23.2	27.8	23.9	C	Acid rain is rain or snow containing fine powder of sulphur
21.4	18.0	7.9	D	Acid rain is acid industrial pollution which has evaporated and thereafter precipitated
6.6	0.8	0.4	E	The increasing acid rain in southern Norway has no effect on the water course
1.6	1.2	0.4	()	

7. AN ANIMAL CONSISTS OF MANY ATOMS. WHAT HAPPENS TO THE ATOMS AFTER THE ANIMAL IS DEAD?

9		
15.3	A	They disappear
11.2	B	They stop moving
6.7	C	They are destroyed
50.4	D*	They do not disappear, but are combined to new compounds
15.5	E	They disintegrate and are then combined to other atoms
0.9	()	

Table 3 COMMON MISCONCEPTIONS IN ENVIRONMENTAL CHEMISTRY (% of students in lower and higher grades supporting the ideas) Notation of items given.

a.	1A	Chemical substances do not belong in nature. They are man-made (31-11 %)
b.	1B	Natural substances are not harmful (13-1 %)
c.	1C	Chemical substances are usually harmful (23-4 %)
d.	2D	Poisonous compounds are destroyed when heated (39-12 %)
e.	3B	Poison is an inherent property of a substance, transmitted during a chemical reaction (18-7 %)
f.	4C	Oxygen is the sole component of air (49-5 %)
g.	5B	SO_4 is the formula for sulphuric acid (62-8 %)
h.	6C	Acid rain consists of powder of sulphur (23 %)
i.	7A&C	Atoms are not conserved (22 %)

We have investigated whether there were any relationship between a specific misconception and the students' achievements in science as demonstrated by the average total scores on the science tests in SISS. For all four samples there were some 20-35 % higher scores recorded among the students selecting the key answers and those choosing either of the alternative beliefs. We have also examined whether there was any relationship between students' confidence in science and technology and their choice of alternatives. There were no such differences. On a whole Norwegian students were rather positive to the impact of science and technology on society. As to students interest in chemistry there were a statistical significant difference of 5-10% between a group of students chosing an altenative belief and the group selecting the "correct" answer.

The most striking alternative ideas revealed in Table 2 are listed in Table 3. May these ideas count for misconceptions?

True misconceptions have certain characteristics in common. They are shared by many people, are not in accordance with conceptions held by experts, are resistant to change and are important in the student's belief system. The alternative ideas referred to in Table 3 are all shared by many students. They are definitely not the concepts held by chemists. The ideas may be said to be peristent even though we have not been monitoring the same sample of students of a period of time. Young and old students all share the ideas. As to whether the alternative ideas are imbedded in the students' conceptual ecology and will foster further mistakes and erroneous beliefs, you cannot tell by this multiple-choice study. — In my opinion, what we have disclosed are real misconceptions in environmental chemistry.

Our transversal survey of students of different ages shows that the misconceptions are most preponderate in the younger students indicating that instruction in chemistry has had a (certain) effect and has caused conceptual changes. The older the students are and the more science lessons they have had, the less pervasive are the misconceptions, but some students still stick to them.

Limitations of study

As mentioned earlier, this study on students' understanding of environmental chemistry is only a small part of a comprehensive study on science education. If the intention had been to detect this spesific domain of chemistry various items of similar context should have been administred.

There are several weaknesses of a survey based on multiple-choice questions. The answers cannot possibly reveal the inner thoughts of students. They have not uttered the ideas themselves, only ticked in a box for the most plausible of five alternatives. A characteristic of a good multiple-choice question is that the distractors should be individually exclusive. I feel to-day that we did not pay enough attention to this when structuring the various items. A look at distractor B and C for item no.1 may illustrate what I mean. A student who chooses distractor C -- "Chemical substances are usually harmful"-- most probably thinks of a natural substance as the opposite. He might therefore as well select distractor B -- "Natural substances cannot be harmful". The "real" percentage of students sharing the ideas of either alternative would then be substancially greater than that given in the table. The limitations of multiple-choice questions and the sort of information we can collect from it, has thus been demonstrated. For a more fully description of the students' ideas about a substance and its properties clinical interviews should be performed. A multiple-choice survey may, however, point to specific areas of interest worthy of further studies.

Discussion

The most important misconceptions may not necessarily be the scientific ones but the students' beliefs on science. Every fifth Norwegian student who leaves comprehensive school, believes that chemical substances are manmade and have nothing to do in nature. If we add that another fifth believes that these compounds usually are harmful, (Norwegian) chemistry teachers have an endeavouring job in head of them. Where to start? I often quote "The air you breathe, the food you eat, the clothes you wear, the house you live in -- yourself' -- everything on earth is composed of chemical substances'" The amount of chemicals available from drug stores, super markets, and petrol stations are almost inexhaustible and exhibites an exiting collection. The chemicals can nourish a discussion on poisons, concentration of a substance, pure stuff and mixtures, safety etc. The start of this chemical-shopping should be food (salt, sugar, mineral water). We probably need to "shift the focus from chemist's chemicals to the chemicals of everyday life" (FENSHAM,1984). We do live in a chemical word to the good and bad.

The origin of the misconceptions in environmental chemistry may be traced from the domain of school science itself or may be found in the environment of the student -- the society by large. Take for example the belief that oxygen is the sole component of air. It is shared by some 40% of Norwegian pre-university non-science students. Instead of fostering a conflict situation and thus generating a teaching strategy for conceptual change, I presume many teachers unwillingly add to the reinforement of the students' misconception. A much used phrase in school in order to emphasize the net gas exchange during respiration is "man inhales oxygen and exhales carbon dioxide".

On the other hand the misconception that SO_2 is the formula for sulphuric acid may have developed from society by large. TV and newspapers in Norway report weekly on outlets of sulphur dioxide and acid rain. A link between SO_2 and acid is easy to see how has generated. This reminds us also of Lavosier's concept in the 18th century of an acid being an oxygen-containing compound. Formal instruction in chemistry for grade 8-10 includes formulas of compounds, non-metals as sulphur and acids and bases with the concepts of hydrogenion, pH and indicators. Somehow the teaching does not seem to have had any great influence on the students' conceptions. It has been shown that concepts like hydrogenions and protons are not present in the cognitive structures of many students of the same age as our students (KEMPA, 1983), and conceptual change is likely to take place only if the new concept is intelligible (1 of 4 prerequisites for accomodation) (POSNER et al., 1982).

Conclusion

If we accept Strike's view that misconceptions belong to the student's conceptual ecology and that his set of concepts affects what he later will find comprehensive and reasonable, then it is worth while to analyse the art of the single misconception, to look for the origin of the idea and to adopt teaching strategies leading to conceptual change (STRIKE 1983). I have in this paper pointed out some common misconceptions among Norwegian students on environmental chemistry. A more thourough discussion will later appear in a report from my own institution.

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Misconceptions in Astronomy

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Astronomy is taught in U.S. classrooms as a normal part of elementary school science (K-6), general science and earth science (7-9). In high school (10-12), it is limited to a small part of some physics courses and to astronomy courses taught in about 10% of the nation's schools. At the college level, Astronomy is perhaps the most popular science course for non-science students, fulfilling science requirements without competing with those students majoring in science or engineering.

Efforts are well underway at Project STAR (Science Teaching through its Astronomical Roots) to increase the amount of astronomy taught in grades 11 and 12, by expanding the offering of astronomy courses at those grade levels. It is our belief that the study of Astronomy helps those students who do not choose to take chemistry or physics to learn basic concepts in science and mathematics. Inspired by the work on student misconceptions in physics, we have undertaken a rigorous program of interviews and testing of high school students to determine their astronomical misconceptions. These results have been used to construct curriculum materials which seem to be effective in dispelling many student misconceptions as well as providing a foundation in spatial reasoning and estimation. A major national test of these materials will begin in the fall of 1987.

I. Videotaped Interviews

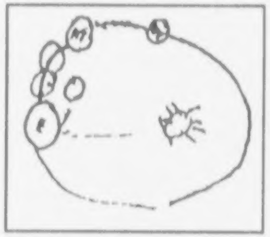
The most rudimentary lessons in astronomy often involve the Earth-Sun-Moon system and the phenomena of day and night, seasons and moon phases. Often these are covered in but a few pages in the text, so that more complex and advanced topics can be covered. We decided to investigate student comprehension of these fundamental topics.

Twenty-five 9th grade students from the local public high school were interviewed in the Spring of 1987. Students were asked to explain these 3 phenomena:

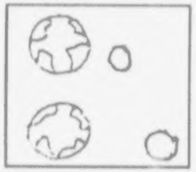
1. Why is it dark at night and light in the day?
2. Why is it hot in summer and cold in winter?
3. Why does the Moon seem to change its shape?

These taped interviews were later examined to create an inventory of students' conceptions. Students were rarely reticent about their explanations and were pleased to explain their ideas with words and drawings.

1. Day and night
   - a. the Earth spins
   - b. the Sun moves around the Earth
   - c. the Moon blocks out the Sun ----->
   - d. the Sun goes out at night
   - e. the atmosphere blocks the Sun at night



2. Winter and Summer
   - a. the axis of the Earth points away from the Sun
   - b. Earth is closer to the Sun in summer ------->
   - c. Winter sunlight "bounces" off objects in winter
   - d. the tilt of the Earth's axis changes its distance.



3. Moon Phases
   - a. Relative position of Earth, Moon and Sun
   - b. Moon moves into Earth's shadow----->
   - c. Moon moves into Sun's shadows
   - d. Moon is lit by Earth's reflection
   - e. Moon is blocked by clouds
   - f. Moon is blocked by planets
   - g. The moon is black and white and rotates.

Over half of the interviewed students had taken or were completing a one-year course in Earth Science (of which almost 25% is astronomy). These students did not seem to get the correct answer any more often than others; indeed, some of the more exotic misconceptions came from this group. They did, however, use many more "scientific" terms in describing their ideas. Terms such as: tilt, orientation, phases, crescent, indirect light, equator, Tropic of Capricorn, planet, orbit, rotation and revolution peppered their explanations.

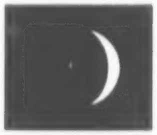
Almost every student drew the Earth, Moon and Sun either the same size or within a factor of 2x of each other's diameters (Earth is 4x Moon, Sun is l00x Earth). Relative distances were even worse, the Sun and Moon were typically drawn within one to four Earth diameters away (Moon distance is 30x Earth's diameter, Sun distance is l0,000x Earth's diameter).

Six students with particularly lucid views were selected as candidates for discussion with their science teacher, who had just completed a lesson on day and night, the seasons and phases of the Moon. In each case, the teacher rated the students on a scale from 1 to 10 as 7, 8, 9 or 10 before viewing the interview. Her ratings after viewing the video interview dropped an average of 6 points to 1, 2, 3 or 4. She expressed shock that her students held these "wild" views and that even the best students harbored deep misconceptions.

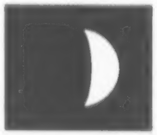
II. Multiple Choice Tests

Multiple choice tests were given to students in grades 9 through 12 in nine high schools within 50 miles of Boston. Questions-were based on student interviews and examples in the literature. Of the 25 items on the test, 6 dealt with the topics of this paper. Thee rest tested for other astronomical misconceptions. The first number is the percentage of the 213 students choosing that answer. The second number refers to the percentage after completing an experimental lesson on the topic. Bold type identifies the correct answer. Full analysis of the data and control groups will be completed in Fall 1987.

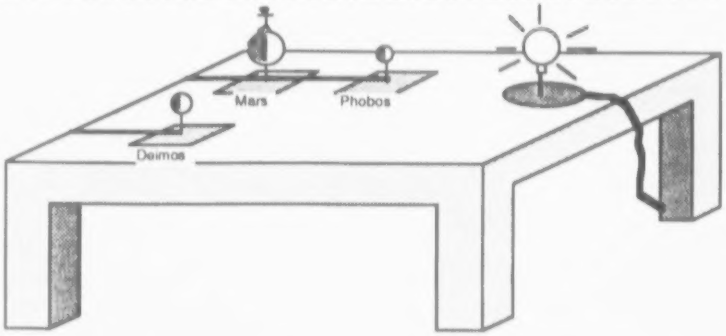
One night we looked at the
Moon and saw:



A few days later we looked at
it again and saw this:

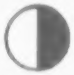


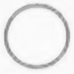
1. What do you think best describes the reason for this change?
   - A. clouds block the Moon 1%
   - B. the Moon moves into the Earth's shadow 37%
   - C. the Moon moves into the Sun's shadow 25%
   - D. the Moon is black and white and rotates 1%
   - E. the Moon moves around the Earth **36%, 60%**

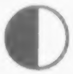


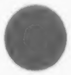
5. The diagram above represents a model of the Sun, Mars, and Mars' two moons, Deimos and Phobos. Please look at the model and determine how each moon looks for the person in the model who is observing from the north pole of Mars.

Deimos (circle one)

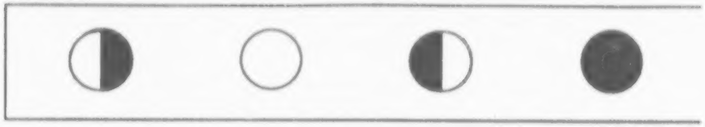

A **45%, 54%**


B 5%


C 45%


D 6%

Phobos (circle one)



A 7% B 5% C 23% D **64%, 79%**

9. Give the best estimate of each quantity:
   a. What is the diameter of the Earth? 10%, 25 %__________
   b. What is the diameter of the Sun? 2%, 9%__________
   c. How far is the Sun from the Earth? 24%, 43%__________
   d. What is the diameter of the Moon? 5%, 16%__________
   e. How far is the Moon from the Earth? 7%, 12%__________

11. Draw a diagram to show what causes the seasons:

show a tilt of earth's axis 43%, 59%

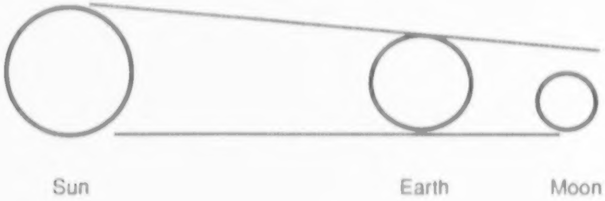
15. How long does it take for:
   a. the Moon to go around the Earth? 30%, 71%__________
   b. the Earth to turn once on its axis? 70%, 79%__________
   c. the Moon to go around the Sun? 10%, 63%__________
   d. the Earth to go around the Sun? 52%, 85%__________
   e. the Moon to turn once on its axis?7%, 39%__________

17. What time could it be if you saw the full Moon on the western horizon? 15%, 28% Explain your answer:

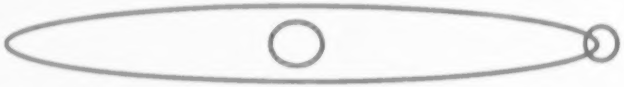
Pondering the answers to these questions, a picture emerges of students with little first-hand experience with astronomical phenomena, poor spatial relations and lack of even the most rudimentary facts about the Sun, Moon and Earth. The typical textbook treatment does little to address these difficulties but require memorization of scientific terms and quantities.

III. Sources of Misconceptions

Interviewed students overwhelmingly attribute their ideas to their schooling. Most have vivid memories of models of the Earth, Moon, Sun and planets hanging over their heads in their elementary school classrooms. These concrete and very visual models seem to become their first and only model for objects in space i.e. all object are about the same size and within a few diameters of each other. This is reinforced by text and trade books which never show objects in their true scale size and distance. Using this incorrect model, the Moon is often in the Earth's shadow, probably 10 days each month.



In grades 7 - 9, much emphasis is given to eclipses of the moon and sun, often using diagrams, like the one above, vastly out of scale. Elliptical orbits are introduced, interchangeable with oblique views of circular orbits.



Students get the idea that the earth's orbit is high eccentric and that at times the earth is very close to the sun and at some times very far.

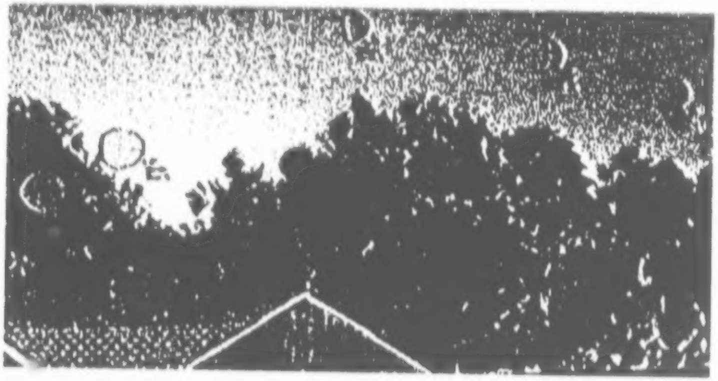
IV. Possibly Therapies.

We have had success in changing student misconceptions by using a three step approach:

1. Students make individual predictions about the outcome of an activity or experiment.
2. Students perform these activities alone or in groups.
3. Students discuss why their predictions did not agree with the observations or results of the activity.

The activities that have been used with high school level students are:

1. Building a scale model of the Earth-Sun System.
2. Building a scale model of the Earth-Moon system.
3. Drawing the appearance of ball circling your head when being lit by a stationary bright light.
4. Graphing the distance to the Sun, length of day and insolation of the sun from almanac data using the same scale for each graph.
5. Make a drawing of the Moon's phases over a 2 week period.



RELATING CONCEPTS TO PROBLEM SOLVING IN THE DETERMINATION OF MEANING IN MATHEMATICS

Jean Schmittau
Cornell University

This study probed the meaning of multiplication for ten subjects who were university students. The study centered on the question: How are subjects conceptualizing the operation of multiplication? Specifically: 1. Is the psychological structure of multiplication prototypic or does it exist in cognitive structure according to criterial attributes? 2. Have formal or school instructed concepts of multiplication interpenetrated the subjects' spontaneous concepts of multiplication? (Vygotsky 1962). And have the spontaneous concepts, rooted in the experience of subjects, informed the formal concepts?

The sample consisted of ten students from Cornell University. Subjects were selected by an iterative process with the goal of maximizing diversity across the following categories: major field, previous mathematics background, and time elapsed since the last formal mathematics instruction. The sample consisted of four males and six females. Two of the subjects were graduate students; the other eight were undergraduates. Four subjects were natural science majors, three were social studies majors, and three were humanities majors. Two of the subjects had taken calculus; one had completed linear algebra; three had gone through pre-calculus mathematics, while two others had completed statistics sequences in addition to pre-calculus mathematics. For two of the subjects high school geometry was the highest level of mathematics course completed. All were informed that their own meanings were of interest rather than the correctness of their responses.

Rosch (1973) demonstrated that the psychological structure of certain perceptual and semantic categories was different from their logical structure. Her findings that such categories were characterized by a core meaning (or prototype) around which other instances of the category were organized challenged the classical view that categories were held in cognitive structure according to criterial attributes. Armstrong, Gleitman & Gleitman (1983) investigated additional categories and obtained similar results which indicated prototypic organization of the categories.

Three categories studied by Armstrong, Gleitman & Gleitman were included along with multiplication in the instrument used to assess prototypicality. Only multiplication was of interest. The first three categories--fruit, vehicle and plane geometry figure--were included primarily in order to function as psychological foils, since it was anticipated that subjects might be inclined to approach the task of rating instances of multiplication without reference to their own meanings, but rather out of an automatic response to the presence of the multiplication sign. It was hoped that working through the first three categories would contribute to the breaking of this anticipated mental "set".

Subjects were asked to rate instances of multiplication for degree of membership in the category, using a scale of "1" to "7". A "1" signified that the instance definitely belonged in the category, while a "7" indicated that the instance was a very poor exemplar of the category or did not belong in the category at all. Subjects were asked to rate the following instances of multiplication:

- 4 x 3
- 2/3 x 4/5
- ab
- (2x + y)(x + 3y)
- (-5) x 2
- (-3)(-2)

-$\sqrt{2}\cdot\pi$

-

A = bh

As in the Rosch (1973) and Armstrong, Gleitman & Gleitman studies, graded responses were accepted as evidence of prototypicality.

Subjects were first asked whether it made sense to rate instances for degree of membership in the category. They were instructed that it did not make sense to do so if the instances were definitely within or definitely outside of the category. Three subjects responded affirmatively to the question, of whom two gave graded responses. Of the seven who responded negatively, four gave graded responses.

Subjects were then interviewed about their ratings. They were first asked the question: "What is multiplication? What does it mean to you to multiply?" After responding, they were asked with respect to each instance of multiplication which appeared on the rating scale they had completed earlier: "In what sense do you consider this (i.e., the instance "2/3 x 4/5", or "ab", for example) to be multiplication?" A flexible clinical interview format was followed in probing the responses, in order to assess the conceptual understandings of subjects and obtain information beyond that of the rating instrument. The subjects' own meanings were elicited, and subjects were consistently directed back to those meanings. This approach was helpful in moving subjects beyond rote responses for which their experience within school settings had conditioned them to expect acceptance. Near the end of the interview subjects were given the opportunity to revise their original ratings. Six subjects chose to do so, asserting that they had never thought about the meaning of multiplication and had, in fact, initially rated instances based upon the presence of the multiplication sign. One subject saw no need to revise his ratings, since he had always actively constructed meaning in mathematics and believed his initial ratings reflected this. Three subjects refused to change their ratings even when confronted with the fact that instances which they had rated "1" had no meaning for them. They bowed to mathematical authority, commenting that the multiplication sign had to "override any kind of feeling I have about it," or "the overwhelming belief I have is to go back and believe what my teachers told me, which is that these were all multiplication."

The table below presents the mean ratings for all instances:

Mean Ratings

Instance	Original	Revised
4 x 3	1	1
2/3 x 4/5	1.5	2.3
ab	1.2	1.9
(2x + y)(x + 3y)	1.2	2.7
(-5) x 2	1.2	2.2
(-3)(-2)	1.2	2.6
$\sqrt{2}\cdot\pi$	2.2	3.4
 A = bh	1.4	2.3
Mean	1.4	2.3

Upon completion of the interview subjects were directed to make a list of relevant concepts which had been discussed during the interview and incorporate these into a concept map. They were thereby required to identify relevant concepts and join them with propositional linkages. The concept map provided an important additional source of information about the manner in which the concepts in question were held in cognitive structure and an important source of corroborative evidence to the verbal and rating data.

Several claims are supported by the data. First, it was apparent that the logical formalist structures of multiplication in terms of definitions and criterial attributes did not constitute the basis for meaning. None of the ten subjects reported meaning based upon criterial attributes, even when they were well aware of them. Algorithms or "rules" were not meaningful, even when these were known by subjects and they were able to use them successfully to find a product. This was especially apparent in the case of fractions, negative numbers and polynomials. Four subjects were able to use the algorithm to multiply two fractions, but did not see this as multiplication. Eight subjects were able to respond similarly to the task of multiplying two negative numbers, but none saw this as multiplication. Three subjects could find the product of two binomials, but did not see any sense in which the process could be considered multiplication. Further, when subjects were unable to remember the definitional algorithm, they volunteered the explanation that it had no meaning for them.

This is a rather surprising finding in light of the fact that the instructional emphasis in school mathematics is overwhelmingly on criterial attributes. Almost no evidence was found that the successive reconceptualizations demanded by this emphasis had occurred. Instead, the fact that subjects gave graded responses to the rating task for instances of multiplication suggests that the psychological structure of the category is prototypic. The instance "4 x 3", rated "1" by all subjects, received the lowest rating of all instances, suggesting that it functions as a prototypic instance for multiplication.

This was further confirmed by subjects' responses to the question "What is multiplication?" All ten gave responses which described multiplication as "three groups

of five objects," for example. In addition, multiplication was described as "a shortcut for addition, since one could count the fifteen objects, or add five plus five plus five, or multiply three times five." Thus, the prototype was described in terms of positive integers, and, in general, the positive integers used were small. One subject mentioned "seeing" a rectangular representation of multiplication accompanying the above description.

This prototype was observed by Ginsburg (1977) as occurring spontaneously in children's informal mathematics. It is clearly numeric rather than geometric. Further, it is numeric in the cardinal sense and may also reflect cardinality emphases in the early school curriculum. However, it does not reflect, and appears to have persisted in spite of, years of school instruction in multiplication along formal and definitional lines. The results of the study lend credence to the view of Rosch (1983) that both criterial attribute and prototypic structures coexist. However, meaning appears to inhere in the prototype rather than in the criterial attributes. Their coexistence may be the result of instruction accompanied by drill along formal definitional lines which fails to adequately link the formal with the prototypic. Thus, the prototypic may persist because it has meaning for the subject, and the formal because its continuance has been ensured through drill and practice.

The persistence of prototypic or featural organization in the face of consistent instructional emphasis on logical definition suggests that featural organization might, in fact, be the predominant psychological tendency. If so, instruction organized around it may be pedagogically more valid than instruction organized around the logical structure of the discipline.

Certainly, it was the case that meaning across numeric and algebraic domains was consistently referred to the prototype rather than to criterial attributes. In better than ninety percent of the cases in which subjects gave evidence of a meaning existing for them in an instance of multiplication, that meaning was linked to the above "prototype."

The ease with which this linkage could be effected differed for various instances. Nine subjects reported substituting small positive integers for "a" and "b" in the product "ab" to link the instance with the prototype. Only five were successful with this approach for the instance (2x + y)(x + 3y). By substituting two for "x" and one for "y", for example, they obtained "5 x 5" to establish linkage with the prototypic instance.

In approaching the product "(-5) x 2" subjects first gave meaning to "-5" as a number possessing both magnitude and direction. They then reported that the product "(-5)(2)" expressed two debts of five dollars each, for example. Six subjects were successful with this task, but only one subject could find any meaning at all in the product of two negative numbers. Subjects could not envision a situation in which two numbers, both of which possessed magnitude and direction, could meaningfully form a product.

In the case of the product of two irrational numbers, two subjects successively approximated the product, making use of the "product of the limits equals the limit of the product." The poor performance of subjects on the product of two irrationals was characterized by the added failure to understand the numbers themselves. This was in contrast to all other types of factors, including fractions and variables, which were well understood. In the case of irrationals, the numbers themselves had no meaning. Subjects commented: "I have no idea what the square root of two is," "I can't remember now what it (pi) is," "I don't know why we have pi," etc.

Parallels were observed between the differentiation of concepts in subjects and the historical development of

847

mathematical thinking. The conceptual difficulties with negative and irrational numbers mentioned above parallel those chronicled in the history of mathematical thought. One thousand years after the development of negative numbers, their meaning was still very much in question, although their usage was established. Negative numbers (together with rules for the four fundamental operations with negative numbers) were introduced by the Hindus early in the seventh century. And yet, negative solutions to quadratic equations were rejected by the Arabs two hundred years later. Many European mathematics during the sixteenth and seventeenth centuries had difficulty accepting negative numbers. The situation was similar for irrational numbers. Two thousand years after the Greeks had struggled with the concept of an irrational number, and in spite of the acceptance of calculations done with irrational numbers, the question of whether irrationals were actually numbers at all still persisted.

A great deal of diversity existed with respect to the conceptualization of the multiplication of fractions. Six subjects could not remember the algorithm for finding the product of two fractions. A common question was whether or not the formula involved "cross multiplication;" i.e., whether a/b x c/d might equal (ad)/(bc) rather than (ac)/(bd). Two subjects were successful in generating the formula by attempting familiar examples such as 1/2 x 2/1. Knowing that the product is one, the subject was able to figure out that the required formula had to yield (1x2)/(2x1). One subject commented that "the rules I forget a lot are the ones that I never understood, that I never was given an explanation for, or never...really shown why." Similar statements were typically made by subjects whenever they failed to remember a rule.

Even when the rule was known or regenerated, most

848

subjects reported that they had no idea in what sense it represented multiplication. One subject tore a piece of paper into three strips. Several minutes later he tore another sheet of paper into five strips. However, he was unable to proceed further to form a meaningful grid using the strips. Another subject drew two circular pie diagrams. On the first she marked off 2/3 and on the second, 4/5. Then she wrote: 2/3 x 4/5 = (2x4)/(3x5) = 8/15. She then drew another circular pie diagram on which she marked off 8/15. While this process illustrated that she understood the meaning of the fractions, it revealed nothing about the meaning of their multiplication. She acknowledged that she was relying on the formula, the meaning of which eluded her.

One other subject used a pie diagram, but in a slightly different manner. She shaded 2/3 of the interior of the circle (actually .67, since she had converted the fractions to decimals). Then she shaded .8 (or 4/5) of the original .67. She saw immediately that this gave a result less than .67. She exclaimed that while she had earlier said that multiplication was a kind of repeated addition, she saw that in the case of fractions it was really subtraction. The result had meaning for her and was one of the few cases where meaning was not linked to the prototype. The inadequacy of pie diagrams to mediate meaning raises a serious question about their popularity for illustrating fractions and their widespread use in textbooks and school classrooms.

Three subjects assigned a rating of three or higher to the instance of multiplication of fractions, and explained that the reason for this rating was that it was necessary to divide before multiplying. Since the rule for multiplication meant nothing to them and they could never remember it, these subjects literally divided the numerator into the denominator of each fraction to produce a decimal. They then multiplied the decimals. This was the standard method they used to multiply fractions. None seemed troubled by the fact that the decimal might only approximate the value of the fraction it represented (as .67 constituted an approximation of 2/3).

Subjects who converted decimals to fractions tended to view them as either the product of two integers, with the "rule" invoked to locate the decimal point, or as "a little more than two" groups of 3.4, for the multiplication of 2.1 x 3.4, for example. Both of these explanations suggest that decimals may be chosen at least in part because they are perceived to be closer to the prototypic representation than fractions--the first because it allows for an integer representation, and the second because it is based on the notion of an approximately integral number of groups of decimal numbers.

Only three subjects reported that the multiplication of fractions had any meaning whatsoever. One was unsure of whether he understood fractions as anything more than "very small numbers," and then only when changed to decimals. Another first tried (unsuccessfully) to add the fractions, and then offered that she thought of multiplication of fractions as division, adding, "I don't know why" and "I don't know why you'd ever want to know what 2/3 times 4/5 was."

The most successful of the subjects drew a rectangle and gridded it into three rows and five columns. He then shaded two of the three rows and four of the five columns, pointing out that within their intersection were eight small squares out of a total of fifteen small squares into which the rectangle had been divided by the grid. The result was 8/15. He then demonstrated that this result was the same as that obtained by using the algorithm for the product of two fractions, pointing out that the product of the numerators simply represented the eight small squares present in the four columns of two squares each obtained from the two shaded rows, and that the product of the denominators represented the five columns of three squares each which were present in the original grid of the rectangle. Hence, there were four groups of two squares represented by the product of the numerators, and five groups of three squares represented by the product of the denominators.

This subject had linked his prototypic meaning through a geometric representation to the formal definition of multiplication of fractions. When one considers the manner in which the rectangular representation functioned so effectively to mediate meaning in this instance, one must question the practice of affording students a few illustrations of this sort in textbooks and classroom presentations before proceeding on to the algorithmic formulation.

It is important to note, however, that such models, if they are to be effective in mediating meaning, require an adequate conceptualization of area. It is also important to note that while subjects tended to simply substitute small positive integers for base and height in the instance "A = bh", such substitution is not sufficient to establish that area has been adequately conceptualized. Several differences were noticed with respect to the manner in which subjects dealt with instance of rectangular area. Five subjects, of whom four were male, gridded the rectangle in question. In addition, these subjects were able to meaningfully integrate area, base and height into their concept maps. One reported that he had gained an insight while constructing the concept map which caused him to make a crossover connection between area and algebra. Another had divided her concept map into relevant and irrelevant mathematics, and, significantly, placed area under the relevant branch while placing the rest of geometry under the irrelevant branch. She

commented that theorem proofs were "irrelevant geometry" and observed, "I guess I think of geometry as area, length times width, and that is relevant."

Data from the remaining five subjects contrasts markedly with the data reported above. The remaining five subjects displayed a combination of the following: failure to grid the rectangle, omission of area, base and height from the concept map and statements concerning area which raised questions about its conceptualization. One subject stated that "area doesn't quite feel like a number." Another was uncertain whether adding or multiplying base and height would give the area of the rectangle. And one subject reported that she saw the area of a five by one hundred foot strip as five rows of one hundred golf balls. Another, when questioned about the area of the rectangle, responded, "I'm not sure what's going on. I'm not certain that it's not multiplication. I'm not certain that it is. Because I have no feeling about what's going on." Comments such as those reported above are entirely lacking in the verbal reports of the first five subjects mentioned.

This data is important, since an adequate concept of area is required not only for the mediation of meaning in the multiplication of fractions, but is advantageous for modeling the multiplication of polynomials and irrationals as well. History reveals a reliance upon geometric models to convey meaning in algebra. In the ninth century, Al-Khowarizmi wrote: "We have said enough...so far as numbers are concerned, about the six types of equations. Now, however, it is necessary that we should demonstrate geometrically the truth of the same problems which we have explained in numbers." (Karpinski, 1915, p. 77). Clearly, meaning inhered in the geometric model rather than in the numerical explanation which preceded it. Henderson (1981) ascribes the avoidance of negative numbers by mathematicians all the way into the sixteenth century in part to the difficulty of finding a meaningful example of three "negative twos" or negative two "threes", but also to the reliance upon geometric models and the inability of such models to provide a direct representation of negative numbers.

While geometric modeling, if it is to mediate meaning, requires an adequate concept of area, an adequate concept of area requires a concept of number sufficiently general to include measurement as well as cardinality. It appears that the prototype of multiplication, based as it is upon a cardinal concept of number, is too limited to permit subsumption, albeit correlative subsumption, of the multiplication of fractions, and perhaps polynomials and to some extent irrationals as well. (Ausubel, Novak & Hanesian, 1978). Fuson and Hall (1981) cite research which indicates that narrowly cardinal concepts of number persist even into adulthood. The data shows that, with the exception of negative numbers, the understanding of subjects was wanting in precisely those instances in which a rectangular geometric model could mediate meaning. There are pedagogical implications to this finding, since at present elementary education does little to stretch the number concepts of children beyond cardinality.

Finally, it may be significant that the subject who gave evidence of the highest degree of conceptual differentiation not only "saw" the prototype of multiplication as a gridded rectangle, but was able to effectively employ a rectangular model to mediate meaning in the areas mentioned above. His analysis of the multiplication of fractions using a rectangular representation has already been reported. It is noteworthy that the last mathematics course completed by this subject was high school geometry, which he had taken seven years ago.

His situation illustrates another finding of this study. The tendency toward meaningful learning in the Ausubelian sense cuts across all of the following lines: gender, major field, previous mathematics background and proximity to last formal mathematics instruction. When subjects were ranked on a rote-meaningful continuum according to the degree to which they gave evidence of having meaningfully integrated into cognitive structure the various instances of multiplication, there were representatives of both genders at all levels--highest, lowest and in the middle range. Further, no one major emerged as dominant in any of the levels, and considerable differences in the mathematics background of subjects appeared at all levels as well. Finally, both the highest and the lowest levels reflect a range of three months to seven years from the last formal mathematics instruction.

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OVERCOMING MISCONCEPTIONS WITH A COMPUTER-BASED TUTOR*

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More than a decade of intensive research (Helm & Novak, 1983; McDermott, 1984) has provided ample documentation that misconceptions are widespread among students of beginning physics (and of other subjects as well); that these misconceptions impede learning even by students who have done well in previous courses; and that these misconceptions often persist even after students have successfully completed physics courses taught by good teachers with good traditional methods. It appears that "deep" misconceptions (those resistant to remediation by straightforward presentation of the correct conceptions) are rooted in strongly-held intuitions. These intuitions often serve us well in everyday life, and sometimes in school work as well. They may not be explicit or even conscious, but they exert a powerful influence, especially in situations not readily broached by application of learned rules. The interesting and difficult question is what can be done to alleviate this situation and help more students overcome their misconceptions.

ANALOGIES IN INSTRUCTION

One approach that has shown promise (Brown & Clement, 1987; Clement, 1987) makes use of selected analogies to help students transfer "correct" intuitions (in agreement with the accepted view of physics) to areas of difficulty. This approach is based on the observation that people have, even prior to any formal instruction, intuitions and concepts that are correct and

856

useful. The strategy is to help them make these intuitions explicit and extend them to appropriate new areas. A misconception for a given problem may be challenged by a correct concept (proposed by the teacher or transferred by the student from a related problem), and the two concepts may compete in the student's mind until, by judicious choice of examples and arguments, the student (with the tutor's help) comes to understand and believe the correct concept. This is in contrast with the traditional approach of simply informing the student of the correct concept. In many instances the latter approach just does not work, because students are unable to forget their intuitions. They may be able to ignore them in easy and straightforward situations, but in more difficult situations students must rely on them. The strategy based on analogies acknowledges students' intuitions, and helps them recognize the appropriate areas of applicability.

BRIDGING ANALOGIES

When a student gives evidence of relying on a misconception in a specific problem (the "target" problem), the strategy calls for bringing to the student's attention a problem that to the expert is analogous, and for which there is reason to believe the student has a correct intuition. Such a problem is called an anchor. The strategy calls for verifying that the student indeed has a correct understanding, and then asking him or her to compare the two problems (anchor and target) and his or her responses to them. To the expert, though not necessarily to the student, the two answers are contradictory.

Finding appropriate anchors is not always straightforward. It requires teaching or tutoring experience combined with empirical trials. The

857

knowledge and logic of the domain expert are not sufficient, because it is the student who must be "anchored" in the cognitive seas. Of course, different students may find different anchors most useful. In a classroom situation this may require some compromises, while in a tutoring situation it calls for some judicious probing.

Even when an appropriate anchor has been established, success is not necessarily at hand. Although the domain expert sees the analogy between anchor and target, in most cases the student does not. One approach would be to just explain the analogy, and hope for the best. But that would be contrary to the spirit of this approach, which aims to have learners construct the necessary connections and intuitions. It would probably also be futile. It is through the effort to resolve contradictory answers to analogous problems that this approach works.

When the conceptual gap between anchor and target is too great, the strategy calls for establishment of intermediate steps, called bridging analogies (or bridges for short), which break the gap into smaller steps. A bridge will have some things in common with the anchor, some with the target. By comparing each with the bridge, one hopes to enable the learner to span the conceptual gap, and to transfer the correct intuition to the target. In practice it has been found that multiple bridges are often necessary, resulting in a chain of analogies. Different learners will require more or fewer - and in some cases different sets of - bridges. When the strategy works ideally as intended, the learner constructs the correct concepts with a minimal amount of "being told."

THE BRIDGING ANALOGIES TUTOR

We have designed, implemented, and tested a computer-based tutor that makes use of this strategy. The Bridging Analogies Tutor is activated when a user answers incorrectly on a target problem. It is interactive in the sense that it uses the pattern of the learner's responses to decide on its next actions. It establishes an anchor, followed by bridges as necessary. In addition to posing a series of problems analogous to the target problem, it asks the user to consider pairs of problems that he or she has previously answered, and to compare his or her answers on these problems. The user is then given the opportunity to change answers. Each of the responses is used to help determine the Tutor's future actions. It is designed to help the learner arrive at correct answers to a succession of bridges until he or she chooses the correct answer on the target problem. The Tutor's presentation to the learner is in the form of text messages and of drawings on the screen. The learner's responses are keyboard entries. In the current version of the Tutor, all the responses are chosen from menus.

A Specific Topic: Forces in Statics

As a concrete example, we will describe the Tutor's operation in the domain of forces exerted by static rigid bodies. A much-studied problem in the research literature is the "book on the table" problem: a book is resting on a table; does the table exert a force on the book? The physicist says yes, the great majority of non-physicists say no: the table is just there, in the way, it's rigid, it's inanimate, it doesn't "know" how to push. Even after a course in physics, large numbers of students still say no (Halloun & Hestenes, 1985)! (The question of the magnitude of that force logically follows; but it has been found that the major barrier for students is whether there is a force at all, and this portion of the Tutor focuses on that question. Another part of the Tutor concerns magnitudes of forces.) This problem is typical of ones in which misconceptions play a dominant role. It is typical also in that while it appears elementary, understanding the principle (actually an application of Newton's Third Law) is crucial in many situations and problems in physics. But in these problems other sources of difficulty are more obvious, and this very basic misconception tends to be overlooked. Furthermore students are not able to articulate their wrong conceptions and thereby alert the teacher to the root of the problem.

The Tutor's memory contains a network of examples (shown schematically in Fig. 1) that it can present to the learner. It is from this network that the Tutor searches for an anchor and for bridges. Actually, this version of the Tutor works with a slightly "harder" target problem - a fly on a road. We chose this because we feared that too many students would get the book-on-the-table problem right, and we wanted to have a sizable sample of potential subjects for the field test. In fact, however, all but one of the subjects in the interviews saw the two problems as analogous, when they had the wrong answers as well as when they had the right answers. The anchor (in this case a stack of books in the hand; does your hand exert a force up on the books?) "works"

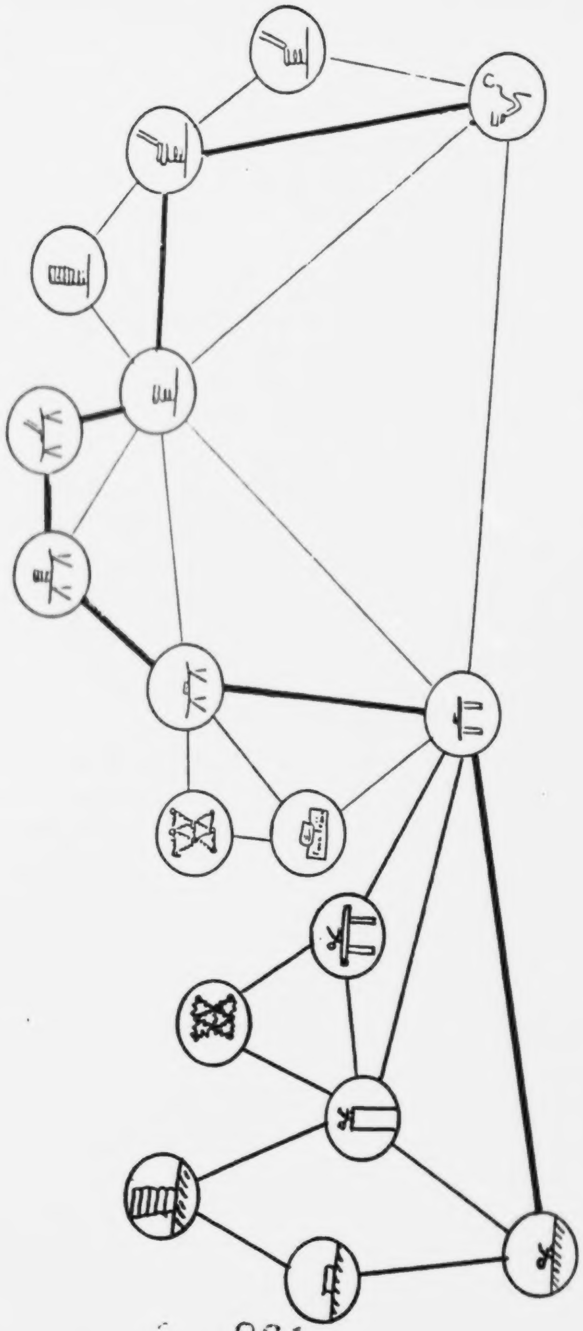


Fig. 1

for essentially all learners - that is, their intuition is correct.

Having established the anchor, the Bridging Analogies Tutor asks the learner to consider why his or her answers to the anchor and target are different, and offers him or her the chance to change answers on either, or both problems. (Fig. 2 shows both some sample dialogue and some representative diagrams). If there is no change in answers, the Tutor splits the conceptual difference between anchor and target, and brings up the bridge case of a book on a spring. (In Figure 1, the various bridges are arranged closer to the anchor or the target as the situations they represent are conceptually closer to the anchor or the target. These determinations were made based on earlier pilot studies.) The question is always the same: does the object on the bottom (in this case the spring) exert a force on the object on top (in this case the book)? The Tutor then asks for comparisons between this bridge and the anchor, and between the bridge and the target, each time reminding the user of his or her last answers to these problems, and giving him or her a chance to change those answers he or she wishes to. Users are asked to make comparisons both when their answers are right and when they are wrong.

The cycle of find-bridge-and-compare is repeated. Each time, the conceptual gap to be split is between the "hardest" (i.e. furthest from the anchor) problem the learner answered correctly, and the "easiest" one he or she answered incorrectly. For example, between the book on the spring and the book on the table, there is the book on a flexible board resting on sawhorses. The entire process is recursive, as

Imagine a medium size textbook resting on a dining room table.

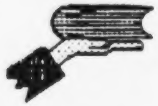
While the book is resting there the table:
A. IS exerting a force up on the book
B. IS NOT exerting a force up on the book

Please rate your confidence in this answer:
blind guess* *not very conf* *somewhat conf
fairly conf* *I'm sure

Imagine that you are holding a textbook in your hand.

While the book is resting there your hand:
A. IS exerting a force up on the book
B. IS NOT exerting a force up on the book

(*** again give an answer and confidence ***)



For "a book in your hand," you said:
your hand IS exerting a force up on the book
(with high confidence).

But for "the book on the table," you said:
the table IS NOT exerting a force up on the book
(with fair confidence).

Explain why these answers are different.

*** Do you want to change your mind on either or both of them? ***

Fig. 2

The Bridging Analogies Strategy

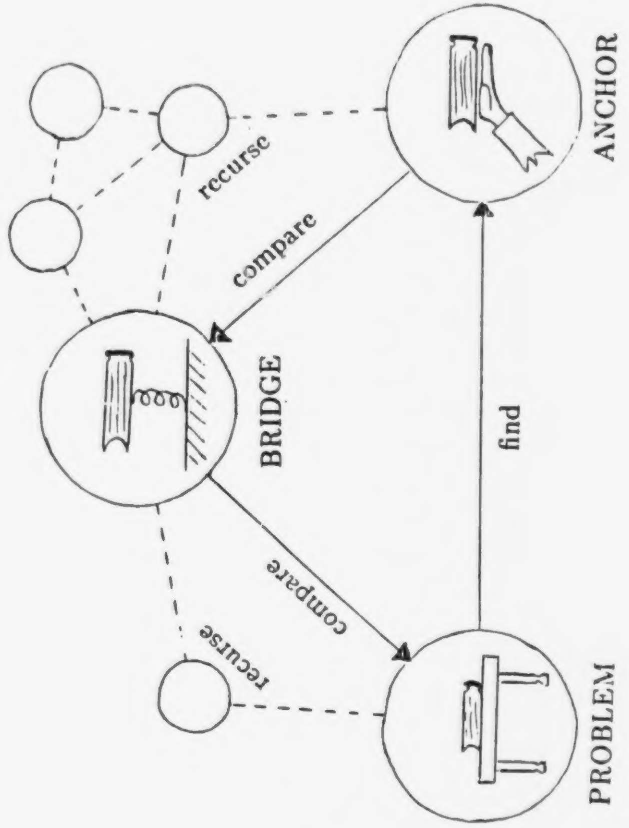


Fig. 3

shown in Figure 3. The heavy lines linking examples in Figure 1 indicate one out of many possible paths an individual learner might take. Of course, the Tutor may run out of new bridges to call up. In that case, it pulls from its memory a stored "hint," more or less directive. This hint is local - it pertains only to the particular bridge problem then under consideration. The Tutor then continues with the find-bridge-and-compare routine.

The strategy of the Bridging Analogies Tutor is constructivist in the sense that it gives the learner every opportunity to decide on his or her own answers, and to change them. Even when the learner is asked for comparisons, there is no implication that either answer was incorrect. This process is time consuming. It works best in areas where traditional methods fail most badly, such as qualitative physics (generally considered very important, but a weak point of physics instruction), where the barriers to understanding are deeply rooted and hidden.

Comparisons

The comparison questions are really at the heart of the Bridging Analogies Tutor. First, they encourage analogical thinking. (We have documented instances where subjects spontaneously use analogical arguments during the tutorial sessions.) Second, they offer the possibility of bringing about disequilibrium in the learner. In the language of Inhelder, Sinclair & Bovet (1974), they confront the learner with two schemes (intuitions) that lead to inconsistent conclusions. Of course, what is a contradiction to the domain expert may not be one to the beginning student. But the Tutor, by probing through comparisons for the point where the student does feel the contradiction, can in many cases bring on an acute cognitive dissonance and with it the impetus for conceptual change. Third, comparison questions provide tests of the stability of subjects' conceptions as revealed by their answers and by the changes in their answers.

To be sure, the comparison questions are for nought unless the learner really thinks about them. Our field trials have been carried out with an experimenter present. The experimenter's main function is to encourage the subject to "think out loud," to provide rationales for his or her answers. How to get learners to think hard about such problems and comparisons without an experimenter present is a challenge we have not yet taken up. Since this project is still in the research and formative evaluation stage, it has seemed justifiable and advantageous to have an experimenter present during the tutoring sessions.

We have also experimented with tutoring sessions involving two subjects rather than one. Our limited experience has given support to our hope that, with the two subjects interacting, the experimenter needs to intervene very little to keep the subjects thinking and talking.

Confidence Levels

The reader may have noticed a passage in the dialogue reproduced in Figure 2 that has not yet been mentioned. After each answer to one of the problems, the subject is asked how confident he or she is of his or her answer, on a 5-point scale. This is an attempt to obtain via electronic means some of the information a human tutor would get via voice inflection, facial expression, or body language. This information is used as input to the decision mechanism of the Tutor, in ways that are too complicated to be described here (Murray, Schultz, Clement & Brown, 1987). As one example, if a subject's answers to two questions are the same but with greatly different confidence levels, the Tutor might ask him or her about just that difference. We have some evidence that just asking about confidence levels leads some subjects to reflect on their answers and on their thinking.

Topics

This report has dwelled entirely on the topic of the existence of forces between static bodies. This was the prototype topic of the Bridging Analogies Tutor, and the one on which we have the most data. In the field tests, we also obtained data on a tutoring sequence about the magnitudes of forces. These data will be reported elsewhere. Other networks are in various stages of design. An important feature of the Tutor is that the problems and associated graphics (in other words, the domain-specific materials) are decoupled from the Tutor's control structure. Thus the modification of existing networks via new examples, or the creation of new topics, requires only the addition of new materials into a text file (and, optionally, new diagrams). Conversely, it is possible to change the control structure of the Tutor, and thereby to change, within limits, the tutoring strategy. For instance, one of the nodes in the static-forces network is a description of the molecular nature of matter and of the springy nature of molecular bonds. Since this version of the Tutor was implemented and tested, we have come to understand that this example is more in the nature of a mental model than a bridging analogy. A small change in the

Tutor's control structure could move this example to the beginning of the tutoring session. Then one could investigate the relative effectiveness of this new sequence and the one we have implemented. This is one of the ways the Tutor lends itself well to continuing research.

FIELD TEST

Subjects were interviewed in one-hour sessions with the Bridging Analogies Tutor, mostly individually, but a few times in pairs. The interviews were recorded on video tape. The computer collected all subject responses (as well as the questions eliciting these responses). The responses were analysed to obtain information on effectiveness of the Tutor and to look for patterns in responses.

The subjects were drawn from basic mathematics classes at the University of Massachusetts. A pre-questionnaire was given to all students in 10 classes, resulting in 180 written responses. The questionnaire included one problem that is a slight variant of the target problem, and another one close to the anchor. Two other questions were included to make the intent of the pretest less obvious. Students who took the pretest were asked to volunteer for the interviews.

Subjects were chosen from among students who answered incorrectly on the target problem and correctly on the anchor. (This is the combination of answers and beliefs for which the Tutor was designed; sixty-six percent of the students answering both questions on the pretest fit this pattern.) Under the direction of the computer tutor, they were led first through the existence-of-forces network, and then, as time allowed, through the magnitude-of-forces network. None of the subjects had taken college physics; fewer than 20% had taken physics in high school. The experimenter's role was limited to making sure that the subjects could activate the Tutor, and that they verbalized their ideas.

RESULTS

The best of human teachers bring to a tutoring task a wealth of experience, knowledge, and ways of assessing the state of a learner's concepts. To hope to simulate all that in a computer environment is an ambitious task, especially when more than simple factual information is to be learned. In building computer tutors, we believe multiple cycles of the sequence design-implementation-testing-evaluation are indicated, not only to build a good working tutor but to learn more about the tutoring process. The results from this field test are reported in this spirit - formative evaluation on the road to more complete and more successful tutoring strategies.

We report results from 13 interviews with 15 subjects (two interviews with pairs) who completed the Bridging Analogies Tutor on the topic of static forces. In addition, we have several partially-completed interviews, which added information to some research questions, and four interviews in which the subjects answered the target problem correctly at the outset, despite having answered incorrectly on the pretest. The latter subjects were automatically passed by the Tutor to the magnitude-of-forces network.

Some questions to which we sought answers in this test are:

1. Can the Bridging Analogies Tutor help students overcome misconceptions?
2. If so, under what circumstances (types of students, types of domains, etc.) does the Tutor work better or less well?
3. What parts of the Tutor (e.g. comparison questions, confidence questions, etc.) contribute most to its success?

Effectiveness

The answer to the first question is yes. We have five "existence proofs" - interviews in which the Tutor succeeded without having to resort to any "give-aways" of answers along the way. Because they were not told the correct answer, we assume that these successes represent a reasoned shift in the students' point of view. This conclusion was supported in general by students' comments during shifts.

All but one of the subjects answered the target problem correctly at the end of the tutoring session, with a confidence of at least 4 on a 5-point scale. (The remaining subject understood the suggested analogies but steadfastly refused to believe them.) This is in marked contrast to their initial responses: at the beginning, all of them answered the target problem wrong, and 9 out of 15 were confident in their wrong answers at least at level 4 .

Most of the subjects needed a hint at some point along the network because they were stuck on an incorrect answer, and the present Tutor had no more bridges to propose. However, 9 out of the 15 subjects arrived at a correct answer to the target problem either with no hints at all, or with hints on problems early in the sequence. These hints enabled them to continue on the network of bridges, thinking about comparisons and changing their answers and confidence levels one by one.

Most subjects needed to pass through the node in the network in which matter is described as being made of molecules with springy bonds (shown in Figure 1). Many indicated with verbal comments that they remembered this from some high school course, but did not spontaneously apply the knowledge to the problems. It is possible that presentation of this information at the beginning of the tutoring sequence would help students arrive at the correct concept faster. However, the only subject who was explicitly asked stated after the session that it was preferable to have considered many problems before being presented the molecular model (see excerpt in the next section). Nevertheless, this is a legitimate question for future research.

The comparison questions alone instigated a change in belief at some point in the network for about half the subjects. This suggests that bridging and analogical thinking are effective instructional strategies for many students, but may not be powerful enough to be used exclusively with all students.

An average of 7.5 example situations were needed to bring subjects to correct answers on the target problem. This supports the idea that many nodes are needed in the tutoring network, and that the anchor and target problems were indeed distant analogies.

Barriers in the Network of Analogies

Analysis of where in the network subjects spent most time or had most difficulty indicates that the analogies between the books-on-the-hand and the book-on-the-flexible-board problems (the right half of the network in Fig. 1) were readily accepted by most (but not all) subjects. In fact, the book on a spring could have served almost equally well as an anchor. By contrast, many subjects had great difficulty accepting the analogy between the flexible board (or any situation to its right in Fig. 1) and the table, which they took to be rigid and qualitatively different from all the other surfaces. The introduction of the molecular model was helpful for some subjects but not needed by others. The formative evaluation of the Tutor clearly indicates that more bridges are needed in this part of the network.

Subjects in Pairs

Among the subjects in this study there were pairs who worked together. Choice and pairing of subjects was determined by times the subjects were available to be interviewed rather than by any information about their physics knowledge. Paired subjects had to agree on all answers they entered into the computer, but they could discuss reasons (and even disagree on reasons) for these answers. Based on our limited sample, we found pairs of subjects to work very well: their spontaneous interaction essentially eliminated the need by the experimenter to remind them to make explicit their beliefs and understandings. Excerpts of one interview are included in the next section.

Other Topics and Questions

The analysis of our data on another tutoring sequence (the relative magnitude of forces related by Newton's Third Law) is not complete. It is clear, however, that the Tutor in its present

version is not nearly as successful for this topic. The problem is that we did not have as good a set of anchors and bridges as in the case of the existence of forces. This underscores the need for extensive research on students' conceptions before implementing a computer-based tutor using the bridging analogies strategy.

Further questions are suggested by our results:

1. Can a computer-based tutor be built that bases its actions on reasons for students' answers? Subjects in our trials gave several types of reasons for their answers, and a tutor that bases its actions on the reasons students use could be more effective than one that only uses students' answers.
2. Can other strategies be implemented on the computer? Choice of strategy might depend on the type of domain or on what is known about the student.
3. Can a computer tutor make reasonable judgements as to when to change strategies?

EXCERPTS FROM TUTORING SESSIONS

We present here excerpts from some tutoring sessions, which will give the reader an idea how the Tutor actually worked with real students. The students' comments were recorded on video tape as they worked under the guidance of the Tutor, and were transcribed later.

The first series of excerpts is from the tutoring session with "Annette". She begins like most subjects, answering the fly-on-the-road problem and the book-on-the-table problem incorrectly, and giving reasons indicative of the misconception that "rigid" stable inanimate objects cannot exert forces. She then answers correctly the anchor, the many-books-in-the-hand problem. Soon after, she answers correctly the very similar one-book-in-the-hand problem, but with lower confidence.

> "I don't know how confident I am any more. I'll put 'somewhat'."

The program asks her to explain her different levels of confidence on the two problems. One explanation might have been that holding up one book does not lead her to imagine exerting a force as vividly as holding up a stack of books. But Annette has been thinking of the previous problems, and it is her increasing doubt about those that comes out as decreasing confidence in the more recent questions.

> "Well, I don't know how sure I am that the table and the ground aren't pushing up, after the last question."
>
> Experimenter: "This, of course, is asking not about the table or the road, it's just asking you about - "
>
> Annette: "But that's why I did [my confidence levels] differently."

This is a good example of how being asked about confidence levels can lead a student to cognitive conflict which may pay off later.

After being presented a few more examples and comparisons between situations, Annette is asked to compare a book on a flexible board (for which she had answered yes - the board exerts a force on the book) and a book on a table (for which she had answered no).

> "[The board] is not sturdy at all, and so it has to be exerting some kind of force up to keep - (pause, then laughs) This is like blowing my whole theory. The book on the board is pressing down on the board, and it's bending it. In order for the board not to bend all the way, it has to exert some kind of pressure, but it kind of makes me think about the book on the table exerting force. I'll tell you why I answered them differently, but I don't know why."

Annette is shown (on screen) a description of the molecular nature of matter, and of spring-like forces between adjacent molecules. She tries to reconcile this information with her ideas on the previous problems.

> "Because if the springy bonds do push back, then that means the table is probably pushing up on the book, but I mean, I don't know." (some hemming and hawing) "Unless we're talking about molecules, I mean, unless we're talking about the book on the table ... would actually mean that there are molecules pushing up on the book." (long pause) "I think that my answer's wrong." (another pause) "But there's a difference between pushing back and exerting a force. Exerting a force just seems so much more powerful. But I guess just pushing back would be exerting a force." (pause) "I think this is wrong, that I should change it."

The above shows a long struggle as Annette gradually puts together the ideas that are finally convincing to her. After she finally decides to change her answer on the book-on-the-table problem, she is asked how confident she is.

> "Well, actually it can't be wrong if the molecules are pushing up - then it has to be exerting some kind of force on the book."

The passages excerpted here show the "wavering points" of a tutoring session during which the subject proceeds from a wrong answer (and a wrong conception of the situation), with a considerable degree of confidence, to the correct answer with complete confidence. This session is typical in that one really cannot identify a single turning point in the subject's belief; rather, the series of problems and comparisons plants the seeds of doubt, and provides a stage on which competing conceptions are compared.

"Laura" and "Tim" worked with the Tutor as a pair. They met for the first time at the tutoring session. They were told that they must discuss and agree on their answers before keying them in. Like Annette, they answer incorrectly on the book-on-the-table problem and correctly on the books-in-your-hand problem. Asked to compare the two, Laura starts to waver, though very tentatively. Tim holds to his misconception.

> Laura: "Maybe because when it's your hand, you can feel yourself doing, you know, pushing a force up on the books, so you're more apt to answer it that way, but the table, now that I think about it - "
>
> Tim: "But the table is the same as the road in that it's free-standing, and the table, I don't think, is exerting any physical energy to stand there - "
>
> Laura: "I know, but it seems that way (pause) It seems as though they should all have been the same answers, kind of. If you answer it that way for the books in your hand, the table must have to exert some kind of a force to keep that book from falling through it, maybe (pause) I don't know."

Throughout this session, there are many long pauses, and sentences that trail off in the middle. Laura is very uncertain, and Tim quite confident in his wrong idea, and they decide not to change. Tim is also confusing force and energy, something we observed often.

Next they are given a bridging problem: a book on a fairly stiff bedspring.

> Tim: "The spring is compressing a little bit, it's just that it's not as stable or as strong as the table. If anything, the book is exerting a force on the bedspring, but not vice versa. So the spring is not - " (puts his finger on the key that would input a no answer)
>
> Laura: (takes a deep breath and holds it, causing Tim to hesitate with his finger on the key) "Although (pause) when the spring is being compressed it's exerting a force in both directions, I'd say."
>
> Tim: "Ahh, I didn't think of that. Umm, yeah."
>
> Laura: "So the spring is exerting a force up on the book, and onto the table."

A few minutes later, the program proposes another bridging situation: the book on a flexible board.

> Tim: "Well, if we're going to stick by our last answer, I think we'd have to say the board is exerting a force up on the book, just because the natural state of the board - when you put a book on the board it bends, and when you take the book off it would bend back, and so its natural state is to be

straight across the sawhorses, and when there's a book on top of it, it's still probably trying to return to its natural state."

Laura: "You know what they're going to say to that one, don't you? They're going to give us the book on the table example."

Laura is anticipating possible future comparisons, but not yet sure enough to propose a change in answers.

The program introduces the molecular model, and asks for a comparison with the flexible board. They had answered both situations correctly, but with lower confidence on the question of a force exerted by springy molecules. The comparison question leads Tim to come up with a spontaneous analogy.

"You can use the pine board as sort of a large example of springy molecules, maybe. We answered the pine board with greater confidence [than the molecules question] because with past knowledge of the pine board in mind our confidence was greater."

As Laura predicted, the Tutor now asks for an explanation of their different answers on molecular bonding and the book-on-the-table problem.

Tom: "Because the table is - uhh - not springy."

Laura: "But the molecules are, and the table is made of molecules."

Tim: "But, depending on the kind of wood, assuming that it's a stronger wood, the molecules aren't as springy, because they said there are springy molecules - "

Laura: "Not as springy, but wouldn't they still be springy, somewhat? They'd have to be."

Tim: "They'd be springy, but not to the weight of a book. Maybe to the weight of a couple of anvils."

Laura: "Hmmm (pause) maybe not as springy to a book, maybe there is a very slight force there - "

Tim: "Yeah."

Laura: "Given even how slight it may be, maybe there's still a force there. I don't know."

Tim: "OK. Well then, maybe we should change our answer on the table."

Despite Tim's attempt to distinguish between the effect of a book and a couple of anvils, they proceed to change their answer on the fly-on-the-road problem as well. In an effort to probe the stability of their answer, the experimenter asked a follow-up question:

"Somebody this morning got to the same point, and said yes, but a fly is really light, and a book is a lot heavier - how can you compare the two?"

Tim: "But if you drop a fly and you drop a book, they both head towards the earth."

Laura: "They both exert a force, it's just that one is stronger than the other."

Tim: "The molecules are so small that a force of dropping a fly on the road would not make a noticeable spring, but it would probably create some disturbance in the molecules."

Tim invokes a mechanism to explain how a "rigid" body can exert a force. Brown & Clement (1987) have argued that this type of reason is more helpful to many beginning students than an explanation based on application of a law of physics.

Aside from being another example of a successful trial of the Tutor, this tutoring session shows the fruitful interactions possible when two students work together on the Tutor. It could be argued that, left to herself, Laura would have obtained the right answer much sooner. It is possible, however, that her efforts to make her ideas explicit to Tim led to much stronger correct conceptions on her part.

General Comments by Subjects About the Tutor

All subjects found it to be interesting and motivating, as indicated by their behavior and their comments. Here are examples of wrap-up comments:

> "I think that it's a good program. It's helpful when it goes back and tells you to look at your answers, and then, once you've thought about it - it forces you to think about it, because it keeps bringing it back, and then if you're totally wrong on something, it tells you, so that you can figure out why you're wrong."
>
> Experimenter: "Tell me about your intuition and your thinking and how it changed during the hour."
>
> Annette: "There are a lot of cases where it changed. It seems like my intuition was wrong almost every time, as far as just basic - you think about a table, you don't think that something's pushing up on a table; you think about a fly on the road, you know, you don't think that anything's pushing up on it, so it kind of - I mean, I have no physics

> background at all, and it's interesting to learn things like that, but at first it goes against everything that your senses are saying."
>
> Experimenter: "Supposing that the order of these things had been turned around, and the program had told you right off the bat: molecules exist, everything is squishy, everything pushes, etc., would that - "
>
> Annette: "No, I think that it's better that you leave it where it was, because I found that coming into this program without any idea - you know, sometimes people will put down what they think people want to hear, and if it gives you that kind of explanation, you're not starting from scratch, which I think is better, because then - It's a really good program, because you write, you put down what you think your answer is without having any clue, and then you put if you're confident or not, and then you go back and it keeps going back saying why, why, why, and it gives you different examples. Then it will say to you this is the way it is. It makes you think at first, instead of giving you a hint at first. I think that's better."

CONCLUSIONS

Not all tutoring sessions worked as well as those with Annette, Laura, and Tim, and not all subjects were as articulate in expressing their beliefs. But overall the sessions were successful enough to make us conclude that this approach is effective, even if it may sometimes require human backup. We recognize that we have not completed the task of building a working tutor. But tests with real students and formative evaluation were necessary to provide the insights needed to design future versions. We now have a long list of desirable improvements. Some require more sophisticated hardware and software, some further cognitive research, some the use of artificial-intelligence techniques. Two of us (T.M. and K.S.) are pursuing this. More rounds of field tests and formative evaluation will be required. Each round is a step toward the goal of an effective tutor, and provides increased understanding of tutoring and learning strategies.

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Understanding Scientific Derivations: A Task Analysis and Constructivist Learning Strategy

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Scientific and mathematical material is often formalistic and presented as a concise sequence of deductive steps. The presentation may be elegant but not necessarily ideal for the learner. Analysis shows that the task of learning from scientific text is complex and often poses difficulties for students. They may be able to follow derivations step by step and handle the mathematical formalism, but are often weak at tracing the esssence separated from detail, and at intepreting the material conceptually in qualitative or physical terms. Even advanced students may resort to semi-rote learning and manipulation of formalism, especially if test questions reward it. A constructivist teaching and learning approach has been tried at undergraduate level. During repeated passes through the material, the essential ingredients are identified, main reasoning steps traced, and dependencies explained. Detail is handled separately. Learners consciously process and restructure the material. The approach has an explicit metacognitive aspect which gives many students an appreciation of the nature of real understanding for the first time. Examples are available from various areas of physics.

A. INTRODUCTION AND OVERVIEW

Scientific and mathematical derivations often pose difficulties for students in terms of real understanding, even up to graduate levels.

Some examples from physics at various levels are derivations of the following equations:

$s = ut + {}^1/_2 at^2$ in kinematics,

$P = \rho gh$ pressure at depth h in a liquid,

$E^2 = p^2c^2 + m_o^2c^4$ in relativity,

$\lambda - \lambda' = \frac{h}{mc}(1 - \cos\phi)$ for the Compton effect,

$P/_T = (\partial\sigma/\partial V)_{U,N}$ volume dependence of entropy.

A derivation usually consists of a sequence of steps, starting from particular assumptions and information, drawing on general physical laws, using mathematical techniques, and finally deriving the desired result. The process of understanding generally involves both conceptual physical reasoning and mathematical formalism.

This paper concentrates mainly on scientific derivations of reasonable length and complexity, at advanced undergraduate university level, such as the last three examples above. However the ideas would certainly apply more generally.

Although students may be able to follow textbook derivations step by step, and perform the algebraic manipulations, they may acquire only a surface short-term understanding. One criterion for understanding and assimilation would be that the student be able to reconstruct mentally the main ingredients and procedures, thereby generating the result independently, without the text, some time later when short-term memory has faded. Balanced understanding also requires conceptual interpretation, so the student should be able to answer qualitative questions about the physical behaviour represented by the theory.

Students are often weak at tracing the essence of the material, separated from the detail, and may rely on semi-rote learning and manipulation of formalism. They are often unaware of the deficiencies in their understanding, not knowing that considerable processing, analysis, ancillary knowledge, and resynthesis of the material is needed to make the material "their own".

Text is possibly the most important source of scientific knowledge. Learning from scientific text is thus one of the most important modes of learning, if not THE most important, in the real world. It is crucial that students learn how to learn independently from text and emerge from their university education able to do it well, and for this to occur explicit guidance and modelling is valuable. The task of constructing a thoroughgoing understanding of a scientific derivation is much more complex than is generally recognised, and is not usually taught explicitly. It is not educationally effective to leave students to "pick up" these skills somehow while focussing on the material; some students will, while others will never quite manage.

Scientific text has certain characteristics and limitations as a learning medium. These play a large role in determining how students learn, how they view science, and what they deem understanding in science. Certain active processing skills are necessary in studying scientific text for real understanding. Those experienced in the topic may not fully appreciate the difficulties facing a beginning learner, since experts have large stores of compiled knowledge and skills which are mostly tacit and are brought to bear automatically. Thus instructors may provide little explicit guidance to the learner in the processes needed to understand science.

A 'constructivist' approach to understanding scientific derivations from text is briefly as follows: The text must be actively processed by the learner in several passes. The derivation must be analysed, ingredients identified and characterized, essence separated from detail, main lines of reasoning traced, links visualized between various pieces within the material, connections made to knowledge outside the presented material, and heirarchical structuring visualized. A conceptual understanding must accompany the formalism; the material must be interpreted, qualitative behaviour understood, and physical insight into the system developed. Explanations, advance organisers and instances must be added as needed by the learner. The material is then resynthesised by the learner, probably in modified form. A self-questioning strategy, and metacognitive awareness is valuable in this processing. This approach will be elaborated in a later section.

We first turn to the characteristics of scientific text, and analyse the task of learning from it.

B. UNDERSTANDING SCIENTIFIC MATERIAL: TEXT CHARACTERISTICS AND TASK ANALYSIS

We will characterize the nature of scientific text and the task of learning from it. Some of the aspects listed below are inherent in the text form of presentation, some arise in the trade-off between brevity and explanation, while other aspects depend on the writing style and didactic approach of the author.

1. Concise.

Text treatments tend to be concise, particulary at higher levels, with non-essentials omitted. The degree of explanation depends on the author. Few cognitive links are presented either within the material or to items outside the material. Learners will have to provide their own links to assimilate the material.

2. Mathematical.

Mathematics is an important part of the 'language' of science. Mathematical formalism has a high information density, and must be interpreted. A single graph or equation contains a great deal of information, and tacitly assumes considerable background and interpretion skill by the reader. A long time may be required to process a few lines. Certainly the material cannot be 'read' in the same way as ordinary text.

3. Scientific writing.

Scientific writing is different from everyday writing, both in conciseness and structure. Here is a sentence occurring in a derivation of the equation $P = \rho gh$: "Consider the equilibrium of an imaginary cylindrical element of fluid, of thickness dh and cross-sectional area A, at a depth h in an incompressible fluid of density ρ, as shown in the figure." To follow this requires careful mental processing and imagery.

3. Pyramidal; prerequisites.

Science is a pyramidal subject - one layer of concepts, knowledge and skills is built on another. Texts assume a prerequisite knowledge and skill base, and unless students possess this, it is difficult for them to adequately understand new material. They will have to go outside the material to develop what they need. This may not happen easily, since it is an additional time-consuming task.

4. Few advance organisers or questions.

Texts may not provide much in the way of 'advance organisers', advance instances, or lead-in questions, which could 'set the scene' for the learner, provide a context, and pose the questions which the section is proceeding to answer. This makes it more difficult for learners to place the new material in a broader context and relate it to general and specific aims. The learner will have to attempt to do this after reading the section, and then re-read.

5. Few advance instances.

Generalities and abstractions may not initially be meaningful without concrete instances on which the learner can hang them.

6. Surface features of text: a learning danger.
The apparent visual dominance of the text by mathematical equations, with conceptual understanding and interpretation only implicit, is a danger to learning if it leads the student to think of understanding science as mainly mastery of the formalism.

7. Linear sequential presentation.
Text by its nature gives a linear sequential presentation. Learning however usually involves backtracking, jumping back and forth, linking different items, selecting out, grouping together, etc. Mental rearrangements and cross-links are needed in learning.

8. Essence and detail.
Main ideas (eg principles) and details (eg equation manipulation) have similar apparent status in text format and are not automatically distinguished, except possibly by cueing language. Heirarchy is not immediately evident. The learner will have to identify and characterize items, and construct a mental heirarchy in which to place them.

9. Content and process.
Scientific treatments by their nature tend to focus on the subject matter ('content') rather than on the learner or the processes of understanding. 'Acquisition' of presented material may seem to be the aim. The learner will have to master the processing, without much guidance.

10. Non-interactive.
Text is clearly non-interactive. The form of the medium may thus tend to 'suggest' passive following and absorption of presented material, rather than active processing.

11. Qualitative reasoning, physical insight.
Qualitative reasoning and physical insight are much neglected in conventional texts, compared to the formalism and procedural aspects. The understanding this develops is one-sided: it is not unusual for a student who has mastered the formalism to be stumped by quite simple qualitative conceptual questions on the same material. The conceptual side will have to be constructed by the learner in text processing.

12. Interpretation.
Texts offer varying degrees of interpretation. Most give little interpretation of stages in the derivation. The result is usually discussed briefly, possibly followed by application to examples. Interpretation is crucial to full understanding. The learner needs to spend time considering the form, meaning and implications of results, and the reasons for the dependencies exhibited.

13. Deductive/inductive.
Text presentations tend to be deductive rather than inductive. They tend to work top down, from presented generalizations (theory) to consequences in specific cases. The need for the theory is not always clear. It is as important to ask what the questions are as to know the answers.

14. Finished product.
Texts tend to present the polished finished product of human endeavour. Arnold Arons has noted that we present students with cut flowers rather than let them see the plants growing.

15. Approach.
Text approaches, particulary at higher level, have a mainly subject-matter focus. Approaches are not usually from a pedagogical, historical, cognitive, or discovery perspective. If some of these alternative perspectives are useful in developing balanced understanding, the student will have to provide them.

16. Traces of invention.
The "draft" thought processes involved in creating the derivation (starting points, aims, possibilities, approaches, backtracks, tries, checking, etc) are eliminated. The student sees no traces of invention. Yet to fully understand the material, the student needs to have tried to develop it.

17. Authority in science.
The message conveyed is that of the authority of the text and the correctness of the material. The student usually does not question this, not the presentation approach.

18. Booklearning.
Booklearning can often seem isolated from the real world it is supposed to represent. Students may be unable to handle questions using real phenomena even after book study.

19. Authors: content-experts.
The authors of textbooks, and those who teach from them, are most often content-experts, who may or may not have pedagogical expertise. They possess direct and ancillary compiled knowledge and skills in the topic, but much of this is tacit. The student has a greater task than the author may realize.

The list above shows the limitations of text for learning, and indicates that much active processing of the text by the learner will be necessary to construct understanding.

Note that although there are many common limitations of text for scientific learning, it is not necessarily suggested that authors modify the presentation to provide as much learner support as possible, in the way of explanations, background material, discussion, interpretation, process or metacognitive discusssion etc. Certainly this would be helpful, since at present the balance seems the other way. However too much might obscure the material - and more importantly it is the individual learner who ultimately must construct the knowledge from the text, dissecting it, providing background knowledge, reassembling it, interpreting it, etc. It is this skill that is crucial to develop. Furthermore, even if some texts gave comprehensive content and cognitive discussions, most would not, and the independent learner must be able to handle these.

C. LEARNING, TEACHING AND TESTING TENDENCIES

The abovementioned characteristics of scientific texts give rise to certain learning, teaching and testing tendencies. Ideally students and teachers should be aware of this.

C1. Learning tendencies

Some of the following learning tendencies may sound familiar, particulary for students studying for conventional course examinations. Learning may be semi-rote, with considerable recourse to memory. A surface knowledge may be acquired, which is not yet integrated into the student's framework, and will not pass into long-term term memory. The perspective may be formula-centred, with the student worried about reproduction of algebraic steps. Qualitative reasoning, physical insight, and interpretation will be weakly developed. Thus the understanding will be incomplete and unbalanced. (This can be revealed by innocent non-formalism questions). The student may be able to follow the material as presented, but not generate the main features independently some time later. Trees and forest may be unseparated in the student's mind, and hence the student may have the impression of lots to be remembered in a field like physics, rather than just a few powerful principles and laws.

The situation described also tends to give the student a particular (and unfortunate) view of the nature of science, and of the nature of understanding in science. It is also unlikely to provide much aesthetic or intellectual satisfaction, nor the motivation that derives from this.

Students may not in fact be aware of the shortfalls in their own understanding. Unjustified satisfaction with their level of understanding may be reinforced if they pass conventional examinations which emphasize formalism and memory.

C2. Teaching tendencies

The inherent features and limitations of scientific text could in principle be overcome by appropriate instruction and explanation. The explanatory function of a teacher would be an important aid to the text. However, here also there are certain tendencies which may thwart this potential.

Classes may be large, and the conventional lecture method consists mainly of "delivery" of material. Teachers, particularly university lecturers, tend to teach "by the book". There is pressure to 'cover' a syllabus. There is also a tendency to teach as tone has been taught, and to teach what is easiest to teach (eg the formalism), neglecting the harder and less visible aspects (processes). When taught by content-matter experts the approach will naturally be content-centred. The teaching will in fact probably match the book derivations and end-of-chapter exercises quite well. To compound all this, lecturers have little training in education or their teaching task. A good instructor will have to be aware of these tendencies and work consciously to combat some of them.

C3. Testing tendencies

There are also certain testing tendencies which prevent the development of balanced understanding. The type of questions used in tests strongly affects learning.

It is hard to test well at higher levels in science in a conventional examination format. When one is restricted to say 20 minutes per question in an intermediate electromagnetism course, one must usually ask something fairly routine. Hence bookwork and formula-centred problems may predominate. (A different exam format, eg take-home, could overcome this). Pragmatically, one also tests what is easiest to test and to mark. Students' learning objectives will be formed by the type of questions they see in the exams. If one wishes to encourage a different emphasis and type of learning, one has to work hard at devising assessment to match this.

D. LEARNING APPROACH: ANALYSING, TRACING THE ESSENCE, QUESTIONING, RESTRUCTURING

The recognition of the abovementioned problems in learning and teaching science, particularly from text, prompted the development of a learning approach for scientific derivations.

The approach taken is basically constructivist. The learner, using the text and other resources, must construct the material in his or her own mind. Such a construction of knowledge and understanding will involve active processing and reconstruction of the material. A scientific derivation must therefore be analysed, interpreted, added to, and resynthesised. The process will involve posing questions at every stage. Reflection on and management of ones own cognitive processes is important. There will be several passes through the material, focussing on different aspects each time. The first pass may well be linear, to get an overview of what its all about. In subsequent passes one will purposefully jump around.

Aspects of the process are given below. In practice, many of these processes will become tacit and automatic rather than conscious. We have in mind rather a lengthy derivation involving several inputs.

* Analysing.
Taking apart the derivation, identifying and discriminating its various 'ingredients'.

* Characterising.
Characterising ingredients by nature and function, eg as general principle or law, assumption, definition, relationship, manipulation, constraint, special case, knowledge item, etc.

* Tracing the essence.
This involves separating essence from detail, and tracing the main lines of the derivation. Detail is set aside at this stage. One builds mental lines across the text material, linking central items, seeing how they relate and contribute. One constructs an overview of the heirarchical organisation of the material. Processing is easier if one groups coherent parts of the derivation, and 'chunks' items - eg one may treat three lines of pure mathematical manipulation as a chunk.

* Links to external material.
It may be necessary to link to ancillary material not explicitly in the presented text in order to complete one's own picture.

* On hold.
It may not be possible for a particular learner to understand certain aspects of a derivation without going outside of the material to get necessary knowledge and skills not yet possessed, or obtaining help. The learner can either take time to do this, or can temporarily accept certain things pending this, in order to get the gist of the rest.

* Scene-setting.
One may need to set the scene for oneself by placing the material in a broader context and specifying both broad and specific aims.

* Tracing backward - origins and dependencies.
By working backward through the material, one can trace the origins of selected features of the result, and can explain the form and dependencies of the result. For example one might wish to explain the origin of the (ut) term in the kinematic equation $s = ut + {}^{1}/_{2}\,at^{2}$, or explain why t appears squared in the second term. Or one might wish to exlain why the mass does not appear in the expression for the period of a simple pendulum.

* Creation of instances.
To provide oneself with concrete examples, one may create specific instances to accompany the general theoretical treatment.

* Qualitative reasoning and physical insight.
A conceptual grasp of the material involves the qualitative behaviour of the system, eg knowing how a change in one quantity will affect another, without having to go through all the mathematical calculations. Physical insight into the behaviour of the system should enable one to predict the general features of the result without recourse to formalism.

* Interpretation and implications.
One should be able to interpret the result, and note its implications.

* Restructuring.
The analysed material can be modified and restructured, possibly with a different ordering, emphasis or approach, and notes or explanations added, to suit the personal cognitive needs of the

learner. This can be done mentally during processing, and also in writing. Very often, a rewritten restructured derivation, with personal explanations, is the most valuable tool for study or reference at a later date. The act of rewriting helps to make the material 'one's own'. This version need not, and preferably should not, contain all the details; it should be an explanatory ouline, emphasizing the essence. Note that in the process of restructuring, the 'centre of control' of the topic shifts to the learner, rather than being with the text. This is important for active, lasting learning.

* Self-test.
One can test one's own understanding by setting the material aside, and trying to generate the essence mentally. Unless one can do so in a few minutes, (and not from short-term memory), one has not yet properly understood or asssimilated the material. Eventually, the material must come out of oneself if it has been properly intergrated into one's knowledge structure. Also the act of trying to regenerate independently is itself a valuable learning tool during processing.

* Teaching.
Real or imagined teaching and explanation of the material to another person is valuable in developing and checking understanding. Gaps come to light.

Note that much of the above processing for understanding is done tacitly by experts, with only partial conciousness of their own thought processes. The main conscious focus is on the material, and it is only when experts find new material difficult that metacognitive awareness and strategies become more consciously deliberate. Students on their own may or may not develop good strategies for learning, and if they do it may take many years. It would seem educationally more efficient to provide students with explicit guidance on how to do it.

During instruction, the instructor can ask questions aimed at bringing out the various aspects of understanding. These questions would form the model for self-questioning by the student in future. If study assignments are given, questions about the material can form part of the assignment, to promote thought about particular aspects, guide the study, and help students assess their own understanding. Mini-problems can also be set to accompany study, each of which focusses explicitly on a particular aspect of understanding, (eg explaining physically a particular dependence in a result, or taking a specific instance to help concretize the general).

894

E. SELF-QUESTIONING GUIDES

Self-questioning guide forms have been devised to help students develop learning skills for scientific derivations. It was found that discussion of the issues during discussion of particular sections did work, but was soon forgotten; printed guide forms had more impact and permanence.

Note however that such guide forms are NOT intended to serve as a mechanical aid to learning or teaching, simply to be "filled out". This would go counter to the constructivist model of learning that is being advocated. The forms are general guides, not yet adapted for a particular topic or group of learners; they can function as models for teacher and student to produce modified specific guides. Ideally, the student would devise his or her own self-questioning guide for the particular topic at hand. It is also not appropriate for teachers to look at such forms as a packaged teaching device; it is important that they construct their own guides to match their own particular teaching situations.

The general guides are shown on the following pages.

895

SELF-QUESTIONING GUIDE 1:

1A. SCENE-SETTING AND AIMS:

The place of this in the broad picture is . . . .

What are the broad questions and problems of interest in this field? . . . .
And the specific questions in this topic? The problems?
. . . .
Aims? . . . .
Generally, how would one go about tackling this? Where would one start? What approach? Using what? . . . .
. . . .

1B. ANALYSIS AND TRACING ESSENCE

To do the above, we START from the following: ("ingredients").

ITEM	CHARACTER (eg law, constraint)
1	. . . .
2.	. . . .
3.	. . . .
etc	

And IF

1.	which means . . . .
2.	which means . . . .
etc.	

THE INTENDED GENERAL METHOD BEING TO:
. . . .
. . . .

THEN, by doing . . . .
and . . . . (basic steps/procedures)

IT FOLLOWS THAT
. . . . (Result)

Note that such an analysis scheme leaves aside all details and mathematics, but traces the essential ingredients and lines of reasoning.

SELF-QUESTIONING GUIDE 2

EXPLANATION OF RESULT

Tracing backwards through the derivation, to explain the origin, form and features of the result.

*Thus this result ARISES FROM (and depends on) . . . .
and . . . .
and . . . .

(Identify assumptions, relationships, restrictions, special cases, knowledge items etc)

*From whence cometh . . . . . . ? (particular feature of result) . . .
. . . . (trace back)

*We note that the DEPENDENCY of . . . . on . . . .
. . . . . is of the form . . . .
Which arises basically because . . . . and . . . .
. . . .

*We see that IF. . . . . (changed situation)
then . . . .

*The REASON THAT . . . .
is essentially because. . . . .

SELF-QUESTIONING GUIDE 3

INTERPRETATION

• QUALITATIVE REASONING
This result has the features . . . . and . . . .
which mean that . . . . and . . . .
The dependence on . . . .(variables)
is . . . .(eg inverse, squared etc),
which means the type of behaviour is. . . . .

•PHYSICAL INTERPRETATION
The physical interpretion of the result is that . . . .
.

•IMPLICATIONS
The implications of the result are that . . . .
and . . . .

•SPECIAL CASES
Interesting special and limiting cases are . . . .
and . . . .

•PHYSICAL INSIGHT
Does the result and its dependencies make physical sense? . . . .
Explain. . . . .
Could its general features be obtained by physical reasoning? (ie without the full mathematics). . . . .

•CREATION OF INSTANCES
Illustrative specific instances would be . . . .
and . . . .

•ARITHMETIC ILLUSTRATION
Arithmetic or ratio examples to highlight the dependencies might be . . . .

SELF-QUESTIONING GUIDE 4

4A. THE SUBJECT-MATTER APPROACH CHOSEN

The approach chosen here by the writer is . . . .
. . . .
. . . .

Alternative approaches would be . . . .
. . . .

with the features/consequences that . . . .
. . . .

4B. THE PRESENTATION

The author's way of presenting the topic was . . . .
. . . .
For the learner, this had the features . . . .
. . . .(good and bad)

An alternate presentation, or ordering, would be
. . . .

I liked/didn't like/ would have preferred . . . .

The way I would teach it would be . . . .

NOTES:

1. It is clearly not possible to go through such a comprehensive type of analysis and reconstruction for each derivation in a course. However it is desirable to do it for a few selected topics, so that the learner becomes aware of the many aspects involved in constructing real understanding in science, and has a model on which to base his or her own processing of material in the future. The explicit metacognitive aspect of the self-questioning approach gives many students an appreciation of the nature of real understanding for the first time.

2. Assessment must go hand in hand with the type of understanding emphasised in the course. If the aspects of understanding considered in this paper are considered important, then questions should be devised to assess, reward and encourage them, otherwise examinations will belie objectives, and much teaching effort may be in vain.

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ERRORS AND MISCONCEPTIONS IN COLLEGE LEVEL THEOREM PROVING

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INTRODUCTION

In this paper we describe a number of types of errors and underlying misconceptions that arise in mathematical reasoning. Other types of mathematical reasoning errors, not associated with specific misconceptions, are also discussed. We hope the characterization and cataloging of common reasoning errors will be useful in studying the teaching of reasoning in mathematics.

Reasoning in mathematics is no different from reasoning in any other subject. Mathematics does, however, contain many exceedingly long and intricate arguments. The understanding of such arguments is essential to the correct application of mathematics as well as to its continued development. Indeed, in mathematics it is the proofs that provide the strong consensus on the validity of basic information that is characteristic of a science. From this point of view, the idea of proof in mathematics corresponds to those special techniques of observation and experimentation which are essential to the other sciences.

The examples presented here arose as student errors in a junior level course in abstract algebra, taught several times at universities in the United States, Turkey, and Nigeria. A principal goal of the course is the improvement of reasoning ability. Before giving a brief description of the course and the methods used in teaching it, we will comment on a widely held conception of the nature of mathematics and its teaching. The inadequacy of this concept interferes with a student's ability to benefit from mathematics instruction and, in particular, delays improvement in his reasoning.

THE STATIC VIEW OF MATHEMATICS

One way to view lower division college mathematics is that it consists of (1) concepts, (2) algorithms for solving problems, and (3) implementations of these algorithms, including the selection of the proper algorithm. As part of this view, the correctness of an answer depends entirely on selecting the right algorithm and on implementing its steps correctly. We will call this the static view because it suggests that most important work on this level depends only on the implementation of unchanging algorithms. Increased mathematical competence is seen as equivalent to knowing more algorithms.

Here is an example of an algorithm as we mean it in this context. Consider a continuous function f defined on an interval [a,b]. To find the maximum value of f on [a,b]: (1) find the derivative f' of f, (2) set f'=0 and solve the equation, (3) find where f' does not exist, (4) evaluate f at all of the above points and at a and b, (5) the largest of these values is the maximum.

Of course this simple view of the nature of lower division college mathematics leads to a simple view of how it should be taught. The concepts should be explained, the algorithms should be provided, and the implementation of the algorithms should be practiced.

We think this is the conception of mathematics and its teaching held by many college students. We do not claim that they consciously ascribe to a description such as this or that they think about this topic at all. Rather, we note that they act as if they did and that they do so persistently.

Any teacher of lower division college mathematics can observe this by asking students to solve problems for which they have not been provided algorithms. Even if the teacher explains that this will require combining familiar techniques used on previous problems, many students will be disturbed. Often students who are reluctant to characterize mathematics or its proper teaching are definite about what it is not; it is not solving problems without instructions.

Turning now to the textbooks used in teaching lower level college mathematics, we see that their structure is in agreement with the static view. Emphasis is placed on explaining concepts and algorithms clearly and on sample solutions and practice problems. It is true that a scattering of theorems and proofs is usually included. Most teachers, however, think this material is more or less ignored by students and it is tempting to think authors agree. Often theorems are written in a style which makes them shorter and more memorable but harder to link to their proofs. It is second nature for mathematicians to expand such writing, adding missing quantifiers etc., so that they can understand the proofs. Such amplification is beyond most students who have no training or practice with it. Indeed we have occasionally found a theorem in a calculus book which could not be unambiguously understood by anyone not already familiar with the theorem.

INADEQUACY OF THE STATIC VIEW

Problems arising outside a mathematics course often do not match exactly the algorithms covered in the course. This is inherent in the general nature and broad applicability of mathematics and cannot be avoided merely by expanding the list of algorithms covered. Applications of mathematics, even on the lower division college level, will often require the creation of a new algorithm, at least in the sense of altering and combining familiar procedures.

Since, even without instruction, everyone has some ability to adapt algorithms to new problems, we will describe an example illustrating the impoverished level of this ability in a typical first calculus class. It is this that renders the static view of mathematics inadequate.

Consider two related problems: (1) Given a curve and a point on it, find where the tangent line at that point crosses the x-axis. (2) Given a curve, find a point on it so that the tangent line at that point passes through the origin. The solutions to these problems involve the same techniques: finding and evaluating derivatives and formulating and solving linear equations. Suppose an algorithm for the first problem is provided in class and the second problem is not mentioned. Our experience suggests one can expect at least 75% of the students to be able to work the first problem and less than 20% the second.

Clearly, if mathematics is to be widely useful, the static view of it is inadequate. Even on the lower college level, mathematics should include the creation of algorithms, at least in the sense of combining and altering known techniques. Since correctness of a newly created algorithm cannot be ascertained by appeals to authority, reasoning should also be regarded as an integral part of mathematics and its teaching. We do not mean to suggest that reasoning on this level should be in the form of proofs, but some examination of the correctness of algorithms should be included.

Perhaps this point can best be illustrated by looking outside mathematics to programming courses. Students are required to produce their own algorithms and it often happens that these algorithms do not perform as expected. As a result, the validation of programs through testing is regarded as an important and necessary component of the discipline.

The inadequacy of the static view of mathematics not only limits the usefulness of early college mathematics courses, it also inhibits the development of informal reasoning skills. These provide the foundation for the more formal reasoning required in proofs.

THE ROLE OF PROOFS

Proofs are not only essential to the development of new mathematics, their proper reading is an integral part of the understanding of advanced topics. Reading a proof is much more active than generally supposed. Subtle questions must be asked and answered. It is difficult to know when a student is doing this correctly, but it is easy to see if he has written a proof correctly. Since skills in reading and writing proofs are interdependent, often students are asked to write proofs as the only clear indication of their understanding of a topic.

ABSTRACT ALGEBRA

Abstract algebra, the course from which our examples are selected, is a three hour per week year sequence usually taken in the third year of college. At this stage in a student's education the rapid development of reasoning skills is important and we have taken it as a major objective of the course.

Abstract algebra is a particularly good vehicle for teaching reasoning because the proofs are less complicated than, say, advanced calculus, and the notation is not particularly complex. It is also useful that students have not seen this topic before and it is difficult for them to lift proofs from textbooks, so they must rely on their own reasoning and ideas.

THE SOCRATIC-MOORE METHOD

We provide the students with a short set of notes containing definitions, theorems, problems, and occasional examples, but no proofs or solutions. We do not give formal lectures although we answer questions. Students present their work in class and we provide evaluations, detailed criticisms and suggestions [1].

In the mathematical community this technique is sometimes called the Moore method after the late R. L. Moore who practiced it with remarkable success (Forbes, 1971). Of course in a broader setting this sort of teaching has a long history and calls to mind the methods of Socrates. There are many versions of this method, and seemingly insignificant variations in it may greatly alter its effectiveness.

REASONING ERRORS

Students make a great variety of reasoning errors in attempting proofs. We feel some of these errors are based on underlying misconceptions, while others, although repeatedly observed, are of a technical or other nature. Students persist in making both types of errors [2]. For the former, we offer our views as to the possible underlying misconceptions, that is, we give a general rule or idea which, if believed by a student, would result in that type of error. These errors are taken to have a rational basis, and we comment on how they might come about [3]. For the latter, we sometimes speculate on the underlying causes of the errors, but do not see them as conceptual in nature. Each type of error is illustrated with one or more actual student "proofs" [4].

REASONING ERRORS BASED ON MISCONCEPTIONS

M1. Beginning with the conclusion, arriving at an obvious truth and thinking the proof is complete. Of course, this provides a valid argument if and only if the steps are reversible.

The misconception consists in thinking that one valid technique of proof begins with the conclusion and ends with a known fact. However, this is not acceptable as it is often difficult to arrange a proof into a sequence of discrete steps, each of which can easily be checked for reversibility. This misconception may have arisen from methods learned in secondary school for verifying trigonometric identities and solving equations. Also, since a good heuristic for discovering a proof is to analyze the meaning of the conclusion, college students may have seen this presented in class, along with a statement that all steps are reversible. They, thus, could easily be confusing discovery with proof.

Example

Theorem: Let G be a group such that for all g in G, $g^2=e$, where e is the identity of the group. Then,

(i) for all g in G, $g=g^{-1}$ and,

(ii) G is commutative.

"Proof" of (ii) having proved (i): To show G is commutative means, for all a and b in G, it must be that ab=ba. Multiplying ab=ba, by the appropriate inverses, and using part (i), one gets a=bab, and b=aba. Now,

(1) $b^*=aba^*=(bab)ba=ba(bb)a=baea=baa=be=b$, and

(2) $a^*=bab^*=(aba)ab=ab(aa)b=abeb=abb=ae=a$.

Comment

There are no errors present in lines (1) and (2). In this case, the steps indicated with a "*" are not reversible; they are equivalent to $ab=ba$.

M2. Names confer existence. This error occurs from failing to distinguish between symbols for things whose existence is established and symbols for things whose existence is not established. Often this error occurs when a student attempts to solve an equation, without questioning whether a solution exists. We asked a class of precalculus college students to solve the equation $\cos x=3$ in order to test whether they could apply the fact that the range of $\cos x$ is $[-1,1]$. Most tried unsuccessfully to manipulate the equation to find x and were unable to reach the proper conclusion.

A different example of this type of error occurred when an abstract algebra student attempted to prove that a semigroup in which the equations $ax=b$ and $ya=b$ always have solutions is a group. One must first establish the existence of an identity element, e, and then show that each element, g, in the group has an inverse. The student attempted the second part by contradiction, supposing that g had no inverse. Then, in the very next line, he used the symbol g^{-1}, which he just assumed didn't exist, and made calculations with it.

The underlying misconception is that names always represent existing things. Writing $\cos x$ or g^{-1} seems to confer existence on and the right to manipulate the symbol. Perhaps this comes from secondary school algebra where x is referred to as "the unknown", that is, as something which exists and should be found.

If one asks an abstract algebra student to prove that the equation $ax=b$ has a solution in a group, he will often proceed as follows: $ax=b$, so $a^{-1}ax=a^{-1}b$, so $x=a^{-1}b$. He does not realize that by manipulating the equation he is tacitly assuming x exists. Of course, he should produce a group element, in this case $a^{-1}b$, which when substituted in the given equation yields a true statement.

One final simple-minded example illustrates the difficulties that can occur. Given $(x+3)(x+2)=(x+1)(x+4)$, one can conclude $6=4$, by supposing there is such an x.

M3. Apparent differences are real. This error occurs when things which have different names are taken to be different. The underlying misconception is there is a one to one correspondence between names and mathematical objects.

Although students realize that the same real number can be written in many different ways, for example, $1/2 = 2/4$ or $3 = 1+2$, often it does not occur to them that two different abstract expressions may represent the same thing. This happens even though they know two apparently different trigonometric expressions can be equal from having verified identities.

Example

Theorem: If a commutative group has an element of order 2 and an element of order 3, then it must have an element of order 6.

"Proof": Let g be the element of order 3 and let h be the element of order 2. The $g^3=e$ and $h^2=e$ where e is the identity of the group.

Consider the subgroup generated by hg. Since $h^6g^6 = (h^2)^3 (g^3)^2 = e^3e^2 = e$, this subgroup is $\{hg, h^2g^2, h^3g^3, h^4g^4, h^5g^5, h^6g^6\}$ which simplifies to $\{hg, g^2, h, g, hg^2, e\}$ using $g^3=e$ and $h^2=e$. So hg has order 6.

Comment

Something is missing here, namely an argument showing that the 6 symbols hg, g^2, h, g, hg^2, e represent 6 distinct elements. This can be shown, but one shouldn't assume the student can show it or is even aware that he must.

Example

Lagrange's Theorem: Let G be a group of order n. Let H be a subgroup of order m. Let r be the number of distinct right cosets of H in G. Then n=rm.

"Proof": Let the distinct right cosets be H, Hg_1, ..., Hg_{r-1}. These form a partition of G into equivalence classes. Let $H=\{h_1,\ldots,h_m\}$. Then $Hg_s=\{h_1g_s, h_2g_s, \ldots, h_mg_s\}$ has m elements in it. Thus G is partitioned into r classes, each with m elements, so G has n=rm elements.

Comment

Again, the student has concluded there are m distinct elements by counting up m distinct symbols. He is tacitly assuming the symbols represent different elements.

M4. Using the converse of a theorem. This is a classic and extremely persistent reasoning error. The misconception consists in equating an implication and its converse. Even students who have been taught that there is a difference make this error, especially when a theorem is complicated.

The basis for this misconception seems to be the imprecision of everyday language. People often use the "if, then" construction when they mean "if and only if." When someone says "if it rains, I won't go", he often also means, but doesn't explicitly say, "and if it doesn't, I will", which is logically equivalent to the converse. Mathematicians and textbook authors may reinforce this confusion when they state definitions using "if", but actually mean "if and only if".

Example

Theorem: Let G_1 and G_2 be two groups contained in a semigroup S such that $G_1 \cap G_2$ is nonempty. Then $e_1=e_1e_2e_1$, where e_1 is the identity of G_1 and e_2 is the identity of G_2.

"Proof": By an argument with another type of error, the student concludes $e_1=e_2$. Then $e_1e_2=e_1e_1e_2=e_1e_2e_1$, so $e_1e_2=e_1e_2e_2=e_1e_2e_1e_2$, so e_1e_2 is an idempotent. The identity of a semigroup is an idempotent. Therefore, $e_1e_2x=x$ for all x in S. Let $x=e_1$, $e_1e_2e_1=e_1$.

Comment

The student has used the converse of the theorem that the identity is an idempotent in his penultimate line, not to mention the fact that the semigroup under consideration was not given as having an identity. As is often the case, this student has made multiple errors.

M5. Real number laws are universal. Students who take abstract algebra at the junior level have very little idea that mathematics deals with objects other than geometric configurations and real and complex numbers. Thus, the examples of abstract concepts like group must be rather simple, and we tend to stick to real and complex numbers, matrices, and functions with which they have some familiarity. Even this limited collection of examples is rich enough for students to see that not everything behaves the way the more naive students expect. What they expect is really a misconception, namely, that the rules they know for dealing with real numbers are universal.

Example

Theorem: A semigroup can have at most one identity.

"Proof": Suppose the semigroup has two identities, e and e'. Then $es=s$ and $e's=s$, so $es=e's$. Hence $e=e'$.

Comment

When it was pointed out to the student that the last step of his argument amounted to cancelling the s, and he was asked why that was permissible, he said, "That's not cancelling; that's logic."

Example

Theorem: Given a group G of finite order n. For each g in G, $g^n=e$ where e is the identity of the group.

Further, if H is a normal subgroup of order r, then for each g in G, g^m is in H, where $m=n/r$.

"Proof" of the second part: From Lagrange's Theorem, we know the number of distinct cosets of H in G is m. Now $g^m=g^{n/r}=(g^n)^{1/r}$. By the first part, $g^n=e$, so $g^m=e^{1/r}$. This is followed by an argument attempting to show $e^{1/r}$ is in H.

Comment

It apparently never occurred to the student to question whether the 1/r power of an arbitrary group element made sense; after all, there is nothing strange about taking the r-th root of a positive number.

Example

Theorem: A group G in which every element is of order 2 is commutative.

"Proof": Let g, h be elements of G. By hypothesis, $(gh)(gh)=1$ and $(hg)(hg)=1$, where 1 is the identity of G. Then $(gh)(\cancel{g}h)=(hg)(hg)=h(\cancel{g}h)g$ by the cancellation law for groups. So $gh=hg$.

Comment

Here the cancellation law for groups was not used correctly. The error could easily have resulted from thinking of the cancellation law in the real numbers, where cancelling this way is allowed, and even, routine.

M6. Conservation of relationships. This type of error occurs when students act as if doing the same thing to both sides of any relationship preserves the relationship. For example, given $h\neq k$, they will conclude $gh\neq gk$. They will do this in an abstract algebra proof, even though they are aware that in the real numbers, one must know $g\neq 0$. They may also be aware that in matrices, one must know g is nonsingular to conclude $gh\neq gk$.

This seems to be an instance of improper generalization from past experience. In secondary school algebra, one can say if $a=b$, then $ac=bc$, and if $a<b$ and $c>0$, $ac<bc$. The misconception is that relationships can be preserved by operating on both sides in the same way.

As a variant of this, we note that occasionally students will act as if expressions are preserved, even when there aren't two sides. Students will simplify the polynomial $4x^2+2$ to $2x^2+1$.

M7. Element set interchanges. Student understand statements involving elements more easily than equivalent statements about sets. Our beginning abstract algebra students had difficulty making proofs when the notion of a subsemigroup T of a semigroup S was defined to be a nonempty subset T of S such that $TT \subseteq T$, the product of two sets having been defined previously. However, when a subsemigroup was defined to be a nonempty subset T of S such that for all a and b in T, ab is in T, students made better proofs.

This strategy of delaying potential confusion is helpful, however, set concepts must eventually be introduced. The usual definition of a normal subgroup H of a group G is given in terms of the equality of left and right cosets; that is, a subgroup H is normal if and only if $gH=Hg$, for all g in G. Students often covert this to $gh=hg$; they incorrectly think they can merely substitute h for H. Even when told explicitly that $gH=Hg$ means that given gh in Hg, there is an h' in H so that $hg=h'g$, they revert to writing $gh=hg$ in making proofs.

The misconception is that information about a set is interchangeable with information about a typical element of that set. So each time the set H appears, it is permissible to replace it by h.

OTHER ERRORS

E1. Overextended symbols. This error occurs when one symbol is used for two distinct things, often because the distinction was unobserved. Such errors can indicate an incomplete grasp of a mathematical structure, such as group, and first appear when the structure is used several times in the same setting.

Example

Theorem: Let G_1 and G_2 be two groups contained in a semigroup S such that $G_1 \cap G_2$ is nonempty. Then $e_1=e_1e_2e_1$, where e_1 is the identity of G_1 and e_2 is the identity of G_2.

"Proof": There is an element g in $G_1 \cap G_2$; $g=e_1g$ and $g=e_2g$, so $e_1g=e_2g$. Since g is a group element, g^{-1} exists, and $e_1gg^{-1}=e_2gg^{-1}$, $e_1e_1=e_2e_2$, $e_1=e_2$. Multiplying on the left and right by e_1 yields $e_1=e_1e_1e_1=e_1e_2e_1$.

Comment

The error consists in not distinguishing the two different kinds of inverses that exist. There is an inverse of g in G_1, one might call it ${g_1}^{-1}$, and an inverse of g in G_2, one might call it ${g_2}^{-1}$.

In the same way, students often fail to distinguish between equivalence classes coming from different equivalence relations.

Example

Theorem: Let G be a group, H a subgroup of G, and K a normal subgroup of G. Then $F:HK/K \rightarrow H/H \cap K$ defined by $f(hkK)=f(hK)=h(H \cap K)$ is an isomorphism.

"Proof" that f is one-to-one: Suppose $f(h_1K)=f(h_2K)$. Then $h_1(H \cap K)=h_2(H \cap K)$ so $[h_1]=[h_2]$ so $h_1K=h_2K$.

Comment

What is needed here is two different symbols for the two different equivalence classes, such as $[\ ]_{H \cap K}$ and $[\ ]_K$. One then sees that an additional argument is required.

E2. Weakening the theorem. This error occurs when what is used is stronger than the hypothesis or when what is proved is weaker than the conclusion. Often a student thinks it's clear he has proved the theorem. Adding to the hypothesis is a well-known technique of practicing mathematicians. If one cannot prove a conjecture as it stands, one can add to its hypothesis and attempt to prove a weaker result. However, students rarely realize they are proving a weaker result.

Errors of this kind occur when a student tacitly assumes a group is finite although nothing is stated about the order of the group, when a student assumes a semigroup has an identity although that is not given, or when a student assumes a group is cyclic.

Example

Theorem: Given a semigroup S with identity 1, and left cancellation. The S has only one idempotent, 1.

"Proof 1": Let's suppose we have a group with 1. Let e be an idempotent. Then ee=e. Multiplying by e^{-1} gives $e^{-1}ee=e^{-1}e=1$, but $e^{-1}ee=1e=e$, so e=1.

Comment

What the student has actually shown is that a group can have only one idempotent, the identity. The hypothesis was strengthened. Another student weakened the conclusion as follows.

Example

"Proof 2": Suppose there are two identities s and t, then as=a and at=a, so by left cancellation s=t. This unique identity is an idempotent as a·1=a for all a in S, so in particular 1·1=1.

Comment

This student has shown the weaker result that a semigroup identity is an idempotent, rather than that a semigroup identity is the only idempotent, given left cancellation.

E3. Notational inflexibility. This error arises from an inability to adapt notation from one context to another. As is customary in abstract algebra, we use multiplicative notation in all definitions about noncommutative groups and semigroups. However, additive examples are considered in class, and commutative groups, especially in the case of rings, are written in additive notation.

For example, the cyclic subgroup generated by the element h in a group G is defined to be $H=\{h^n:n \text{ is an integer}\}$. On being asked to find the cyclic subgroup of the additive reals generated by 1, students occasionally answer $H=\{1^n\}=\{1\}$, which is incorrect. In additive notation, the cyclic subgroup generated by h is $\{nh:n \text{ is an integer}\}$.

A similar error occurred when a student was asked to find the kernel of a homomorphism. The group was the nonzero reals under multiplication and the function was defined by $f(x)=|x|$. One student wrote: $K(f)=\{x:f(x)=0,\ x \in R-\{0\}\}=\{x:|x|=0,\ x \in R-\{0\}\}=\emptyset$. He used the additive identity, 0, in place of the multiplicative identity, 1. The student should have suspected his answer was incorrect, as a previous theorem stated that the kernel is always a subgroup, and hence, nonempty. Students often fail to notice if their answers are reasonable, that is, in agreement with previously obtained results.

E4. Misuse of theorems, other than the converse. In applying a theorem, an error of this type arises from misunderstanding or partly neglecting the hypothesis or misinterpreting the conclusion. A student's inability to read precisely [5] combined with a desire to finish the proof quickly may result in such mistakes. This error has similarities with E2. In this case, the theorem being used is misunderstood, and in E2, the theorem being proved is misunderstood.

One student stated that if a group has subgroups of orders r and s, then it must have a subgroup of order rs, which is false. The student was probably thinking of the following: If a group has cyclic subgroups of orders r and s, where r and s are relatively prime, then it has a subgroup of order rs.

Another student stated that a subsemigroup H of a group must necessarily be a subgroup, forgetting the additional requirement that H must contain the identity and the inverse of each element. A counterexample was well within the student's mathematical experience; the open unit interval is a subsemigroup, but not a subgroup, of the positive reals under multiplication. Once again, the student has not checked to see if what he claims is reasonable.

E5. Circularity. This error consists in reasoning from a statement to itself. Often the reasoning is from one version of the conclusion to another, already known to be equivalent to the first.

Example

Theorem: Let G and K be groups. If $f: G \to K$ is an onto homomorphism and H is a normal subgroup of G, then f(H) is a normal subgroup of K.

"Proof": It has already been proved that f(H) is a subgroup of K, so it remains to show that $kf(H)k^{-1} \subseteq f(H)$ for all k in K. Now $kf(H)k^{-1} \subseteq f(H)$ implies $kf(H) \subseteq f(H)k$. We also need to show $f(H)k \subseteq kf(H)$. This implies $k^{-1}f(H)k \subseteq k^{-1}kf(H) = f(H)$, which in turn implies $kk^{-1}f(H)k = f(H)k \subseteq kf(H)$. Thus, $f(H)k = kf(H)$, so f(H) is normal in K.

Comment

The student began with the conclusion. In the first part of the argument, he went from one equivalent version of normal to half of another. He then took the remaining half and got back where he started. Each instance is an example of circularity. The student doesn't understand what constitutes a proof but realizes arguments progress from one piece of information to another.

E6. The locally unintelligible proof. In this error neither the proof as a whole nor most individual sentences can be understood. The format is acceptable, the words "theorem" and "proof" occur together with many symbols, and most sentences are syntactically correct, however, the assertions are incomprehensible or incorrect. Such "proofs" may be first approximations at imitating textbook and classroom proofs [6].

Example

Theorem: A commutative group which has an element of order 2 and an element of order 3 must have an element of order 6.

"Proof": Let G be a commutative group, $H \subseteq G$. Let $g_1 \in G$, $g_2 \in H$, $g_3 \in G$. According to Theorem 55 of the notes (on using a subgroup H to define the right coset equivalence relation on G), $Hg_1 \subseteq G$.

Then $Hg_1=g' \in G$. Let a be the order of g', which is the number of distinct right cosets of H in G. Let b be the order of g_2, c be the order of g_3. Since $g_3 \in G$, c is the order of G. We want to show c=ab. According to Lagrange's Theorem, the order of a group is equal to the order of a subgroup times the number of right cosets of that subgroup in the group. Therefore, c=ab. In this theorem a=2, b=3, c=2·3=6. This gives an element of order 6.

Comment

On the surface this appears to be a proof. It starts and stops in an expectable way and quotes Lagrange's Theorem correctly. It is also syntactically correct, except for one small place. However, it is impossible to find any basic underlying idea which the student might have started with or to follow the individual sentences [7].

The student may have selected Lagrange's Theorem almost arbitrarily [8] and tried to develop it into a proof.

E7. Substituting with abandon. This error consists in obtaining one statement from another using an unjustifiable substitution. One fixed element is replaced by another unequal fixed element. Of course, it is permissible to substitute for a universally quantified variable; and perhaps, this error results from confusing the two situations.

This error often occurs when a student attempts to prove a theorem which begins "For all s in S, ... ". The standard way to start the proof is to write "Let $s \in S$." With these words, s becomes a fixed element, and one is no longer free to substitute for s. Occasionally, to emphasize this point, one writes, "Let s be an arbitrary, but fixed element of S."

Examples

Theorem (Cancellation Law): Let G be a group. Let g, h, k be elements of G. If gh=gk, the h=k.

"Proof": Let e be the identity of G. Substitute e for g in gh=gk. The eh=ek, so h=k.

Theorem: Let G be a group with identity e. If g, h, k are elements of G so that gh=e=hg and gk=e=kg, then h=k.

"Proof": We have gh=e=hg. Since $k \in G$, substitute h for k to get gk=e=kg. ...

Comment

In the first theorem, we have this error in its purest form; g was arbitrary, but fixed from the beginning, but the student substituted e for g, believing g to be a universally quantified variable. In the second theorem, both h and k are arbitrary, but fixed elements from the start; the only way to substitute one for the other is to know they are equal.

Example

Theorem: If G is a group of order n, then $g^n=e$ for all g in G, where e is the identity of G.

"Proof": Since G is finite, and one wants to show $g^n=e$ for all g in G, one can choose n to be zero or a value which gives the identity element.

Comment

The student has failed to realize n is given as an arbitrary, but fixed positive integer in the statement of the theorem. He is not free to choose it, rather he must prove something about it.

E8. Ignoring and extending quantifiers. This error results from failing to notice restrictions on variables. Often a variable is thought to be universally quantified when it isn't.

Example

Theorem: Given a group G and an element a in G. Then $H=\{g \in G: ag=ga\}$ is a subgroup of G.

Comment

One student concluded incorrectly that H was commutative because $ag=ga$, failing to notice that a is not a universally quantified variable.

Another student proved H was closed under multiplication as follows. Let $g, h \in H$. Then $ag=ga$ and $ah=ha$. Then $a(gh)=(ag)h^{*}=h(ag)=(ha)g^{*}=g(ha)=(gh)a$. In the equalities marked with a "*", the student assumed that since h and g commute with a, they also commute with ag and ha, respectively. He failed to notice that a is fixed, and not universally quantified, in the definition of H.

E9. Holes. This type of error consists in claiming that a statement follows immediately from previously established results when in reality a considerable argument is required.

Example

Theorem: Given a group G of order n and a normal subgroup H of order r, then for all g in G, $g^m \in H$ where $m=n/r$.

"Proof": First it was shown that $g^n=e$ for all g in G, where e is the identity of G. Then $g^n=g^{mr}=(g^m)^r=e$. Letting $k=g^m$, one gets $k^r=e \in H$. *** So $k \in H$.

Comment

The hardest part of the argument goes where we have inserted stars. This student grasps the concept of proof and has a reasonable overall approach, but has difficulty distinguishing between statements which follow immediately and those requiring justification.

E10. Using information out of context. In this type of error information from one argument is improperly used in another, often because identical symbols appear in both. This error is most likely to occur when proofs are organized into independent sections, for example, in theorems involving case analysis, set equality, or equivalence of statements. In such situations, a student may unjustifiably transfer information from one section to another. The next example is rather unusual in that the error involves two theorems, rather than two independent sections of one proof.

Example

Theorem (Cancellation Law): Let G be a group. Let $g, h, k \in G$. If $gh=gk$, the $h=k$.

"Proof": Since G is a group, it has an identity e and $gh=e=hg$. (A previous theorem with the letters g and h is invoked here.) Since $k \in G$, we can substitute k for h and get $gk=e=kg$. Then, by the previous theorem on uniqueness of group inverses, $h=k$.

Comment

Not only has the student used a piece of a previous theorem out of context in the first line; he has also made a substitution error (E7). This serves to illustrate that several of the reasoning errors described in this paper can occur within a single proof.

ANOTHER PERSPECTIVE

The reasoning errors analyzed above have been classified according to whether or not they are misconception based. It is also possible to classify reasoning errors according to their logical characteristics, that is, according to whether they arise from difficulties in generalization, use of theorems, notation and symbols, nature of proof, or quantification. We summarize these two classifications in the following table and note their independence.

ERRORS	Misconception Based	Other
Generalization	M5, M6	
Use of Theorems	M4	E4
Notation and Symbols	M2, M3, M7	E1, E3, E10
Nature of Proofs	M1	E2, E5, E6, E9
Quantification		E7, E8

GENERAL COMMENTS

The analysis and classification of reasoning errors presented in this paper suggests a number of questions:

(1) How complete is the error list? It would be useful to have a list sufficiently complete that each incorrect student proof contains at least one of the errors.

(2) How does each of these reasoning errors arise and how could it be prevented?

(3) Is the making of one type of reasoning error correlated with making others? Perhaps students who make a particular type of error always make another type.

(4) Which types of reasoning error occur most frequently?

(5) Do certain types of reasoning errors occur more in one course than another, for example, in algebra as compared with topology?

(6) Are any of these reasoning errors correlated with particular sections of students' earlier coursework?

If lower division mathematics courses were to ignore the static view and include significant instruction on creating and validating algorithms, it is possible that reasoning would be improved, as well as applications extended. Evidence concerning this point would be useful.

Finally, we note that there is remarkably little correlation between the reasoning errors we have observed and classified and the topics emphasized in an introductory logic course or even in one of the newer courses as on transitions to advanced mathematics [9].

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1. A detailed description of this method is given in Selden and Selden (1978, p.69-71).
2. This agrees with the general research on misconceptions (Novak, 1983, p.2).
3. An assumption shared by Confrey. (1983, p.30, p.25).
4. Occasionally extraneous material was edited out.
5. Lack of precision and accuracy have been observed in first year university students' attempts to solve physics problems (Mehl and Volmink, 1983, p.228). Lin (1983, p.202) has suggested that beginning physics students are unaccustomed to the necessary precision of expression.
6. Students indicate they solve mathematics problems by imitating textbook examples (Confrey, 1983, p.23).
7. The authors have made a painstaking attempt to find some basic underlying idea, plus a line-by-line analysis of this "proof" (Selden and Selden, 1978, p.78-9).
8. In making this "proof", the student appears to have acted in an unsystematic and somewhat impulsive way (Mehl and Volmink, 1983, p.228). He does not take what computer scientists would call a top-down approach.
9. We recently taught such a course several times and found the treatment in a typical text (Smith, Eggen, and St. Andre, 1986) only superifically related to these reasoning errors.

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Raising Laboratory Learning to a Conceptual Level: Does MBL Help?

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Introduction and Rationale

Everybody has a chemistry lab story: the time someone ignited a flammable, provoked a violent reaction, mouth-pipetted something dire, or otherwise flaunted safety and sense with dramatic result. I'd like to tell a different kind of lab story, a work in progress, and one which will have a happy ending if an increasing number of students grow up remembering more of chemistry than white powders, colorless liquids and one big bang.

I shall set this story within the realm of the Computer as Lab Partner Project. This project grew out of my interest in laboratory learning, an interest which developed throughout the course of my involvement with science education: first as a student, then as a teacher, and finally as a researcher. In the Computer as Lab Partner Project I had the great good fortune to collaborate with project members Marcia C. Linn, Doug Kirkpatrick, Rafi Nachmias, Yael Friedler and John Layman, and received invaluable assistance from faculty at the University of California Berkeley and from researchers at the Technical Education Research Centers (TERC) of Cambridge, Massachusetts. The synthesis presented here, however, is mine and does not necessarily reflect the views of any of these scholars nor of the funding agencies: Apple Computer and the National Science Foundation.

The reason most people have at least one exciting lab story is that the lab is an integral part of science instruction. When researchers and practitioners list goals or objectives for laboratory experience the language varies but it generally boils down to something like this taxonomy of laboratory skills (Kempa & Ward, 1975):

1. Planning and design of an investigation in which the student predicts results, formulates hypotheses, and designs procedures;
2. Carrying out the experiment, in which the student makes decisions about investigative techniques and manipulates materials and equipment;
3. Observation of particular phenomena; and
4. Analysis, application, and explanation, in which the student processes data, discusses results, explores relationships and formulates new questions and problems.

This set of skills maps very well onto our current idea of "higher cognitive skills", especially the "procedural skills cycle" of planning, testing and revising hypotheses or ideas.

Although the goal of fostering higher cognitive skills is generally accepted for the laboratory, it is rarely assessed in either teaching or research (Hofstein & Lunetta, 1982). A first step toward such assessment would be an examination of the activities which students perform in lab. I carried out such an examination in the course of evaluating a videodisc lab (Davis & Stein, 1984). Though we were comparing a traditional lab to the same experiment carried out in a videodisc environment, it is also interesting to use the observations to compare an actual lab to the stated goals.

The lab observed was part of an introductory biology course at a major west coast university. The purpose of the lab was to measure respiration rate (oxygen consumption) vs temperature for various organisms to draw conclusions about the effect of temperature on metabolic rate. In terms of the Kempa and Ward taxonomy, we observed:

Planning. Students spent most of the pre-lab time copying each other's summaries of the purpose and procedure of the fully-prescribed lab. There was large-scale inattention to the teaching assistant's preparatory remarks, most of which concerned the procedural logistics of the lab. This is consistent with a cross-cultural study reported by Hitano (1986) which found that American teachers focus pre-lab discussion almost exclusively on lab procedures rather than concepts.

Testing. Testing consisted of the serial execution of instructions contained in the lab book. When students did not understand what was meant by an instruction, a frequent problem, they would seek help from the t.a. This inefficient procedure resulted in numerous errors, some injury and equipment breakage, and a large percentage of off-task time on the part of the students. Little successful autonomous problem-solving was observed.

Observation. Due to equipment breakage during the lab, students were forced to work in groups of 4-5. Typically, only one student from each group was making observations, and these were incomplete.

Analysis. When students pooled their data points, the data presented was incomplete and some data points were clearly anomalous. There was no discussion or analysis of the data, students simply copied it into their notebooks and left the room. The lab was over.

Though admittedly anecdotal, I would argue that this lab was not atypical, but rather is symptomatic of common difficulties real-world laboratories face in trying to promote higher cognitive skills:

1. Students have no input to the design or topic of the investigation, nor are they required to "follow along", that is, to give rationales or make predictions.
2. The lab consists of following instructions, most of which concern the manipulation of materials.
3. Students rely on the teacher for immediate, personalized consultation, which is problematic given the 30:1 student:teacher ratio. Some students therefore will hazard a solution, "hazard" often being the operable word.
4. Results are accepted without analysis. For many students, the goal of the lab is to get the results from someone else.

So there seems to be a "cognitive gap" between the expected benefits of lab and the actual experience. The weakest area in the typical lab are planning and analysis. But these areas are critical because the procedural skills cycle is goal-driven. Two powerful theories of problem solving, Greeno's strategic planning and Larkin's means-end analysis center around choosing goals/ends and working toward them. It is no wonder lab students seem so error-prone since they give little indication of knowing, in terms of the aims of the experiment, what it is they're trying to do or why.

I maintain that in a laboratory assignment, students can be thought to be engaged in a type of problem solving. The problem posed by a lab activity is really twofold, and nested: in order to determine or verify some property of the physical universe ("the conceptual goal") students must carry out an empirical procedure ("the experiment"). The solution of a laboratory problem is not simply obtaining valid results, i.e. running the experiment correctly, but in the final application of those results to achieve the conceptual goal. If laboratory experience is to be used as a tool for conceptual change, the emphasis in lab must shift from rote procedures to the procedural skills of planning, testing and revision which underlie all problem solving.

Another environment thought to facilitate the acquisition of higher cognitive skills is the computer environment. The active, reproducible science lab is very like the interactive, precise and consistent computer learning environment. In addition both environments can be complex and challenging and can provide for multiple solutions. Thus both exemplify the features of cognitive demand identified by one model (Linn et.al., 1985). The most promising computer provisions are thought to be the feedback, which is immediate in both senses of the word (that is, both instantaneous and direct or striking), and the powerful representations, which can be flexible, graphic, real-time and manifold.

These promising features are seen to good advantage in microcomputer-based laboratory (MBL) systems. This is a relatively new class of labware which is not a simulation, but integrates traditional lab apparatus with computer-based data handling. The technology is based on probes, connected to the computer through an I/O port, which record quantities like temperature, light intensity, or pH. The readings can then be displayed directly on the computer monitor. One common display format, shown in **Figure 1**, is a plot of the probe data against time onto axes for which the student has selected ranges. Instead of spending the lab recording data points for later transformation, the student can see an instantaneous graphic display during the lab of the time course of a temperature change, or, in another instance, of the way light intensity falls off with increasing distance from the source.

The Computer as Lab Partner Project (Stein & Linn, 1984) proposed that microcomputer-based labware may be useful in directing students' attention back toward the conceptual goal of the lab by making experimental results both graphic and immediate. Using the computer to forge a strong and immediate feedback link between the process of doing a lab and the analysis of the results might give students an unprecedented opportunity to keep the underlying goal of a lab investigation in sight, to receive and respond to results in terms of that goal, and to modify experimental procedures accordingly.

The Study

The Computer as Lab Partner Project equipped a suburban middle school science lab with 16 computer systems running TERC-prototype "heat and temperature" MBL. A lab-based curriculum emphasizing thermal concepts was designed and evaluated over a three year period. An outline of topics covered in the curriculum is given in **Table 1**. Four classes of approximately 32 students each participated per semester. These classes were all taught by the same teacher, the head of the science department.

Further details of different aspects of the study can be found in analyses of students': graphing skills (Linn, Layman & Nachmias, in press); subject-matter achievements (Nachmias & Stein, 1987); perceptions about computer-presented data (Nachmias & Linn, 1987); and problem-solving processes (Stein, 1986).

The first three analyses listed above proceeded by written assessments, augmented in some cases by semi-structured interviews. To capture students problem-solving processes, I developed a system of analyzing audio-recordings of student pairs (dyads) engaged in carrying out a lab activity. This system categorizes three types of laboratory episodes: off-task, empirical, and conceptual. Episodes encompass statements or actions pertaining to a single goal or a set of closely-related sub-goals. Episodes are typically 1-10 statements long. Off-task episodes are those in which students are pursuing goals unrelated to the lab activity. Empirical episodes are those in which students are pursuing goals related to the lower-level tasks of conducting an experiment (locating and setting up apparatus, monitoring apparatus, recording data points). Conceptual episodes are those in which students are pursuing goals directly related to the analysis of results, or in which students draw conclusions or inferences at a level which deals with scientific principles. ("It's going to boil soon because the curve is levelling off" is an example of a "conceptual" statement; "The temperature is 75 degrees" is an example of an "empirical" statement.) Randomly selected student dyads were recorded in three different semesters during three different substudies: Dissertation Study 1 (Stein, 1987) N = 20 dyads; Dissertation Study 2 (Stein, 1987) N = 15 dyads; Lab Modality Comparison (Stein, Nachmias & Friedler, in preparation) N = 15 dyads. These detailed recordings were all made during lab activities in which students were investigating phase change (or "change of state") phenomena: boiling point of water; boiling point of alcohol; effect of volume on boiling point and time-to-boiling; freezing point of paradichlorobenzene. This data can therefore serve to shed light students' conceptions of change of state and the ways in which they interact with MBL features, curricular provisions and student processes.

Results and Discussion

In striving to make lab activities more conceptual, one consideration is the proportion of lab time students spend in the three categories of lab episode (empirical, conceptual and off-task). These results are shown in **Table 2** for Dissertation Study 1. The most striking result is the very low off-task time. It is difficult to know if the conceptual time represents the hypothesized gain due to MBL feedback unless a comparison study is conducted. The results of such a study are shown in **Table 3**. The comparison indicates that students carrying out the lab activity using MBL were on-task significantly more often than their counterparts who were using thermometers to perform the same activity. There is no significant difference between the groups in proportion of conceptual episodes, though the incidence of such episodes is very low overall.

The nature of student discussion during the episodes reveals more about the factors which lead to successful laboratory learning. Students using MBL consistently spend very little time off-task. In the comparison study, for instance, MBL students set to work much more quickly than the students using thermometers. The computer system may provide a familiar substrate for carrying out a lab, a point of attachment. The start-up routine is constant and familiar: boot the disks, select a menu section, set graph parameters. In addition, it is clear that, having been introduced to manual data collection in the first weeks of the class, the students overwhelmingly appreciate the ease-of-use of the MBL system. Off-task time in the thermometer condition seems triggered by the procedural uncertainty that presaged off-task episodes in the college biology lab. Interestingly, MBL students ask the same number of procedural questions as students using thermometers but seem better able to persist on task. In one taped example, two students are having trouble calibrating their probes. The only computer feedback they receive are cryptic messages like: "error in 1107", yet the students try again and again (4 times in all) to calibrate without seeking help from the teacher. Eventually they succeed and go on to finish the lab.

If empirical procedures still occupy the bulk of student time and attention during MBL labs, at least the vast majority (80-95%) of students have good data to show for their efforts. One MBL feature which seems to contribute to this is the fact that students can see not only their own results displayed graphically, but those of the rest of the class as well. It is clear from the tapes that students monitor each others progress and results closely and remediate problems cooperatively, usually without seeking the teacher's help. This level of success is a very positive result. Since we dealt with all of the eighth graders at this school, all of the eighth graders experienced consistent success in physical science labs. And in terms of ultimate success, you can't reach the conceptual goals without valid data.

It is equally clear that obtaining valid data does not guarantee conceptual success. Many students retained confusions about change of state, even after six experiments. Common conceptions were:

1. Boiling begins when you see the first bubbles in the liquid
2. The same substance can boil at many different temperatures in the lab (i.e., at constant pressure)
3. Boiling point is the time it takes something to boil
4. If you heat something hotter or faster it will boil at a higher temperature
5. Everything boils at 100°C
6. Everything freezes at 0°C (therefore, if it isn't cold, it isn't frozen)

Students did show cognitive gains in some areas. The common graph shapes for boiling substances clearly became a sort of "template". Students could both recognize a correct boiling curve shape, and predict the shape of their boiling curves before and during an experiment. However, as Nachmias and Linn (1987) showed, substantial numbers of students would accept as correct a graph of appropriate shape but inappropriate boiling point (water boiling at 70°C, for example). Student's also recogniz the plateau in cooling paradichlorobenzene as it's freezing (or solidification) point.

As might be expected, students were not able to reason spontaneously from the shape of phase change curves to the mechanisms underlying them. Even with instructional help in the form of conceptual-level post-lab discussion, few students could link boiling and freezing to the more generalized idea of change of state, or to notions of molecular motion or energy. In other words, students gain a certain phenomenological notion of boiling ("it's boiling when the curve is flat"; "if the curve is flat, it's boiling") which is disconnected from the argument of chemistry.

Toward a Model of Laboratory Learning

In order to succeed in solving both the empirical and conceptual problems of a lab activity, it seems as though there are necessary levels of integration. First, the experimental phenomena must be integrated with the data: appropriate observations must be taken, graphs must be correctly labelled. MBL users are prone to fail here if they do not label their curves when recording from more than one probe (this should become a function which the system prompts for). Then the data must be integrated with the variables of interest. In our typical case of two curves, the student must be able to differentiate them meaningfully ("the smaller volume boiled first"; "the water bath cooled normally but the paradichlorobenzene stopped cooling around 50°C for a few minutes"). In general, students were very well able to interpret their graphs, but they sometimes failed to notice significant features. In the paradichlorobenzene experiment, students asked to describe the difference between the curves (paradichlorobenzene and its water bath) concentrated on the first moments when the probes and vessels were equilibrating and failed to make sense of any subsequent questions. Finally, the variables of interest must be integrated with scientific principles. What does it mean that different volumes boiled at the same temperature? Why is a boiling curve flat?

Several instructional provisions in Dissertation Study 2 helped to move students along these levels of integration. The first was giving students a structured observation sheet to fill out during the lab. This produced the lowest level of off-task behavior measured, and helped students link the apparatus to the time-temperature graph unfolding on their screen. A more elegant solution would be a facility for on-screen notation within the MBL system. The second was having students compose their answers to post-lab questions on a word processor. In addition to the draw of computers and the ease of revision, a more subtle mechanism may have been a transformation in the nature of the task: at a keyboard, the shared task became framing the answer rather than writing it down. The third provision which produced a higher level of conceptual answers was parsing the post-lab questions into smaller steps (**Figure 2**). Students laughed when they read question 5a ("What do you have to do to keep something hotter than room temperature?") thinking it ridiculously obvious. But for the next question, about what must have been happening at the plateau, almost all the answers made a certain amount of sense ("It must have been heating itself up"), whereas without the leading question none of the conclusions dwelt on the anomaly of something remaining at 50°C in a 20°C room.

Student interactions also seem to play a role in the success of grasping the conceptual point of the lab. On a second pass through the "empirical" episodes, there appeared to be two different sorts of empirical question. One is an "orienting" question of the "where are the matches?" or "what are we supposed to do?" sort. The other, while concerning a matter of procedure, is more of an attempt to understand or think along with the lab, and is really more of a "why" question than a "what now?" question. Recorded examples of this "constructive" empirical thinking in the phase change labs concern boiling chips (what do they do? what are they made of? what happens if you put them in the wrong place?) or the water bath (what effect does it have?) or the variables of interest (are both volumes supposed to be the same, no, that wouldn't make any sense). Pairs of students who engage in more of this thinking-along with the lab procedures also tend to be more successful at drawing conceptual-level conclusions.

By attempting to chart where and how higher cognitive skills are engaged by laboratory activities, I am working toward a model of laboratory learning that can be used to facilitate conceptual change in classrooms of the not-too-distant future.

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TOPIC	# LAB ACTIVITIES
Scientific Reasoning Skills	10
Introduction to MBL	8
Calibration	2
Graphing Skills	4
Graphing Temperature: Cooling and Heating	12
Boiling and Change of Phase	4
Factors Involved in Heat Flow	6
Measuring Heat Energy	4
Energy and Society	4

Table 1. An outline of topics covered in the 18-week Computer as Lab Partner curriculum. The number of lab activities performed in each topic area is shown.

TYPE OF EPISODE	% OF TOTAL LAB TIME
COGNITIVE	10.3
EMPIRICAL	88.2
OFF-TASK	1.5

Table 2. Cumulative relative percent of student lab time spent in three types of pursuit. Cognitive episodes involve explicit use of scientific principles during a lab activity; Empirical episodes deal with the mechanics of carrying out a lab activity, and Off-task episodes bore no relevance to the lab activity. Time spent in each kind of episode were summed across the 20 dyads recorded, a total of 843 minutes of lab time.

Table 3: Minutes of student lab time spent in three types of pursuit: Empirical, Conceptual, and Off-Task. Student scores were formed by an actual count of audio-recorded episodes.

Group:	MBL	Thermometer	Group Comparison
	(N=7 dyads)	(N=8 dyads)	(Mann-Whitney)
Episode Category:			
Empirical			
% of Time	84.5	66.6	p = 0.001
Mean (minutes)	29.6	17.8	
(Std. Dev.)	(4.4)	(3.3)	
Conceptual			
% of Time	3.5	5.0	p = 0.40
Mean (minutes)	1.2	1.4	
(Std. Dev.)	(0.8)	(0.7)	
Off-Task			
% of Time	12.1	28.4	p = 0.005
Mean (minutes)	4.2	8.0	
(Std.Dev.)	(1.4)	(2.4)	

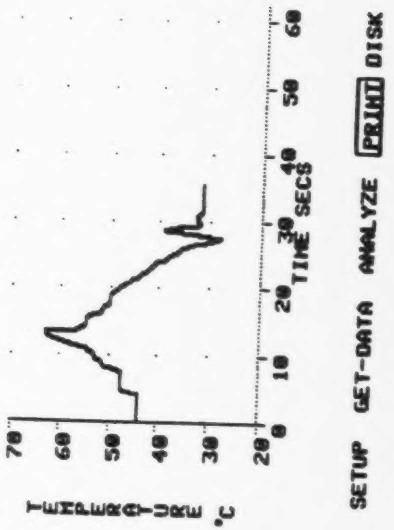

Figure 1. Temperature Grapher Screen Display. The main menu options are listed under the graph. These allow the user to: select or change graph parameters and options; start data collection; analyze points on the graph; save the data in various forms, including superimposing one graph on another; and print. When an option is chosen from the main menu ("Print" is shown selected here), a sub-menu appears in window overlay.

Figure 2:

Lab Report: Cooling of Paradichlorobenzene

Use the Appleworks word processor to answer the following questions about the lab in which you examined the changes in paradichlorobenzene as it cooled.

Discussion Questions:

1a How did the appearance of the paradichlorobenzene change?

1b Did you observe a change of state? Describe it.

2a How did the appearance of the paradichlorobenzene relate to the graph shape? That is, did the shape of the graph change when the physical appearance changed, and if so, how?

Compare the cooling curve of paradichlorobenzene to the cooling curve of the water bath:

3a How are the two curves similar?

3b How are the two curves different?

3c Explain what happened at around 50°C that makes the curves look different there.

4a Describe the change of state that happens when water boils. (When water boils it changes from a _________ to a _________.)

4b Describe what the temperature curve looks like during boiling.

4c What is the similarity between observing water at 100°C and observing paradichlorobenzene at 50°C?

5a We know that normally things cool down or heat up until they reach room temperature. If you want to keep something hotter than room temperature, what do you have to do?

5b What must have been happening to the paradichlorobenzene to keep it at 50°C, which is a lot hotter than room temperature?

5c Since heat is a form of energy, can you draw a conclusion about the role energy plays when a substance changes state?

TRANSIENT ELECTRICAL PROCESSES AS RESOURCES FOR CAUSAL REASONING

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INTRODUCTION

Physics instruction tends to employ steady states rather than transient processes as starting points for model building. Here are some familiar examples from two domains of experience:

DOMAIN	STEADY STATE
Mechanics	Motion of objects at constant velocity
Electricity	Circuit operation at constant current

The motives for adopting such an approach seem reasonable enough. First of all, steady states have simpler formal descriptions than do transient processes. Secondly, they are easier to observe in the laboratory.

There is, however, another reality which must be taken into account in the design of instruction. The fact is that students' initial beliefs about steady states tend to be quite different from the conceptions taught by physicists. Their alternative conceptions in mechanics [1-3] and in electricity [4-7] are tenacious enough to survive instruction at the high school or college level.

This situation suggests the issue I would like to explore this morning: To what extent is the robustness of alternative conceptions about steady states due to lack of opportunities for non-formal reasoning about cause-effect relationships? To what extent might teaching strategies be modified to provide more productive opportunities?

I suggest that misconceptions about steady states are sustained in part by the fact that these states retain no memory of the physical events which brought them into being. Their history-independent character makes steady states inherently susceptible to multiple interpretations, because they are *deficient in information about the underlying cause-effect relationships*. In the absence of compelling evidence favoring the physicists' interpretations, students will feel little pressure to abandon intuitions that have served them well in the past and little motivation to enter the struggle required to make sense of the abstract models favored by physicists.

Transient processes are much more revealing, because they transform initial steady states into final ones. In doing so they *exhibit matter in an active role* -- doing human-like pushing and shoving. Because of this "operative" aspect of phenomena, as Piaget would call it, it seems to me that observation of transient processes offers a more hopeful basis for stimulating causal reasoning. [8]

The implications for the design of instruction are straightforward: (a) First, encourage students to articulate their intuitions about steady-state behavior. (b) Then provide experiments in which observation of transient behavior makes them aware of the underlying cause-effect relationship. Students will then find their initial intuitions in discord with a newly formed conviction, and may come to see more merit in the physicists' interpretation of the steady states.

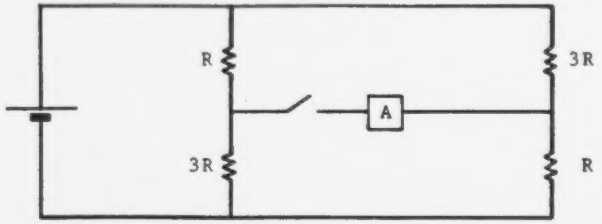
This prognosis is born out by research on the teaching of Newton's First Law, which deals with steady states characterized by motion at constant velocity. The only successful attack I am aware of on the very resilient motion-implies-force misconception in this domain is by Minstrell, [9] whose approach is as follows: (a) Have students articulate their intuition that a constant force causes constant velocity. (b) Then have them perform experiments on motion under constant force supplied by a weight and a towing string.

The observation of transient behavior characterized by accelerating motion led Minstrell's students to the understanding that a constant net force causes changing velocity. The initial intuition was then reexamined in class discussion, and substantially overcome. What seems most important here, in terms of transfer to other domains of experience, is that non-formal causal reasoning which developed from observation of a transient process was crucial for enabling the students to reinterpret steady-state data in expert terms.

My classroom experience with electricity suggests that experimenting with transient phenomena also has the ability to promote conceptual change in this domain. [10] The balance of my presentation will report on efforts to develop instructional materials and methods for investigating two types of transient electrical processes.

TRANSIENT LAMP LIGHTING

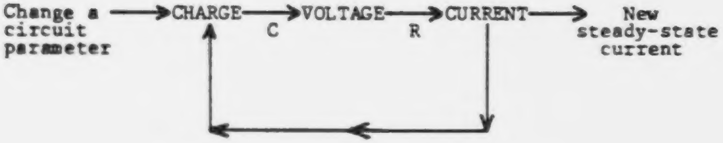
My interest in capacitor-controlled lamp lighting as a way to deal with problems of comprehension of electric circuits began some years ago in a first year college physics course. In a group discussion of problem solving methods, I asked my students to determine the current through the ammeter and resistors in the following circuit after the switch is closed.



These students had already been using principles of quantitative circuit analysis in problems that did not introduce parameter changes, and my expectation was that they would want to discuss ways of calculating the new steady-state current values. To my considerable surprise, however, there appeared instead an intense interest in understanding how the circuit gets to a new steady state after the switch is closed or opened. The group preferred to avoid formal analysis of the final steady state until after its physical origin was clarified by non-formal reasoning. There was a clear sense that insights essential for understanding steady-state behavior are hidden in the extremely fast transient processes that precede the establishment of steady states.

I suspect that the yearning for qualitative causal reasoning which surfaced in this instance was due to the fact that we were dealing with a bridge circuit. For circuits of this type, sequential reasoning [5,11] does not provide the usual comfort because there are no obvious sequential paths. Whatever light may be shed on this speculation by future research, the experience drove home to me the point that experiments on transient processes initiated by parameter changes offer important opportunities for studying the current driving mechanism.

The hidden insights which my students suspected can be brought to light by noting that the wires attached to a resistor constitute a capacitor connected in parallel with it. The rest of the circuit exchanges charge with this "stray" capacitor, while the charges on the wires simultaneously drive a three stage feedback cycle. I shall diagram that cycle by drawing an arrow from each cause to its effect, and shall note the circuit property mediating each effect by capacitance C or resistance R:



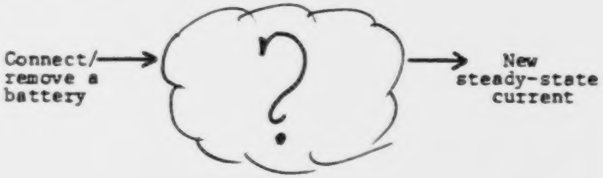
The feedback cycle for each resistor commences whenever a change of circuit parameter unbalances the previous electrostatic equilibrium with the rest of the circuit. The cycle repeats until the current through every resistor has reached a new steady state. The three stages are:

1. Charge-voltage causality -- The charges on the connecting wires cause a potential difference, whose magnitude is determined by the capacitance of the wires.

2. Voltage-current causality -- The potential difference between the wires drives a current through the resistor, with magnitude determined by the resistance.

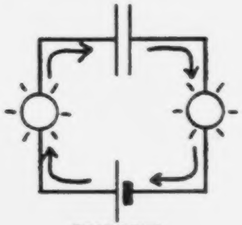
3. Current-charge feedback -- The current through the resistor alters the charges on the wires, which leads to further charge exchange with the rest of the circuit.

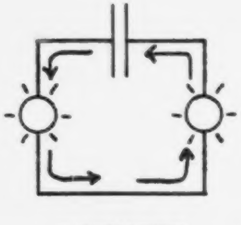
The diagram above gives a clear picture of the way students' experience will be distorted when the transients initiated by parameter changes are too fast for human perception. They will be unable to grasp the feedback mechanism, which is required to make sense of the conception that current is driven through resistors by potential differences caused by the presence of excess charge on the connecting wires. As illustrated in the ammended diagram exhibited below, they will then have no evidence of charge-voltage causality and could easily fail to grasp the causal relationship of voltage to current. They will be aware only of batteries as causes and steady-state currents as effects. Thus, there will be nothing in their experience to disconfirm their initial beliefs in (a) battery origin of current, (b) consumption of current in resistors and (c) sequential reasoning.



A new electricity curriculum that I am developing aims to foster awareness of charge-voltage causality by introducing intentional capacitors into circuits of batteries and light bulbs. The presence of the capacitors builds awareness of this causal relationship by presenting students with visualizable sites where excess charge is concentrated. The enormous capacitance needed to stretch out the time scale of transient lamp lighting for purposes of human perception, combined with the user-friendly packaging required for instruction of novices, has been made available by recent advances in capacitor technology. One can now obtain up to one whole farad in a compact nonpolar unit, and up to .025 farad with the additional feature of extremely low equivalent series resistance. [13]

My preferred way of introducing this approach, which I shall now demonstrate, is by charging and discharging a capacitor through light bulbs connected to both sides of the capacitor. The circuit of choice uses a 4.5 volt battery, #14 light bulbs (about 10 ohms) and a .025 farad capacitor.


CHARGING


DISCHARGING

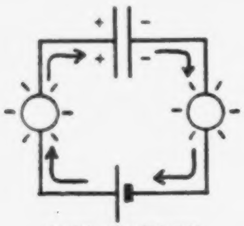
The observation of bulb lighting with this circuit, which is broken at the site of the capacitor, provides evidence favoring a circuitally-directed flow pattern over clashing-currents flow. As illustrated in the diagram at the left above, this flow pattern requires that the direction-of-flow arrows for charging point outward from one of the capacitor plates. Thus it implies that the moving substance is a normal constituent of the passive conducting matter of which capacitor plates are made -- something that does not originate only in batteries. This insight suggests it will be useful to introduce +/- notation with Benjamin Franklin's original meanings: [14]

(+) means more than the normal amount

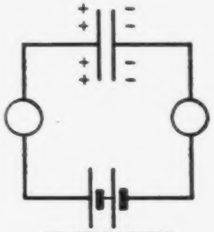
(-) means less than the normal amount

The observation that the bulbs light up during discharging — when there is no battery in the circuit -- leads students to intuit that something becomes stored in the capacitor and then pushes itself back through the bulbs. Plainly, the displaced substance is pushed away from the plate containing an excess amount labelled (+) and back toward the plate containing a deficit amount labelled (-). But what makes it go back?

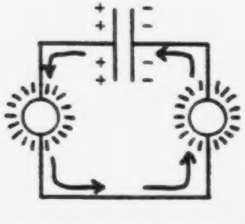
To investigate this crucial issue, I'll place a second battery in series with the original one after capacitor charging caused by the first battery has reached completion. The observation of renewed transient lamp lighting reveals that the moving substance is compressible. I'll then remove both batteries and reconnect the wires. The observation of enhanced lamp brightness during the discharging that follows suggests that the compression is reversible.



MORE CHARGING



COMPRESSION

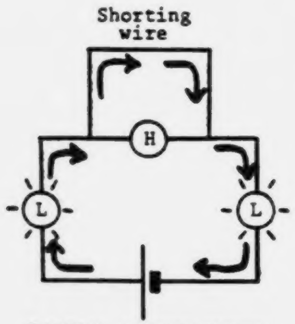


ENHANCED BRIGHTNESS

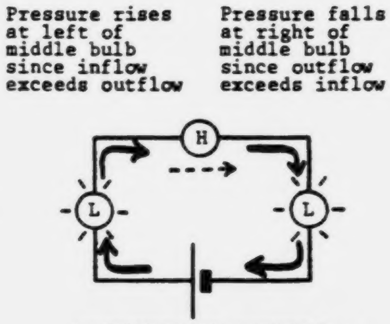
Reversible compressibility suggests to many people an elastic fluid model of the mobile substance. It also suggests the existence of a pressure-like condition in that fluid. The analogy with air pressure leads students to intuit that this "electric pressure", as they like to call it, is HIGH where the fluid is compressed and LOW where it is depleted. This model envisions contact forces and only one type of charge. The concreteness of the pressure-like conception of electric potential makes that idea available early-on for effective causal reasoning. The model must of course be generalized later on to include two types of charge, distant effects due to that charge, and electric potential as a field which can exist in empty space.

Before embarking on a process of generalization, however, it is important to provide evidence for the presence of compressed/depleted mobile substance *in the wires* of an operating circuit. As a preliminary experiment, one might first short-circuit the middle of three series-connected light bulbs and then remove the shorting wire. The resulting fast transient process yields dramatic results if the middle lamp has high resistance and the other two have low resistance. I use #48 lamps (about 40 ohms) for the former and #14's for the latter.

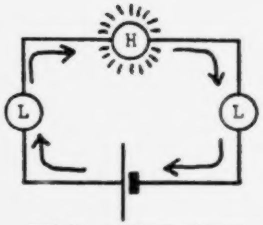
The diagrams below indicate how one might think about the experiment if there is *already* belief in compression/depletion in the connecting wires. The high resistance lamp is labelled H and the low resistance ones are labelled L in these diagrams. The starting point for causal reasoning is the fact that the current through the left and right lamps bypasses the middle lamp as long as the shorting wire is in place. The charge transported by this current will cause "pressure" (= potential) changes in the wires connected to the middle lamp when the shorting wire is removed, and thereby initiate a transient feedback process that changes the amount of current through all three lamps.



INITIAL STEADY STATE:
Middle bulb is off.


Pressure rises at left of middle bulb since inflow exceeds outflow
Pressure falls at right of middle bulb since outflow exceeds inflow
H
L
L

TRANSIENT IN PROGRESS:
Pressure values shift.

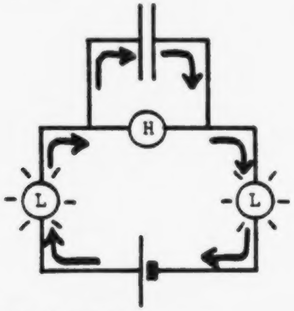


FINAL STEADY STATE:
Middle bulb is on.

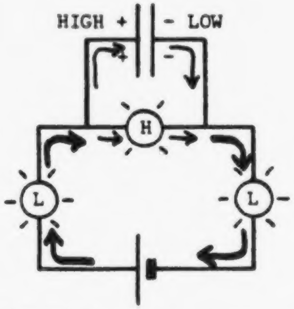
The "pressure" in a wire should continually increase when there is more mobile substance entering than leaving, and it should decrease when there is more going out than coming in. Therefore, the changes illustrated in the upper right diagram above should continually reduce the imbalance of current values through the three lamps. Abstract conceptual feedback is accompanied by *spatially visualizable feedback*, in which events in downstream circuit components influence events in components situated farther upstream. The perception that this feedback is occurring provides effective evidence against sequential reasoning.

Of course, students will probably *not* find it credible that there is more/less than the normal amount of mobile substance in the connecting wires. Most textbooks are silent on this issue, and it is a source of confusion even for experienced teachers. The way out is to have students perform a modified version of the above experiment: Replace the shorting wire with an initially uncharged capacitor that has enough capacitance to slow the transient process down drastically. I use a 1.0 farad or a .47 farad capacitor.

Adding such a capacitor in parallel with the middle lamp can be thought of as adding a very *large amount of metal* to the wires connected to that lamp. Most students will agree that a very *large amount of charge* must then be transferred in order to drive the "pressure" up to the final HIGH value in one mass of extra metal and down to the final LOW value in the other. The transient changes that I have described above should then require a very large amount of time. That would explain why they can only now be investigated on a time scale appropriate for human perception. Here is what one observes over the course of tens of seconds:



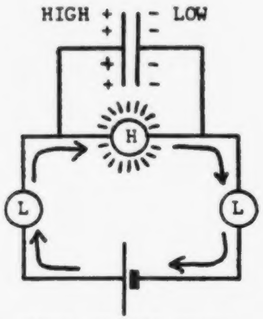
FOR A WHILE --
There isn't yet enough compression/depletion to cause a significant pressure difference in the capacitor plates.



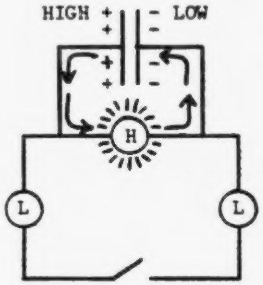
BUT THEN --
The pressure difference across the middle lamp becomes great enough to drive enough current to cause some lighting.

Feedback ceases when the current through the middle light bulb increases -- and the current through the other two decreases -- to the point of equality. The final steady is characterized by current continuity through all series-connected components, no matter how disparate their resistance values may be. Note the requirement that more "pressure" difference must accumulate across the high resistance lamp than across the others, in order to drive the same current through all of them. This provides a satisfying non-formal explanation of the principle of steady-state voltage division.

If the "pressure" values in the capacitor plates also exist in the wires connected to the middle lamp, then removing the capacitor from the circuit after steady-state is achieved should have no effect on the bulbs. This prediction is easily verified. The presence of the capacitor should have a pronounced effect, however, if one removes the battery after the final steady state is achieved. The bulb across which the capacitor is connected should then remain ON while the other two bulbs turn OFF.



FINAL STEADY STATE



BATTERY REMOVED

What drives current through the middle bulb when there is no capacitor connected across it? It has to be the HIGH/LOW pressure difference in the wires connected to the bulb. This can be hard to think about, because only very little charge need be compressed or depleted in a piece of metal as small as a wire in order to drive the pressure way up or way down. To make the point, it is useful to reduce the charging time nearly to imperceptibility by using a series of smaller capacitance values -- say, .025 farad and .005 farad. One can then imagine replacement with even less capacitance, until the only "capacitor" in parallel with the middle lamp is the pair of wires connected directly to it.

The failure of the highly intuitive elastic fluid model embodying contact forces and a single type of charge, and the need for a more abstract model with action-at-a-distance forces exerted by two types of charge, can be demonstrated with a circuit using the following alternative components:

Battery -- several hundred volts (photoflash)
Light bulbs -- A9A (formerly Ne-2E) Neon lamps
Capacitor -- hand-held plates with plastic wrap

I shall have time only for a very brief account.

The compressed air analogy predicts that *nothing will happen* when the capacitor plates are brought together and when they are pulled apart. But the Neon lamps are in fact observed to glow (transiently) when the plates are moved to and fro. This experiment demands that the initial model of electrical causality be generalized to account for *distant as well as local action*.

The fact that lamps connected to *both* sides of the capacitor are observed to light up simultaneously suggests that the plate marked (-) is the site of a *second* type of electrically active matter -- that it is not just a place where active matter is absent. A simple concept that works is to redefine the (+) and (-) symbols as now designating sites where there are *excesses* of charges of "positive" and "negative" type, respectively, with negative charge causing effects *opposite* to those of positive charge.

Experimenting with capacitor-controlled lamp lighting has demonstrated the ability to reduce the incidence of students' belief in (a) battery origin of current, (b) consumption of current in resistors and (c) sequential reasoning. [10] I shall now turn to a consideration of electromagnetic transients in electric circuits. The approach I shall exhibit is recommended for college-age students at the beginning of instruction in magnetic effects of electric currents.

MAGNETIC FIELD PRODUCTION

The transient process that produces a magnetic field when current is turned on in an electric circuit is also too fast for human perception. The result is that students have no way to think non-formally about the *mechanism that brings the magnetic field into being*. There is nothing in their experience about the causal role of an electric field in the process of magnetic field production, and nothing to hint that this electric field is crucial to electromagnetic induction. Textbooks typically state that a changing magnetic field causes electromagnetic induction, and many of them go on to say something like "a changing magnetic field causes an electric field". These purported causes are not described in the context of a non-formal causal model, and it's not very surprising that students have difficulty reasoning with them. [15]

It is plausible causal reasoning, however, to imagine that the magnetostatic field coupled to a current-bearing wire was created by a *radiative process* which is initiated by turning on the current. The stationary character of the magnetic field as normally observed can then be attributed to its being a *nonpropagating superposition* of waves that originate at spatially separated parts of a circuit. The apparent instantaneity of formation can be attributed to a very high speed of wave propagation.

It isn't possible to slow down the transient process in this case, because the waves travel at the speed of light. One can, however, create awareness that magnetic field production is indeed a radiative process and that the emitted waves are electromagnetic rather than purely magnetic. [16] It is only necessary to place the electromagnet one is investigating near a radio receiver. I'll use a simple coil of wire and a transistor radio.

Most people are conditioned to regard a radio as responding to "waves" that travel to it from somewhere else. The coil of wire is the obvious source of these waves. The experiment makes it plain that the envisioned wave emission occurs only when the current is turned on or off -- that is, only when charge is *accelerating*.

What is the structure of the emitted waves? Consider the final purely magnetic steady-state field that one observes in the laboratory. The "right hand rule" structure of that field suggests the local direction of the magnetic field carried by the wave. The electric field should be directed so as to transfer energy from the circuit to the field, which means that the field at the wire should be directed opposite to the turned-on current.

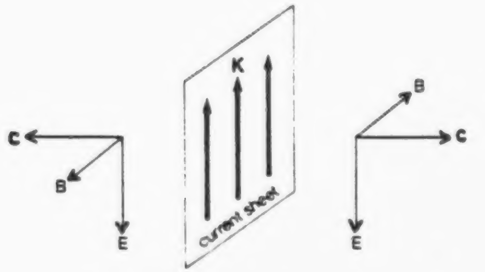
There are then plausible grounds for anticipating that electromagnetic waves emitted by accelerating charge in the magnet wire will have the following local structure:

** The wave velocity (c) should be directed normally outward from the wire, so that propagation is away from the sources of the wave.

** The direction of the electric field (E) at the wire should be opposite to the current, to ensure energy transfer from circuit to field.

** The direction of the magnetic field (B) relative to the source current should be in accordance with the right hand rule of magnetostatics.

The diagram below exhibits this structure for electromagnetic waves radiated from a turned-on infinite current sheet bearing uniform surface current density K.

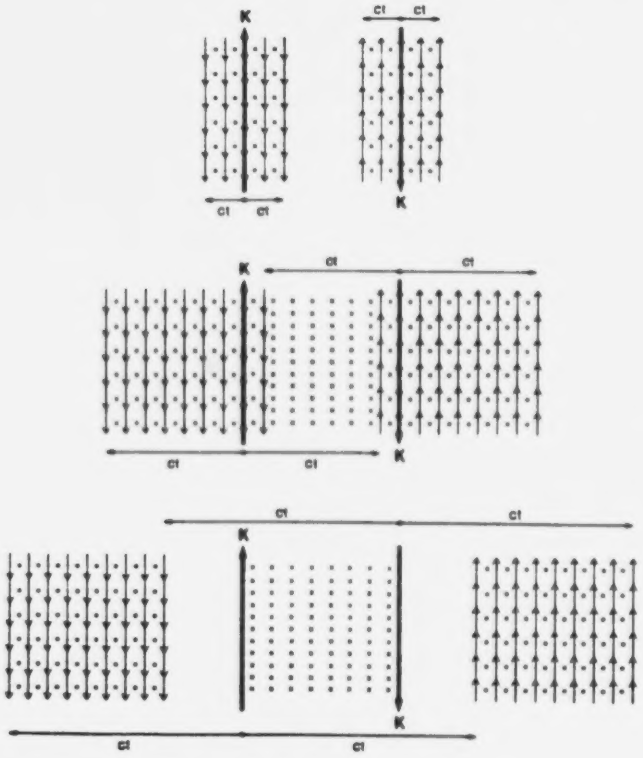


Now, consider field production by an idealized "circuit" consisting of two such current sheets carrying the current in opposite directions. To obtain the clarity of step-shaped wave fronts for the electromagnetic waves that originate at these sheet sources, let the current in each sheet be turned on suddenly to a value K which is thereafter held constant by external means. The acceleration of source charge that occurs when the current is turned on initiates emission of electromagnetic plane waves that propagate away in both normal directions from both current sheets.

The following diagrams illustrate the process of field production by exhibiting the state of superposition at three successive times (t). The first diagram shows the waves propagating independently, with E oriented opposite to K at the site of emission (arrows up or down) and B pointing into the paper or out (crosses or dots). The second diagram shows superposition of waves advancing into the space between the current sheets, with total destructive interference of electric fields and maximum constructive interference of magnetic fields. The third diagram shows the situation after superposition in the space between the current sheets is completed and superposition in the exterior space has begun. Three important events occur at the moment that waves travelling through the interior space reach the other side of the "circuit":

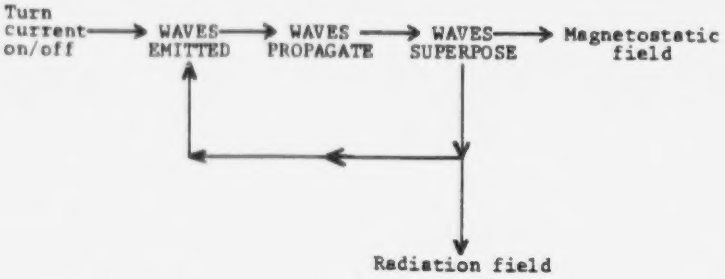
1. The electric field strength at the current sheets drops to zero, with the consequence that there is no further transfer of energy to the waves.
2. The interior space becomes uniformly filled with a purely magnetic field which thereafter remains statically coupled to the current sheets.
3. Total destructive interference with the exterior waves creates a pulse of speed-c electromagnetic radiation having the same width as the circuit.

It is the electric field of this radiation that is detected by the radio receiver.



The field production process represented in these progressive superposition diagrams provides a compelling qualitative explanation of the origin of the magnetostatic field of a turned-on current source. The diagrams make it clear that a radiated electric field is very much involved in the production of a purely magnetic field, and they show that production of electromagnetic radiation is predicted to be an inevitable accompaniment of magnetostatic field production. The role of the electric field in this process provides a vivid exemplification of Lenz's law, and reveals the causal mechanism of electromagnetic induction.

The causality diagram for magnetic field production exhibits a feedback structure similar to that for electrostatically driven transients. There is, however, the added feature that irreversible emission of radiation accompanies the transition from one steady-state magnetic field to another. The detection of that radiation is what makes it possible for students to construct the transient field production process without having to slow it down to a time scale suitable for human perception.



This causality diagram shows how experience with magnetic effects of currents is distorted when the electromagnetic radiation caused by turning current on or off is not detected by a radio receiver. Students will be unable to grasp the feedback mechanism, which is required to make sense of the conception that magnetic fields are produced by a radiative process which is self-limited by superposition. They will then have no evidence of the role of an electric field in this process, and will be unable to construct a satisfactory causal explanation of electromagnetic induction. I have anecdotal evidence for the ability of the radio detection experiment and subsequent discussion of the field production process to improve students' intuitions in this area, but have not yet accumulated quantitative evidence.

CONCLUDING REMARKS

Transient processes appear on theoretical grounds to have a clear advantage over steady states as resources for stimulating causal reasoning and overcoming misconceptions about steady states. Minstrell's classroom experience with accelerating motion, and my own experience with transient lamp lighting and magnetic field production, provide support for this view.

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A Conceptualisation of Students' Meaning of Understanding of Physics in the High School Classroom

by

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Introduction

The past years has seen much curriculum reforms and developments. Despite this tremedous efforts much problems remains with science teaching. Conferences on misconceptions in science learning, like this one, show now and again that the problem of helping students understand the concepts of science in the enviroment of our schools still needs much thought.

Studies of misconceptions in learning science indicate two sources of children's problems. In a study of misconceptions in School Thermodynamics, (Johnstone & others, 1977), it was suggested that what happened in learning thermodynamics in upper grades depended upon what was taught in the lower grades. Students, when presented with a new idea, treated it in three possible ways. They either compartmentalized what they had learned, (i.e., they did not incorporate the new idea with that they knew); or they attempted to attach the new idea to existing knowledge, but made the wrong connections; or else they would incorporate the new knowledge correctly into prior knowledge, and apply clear understanding to the new concept. Misconceptions happened in the second case. Solomon (1983), in her study, indicated that one source of problems in children's misconceptions is that many scientific terms have everyday meanings which then become "conceptual traps". Her conclusions were that the ability to discriminate between the two domains of knowledge, "everyday meanings" and "physics meanings" plays an important role in success in school physics. What Solomon is suggesting, then, is that the context for a student learning physics is more complex than just the nature of the subject matter and the instructional practices. The learner has to contend with previous knowledge and experiences, including those from outside his classroom, in addition to the new experiences and knowledge from the physics classroom.

Studies by Laurilland (1979) suggest another facet in the students' problems in learning. Laurilland had studied what she defines as the "learning style" of the student, that is the way students think about the subject matter and the strategies of students, the way students approached the task of learning. The major conclusions she has made is that students' styles and strategies are context-dependent. The context consists of the nature of the subject matter and students' perceptions of what teaching provides them. These studies among others, suggest two important factors in learning in the classroom: the previous "everyday" knowledge of the students and their learning styles. Learning and learners are complex and active individuals in a learning situation. A question arises: To what extent do these active individuals influence the learning or understanding they obtain from the classroom and equally, to what extent do these classroom events and situations influence the students understanding? What are learners doing in the classroom? How can we help our teachers?

Today, for most teachers, the science classroom have changed from the "traditional didatic" classroom to one where there is at least some student activity part of the time if not most of the time. This is due to the changes made in curriculum in the sixties and seventies.(PSSC, Earth Science, THe Nuffield Sciences in England, the Intergrated Sciences in Scotland, and Intergrated Sciences and Modern Physics, Chemistry and Biology in Malaysia.) Therefore students classroom experiences have now increased. There is not only content to deal with, but the experiences from experiments and their interpretations. Some of the teachers are themselves the product of this "activity" curriculum. Recent approaches to learning have emphaised the active part the experiences of a learner plays in learning concepts.(Novak &

Gowin, 1981; Driver, 1983; Confrey, 1985). But the problem of understanding the concepts and principles of science still remains.

Theories Guiding the Study

To look at the problem of learning physics anew, I choose the conceptual framework of Ausubel (1968), Novak (1977) and Gowin (1981). These three theories see the learner as an active and constructive individual in the process of learning. "Meaningful learning" is a process in which new information has to be linked to some relevant aspect of what the learner already knows (his or her cognitive structure) (Ausubel et al, 1968). However, this linking is not mere connections to past information, but the learner should be taught to form concepts of increasing abstraction or generalisation. (Novak, 1977). The important point here is that the learner does this linking. The learner is no longer thought of as a passive recipient of knowledge, but as a maker of meaning. Learning and teaching becomes a process where both teacher and students actively participate in making meaning. Meaning is negotiated between the student and the teacher or curriculum. Further meaning controls the effort. (Gowin, 1981).

Thus the learner is not merely autonomous, but takes on an active role in deciding his curriculum and the meaning that she/he makes for her/himself. This recognition of the actions and thinking of the learner is significant in the light of the studies on learning mentioned earlier. Student responsibility or active constructive role based on their experiences supports the idea of variations in students' conceptions of the subject matter and the fact that learners have their own styles or strategies for learning based on their context. Based on these three theories and studies on misconceptions and learning the study was formulated.

The questions of the study were: What is happening when students are trying to understand physics in the high school classroom? What do they mean by understanding? How are they trying to understand the lesson? How do they feel? What is important for teachers to know about students in a physics classroom? What is the value of learning about students understanding? How can teaching be in harmony with learning? In this paper I shall not have space to answer all these questions but the first three.

Methodology

Educational research, like research in any field, is an attempt to create new knowledge (Novak & Gowin, 1984, p. 149). In addition to identifying his research procedures and method of analysis of data, the researcher must identify the concepts and theories being used in the research to make observations of the "events" about which data is being gathered. The results of this analysis of the data gathered represents the new knowledge claims made. Further Gowin (1981) suggests that the research takes the form of answering questions instead of proving hypotheses. Gowin has put these different elements of a research in the form of a hueristic Vee (see Novak and Gowin, 1984, p. 3). This concept of educational research was adopted for this study.

The qualitative approach in research recognizes the unique human capabilities for self-conscious reflection and appraisal of experiences. THe use of the qualitative approach and the experiential perspective of learners enable a researcher to respond freely to students' unique set of experiences and to focus on issues which emerge during the investigation as the most significant ones to the learners themselves. The aim is to obtain qualitative descriptions of individuals and their perceptions in situations that are as near real-life situations as possible (Entwistle and Hounsell, 1979). Desriptive data from such research would then have more explanatory purpose than predictive power. (Brophy, 1979) The intentions are to try to explain what is happening in the classroom. In such an "interpretative approach, generalisability can be viewed in terms of how well others understand fairly small contextual analysis and feel that such descriptions and patterns reflect theirs or neighbouring situations."(Avalos, 1982) This methodology provides the means to explore and to probe in a way denied to the "traditional quantitative approach." A richer, more accurate description of student learning could then be obtained.

These two methods and perspectives have been compatible with the conceptual framework of Ausubel, Novak and Gowin, the theories guiding this research. Therefore, in this study, I had taken on the role akin to a sympathetic listener, seeking to understand through empathy and intuition, the perspective or world view of the person or group of persons speaking to me. The procedures of gathering this qualitative experiential data from the students were through the interview method. A set of questions were developed from theory, research reviews and three small projects on students learning. These three projects were carried out with graduate students in physical sciences and engineering, six physics undergraduate students and six non-physics undergraduate students who were attending a physics course for non-physics students. Experiences from these three projects helped to formulate the procedures for the actual study.

Procedures:

Participants in this study were two teachers and their physics students in two schools. They were volunteers. The teachers were senior teachers and noted for their ability to teach well. The choice of such teachers was to remove as much as possible the "poor teaching" factor in students' inability to understand, in order to help focus the interview on students and their understanding physics rather than on the faults of teaching as perceived by the students. Fifty volunteer students were requested with the help of teachers. Students with differing abilities, those considered weak as well as those considered strong in physics, were selected for interviews. All students were then to be attending physics classes in the high school. The choice of schools was based on avaliability to the researcher, that is those schools near and around the Ithaca area. Two schools were chosen: Ithaca High School and Lansing High School.

The interviews were carried out over a period of five weeks in each school. They were conducted during the study hall periods of each student. One interview was scheduled for each student, but students were encouraged to talk with the reseacher whenever they thought of anything that was connected with their learning. Several students took this opportunity and talked several times with the researcher.

Discussions during the interviews were usually about the most recent physics lesson of the student although students were encouraged to give examples and refer to other lessons when necessary. Several lessons were video-taped and "props" such as apparatus, text books were introduced during the interviews to help students recall and explain their learning processes in the classroom. The researcher also attended the same physics lessons as the students to provide herself with a better understanding of the context in which students were trying to understand physics and to enable her to ask more relevant and probing questions.

The following set of questions was used during the interview with high school students of the study. They were:

1. How did you find this lesson? Did you follow what was going on in the class? Can you tell me what it is about? What did you find difficult? How did you feel in the class today?

(This set of questions were aimed at getting the discussion started. They need not all be used. The lesson referred to the most recent lesson that students attended or the lesson that was video-taped).

2. What do you mean when you understand? Or don't understand?

(This question is raised only after students have used the word themselves or suggested that they had understood the lesson. They would then be asked if they understood and if so to then explain what they meant.)

3. What do you do in order to try and understand? What did he say or do that made you understand?

4. What is physics? What do you think you are studying in your class?

These, however, were not the only questions used. Other questions were used as necessary depending on students' responses and suggestions. This included asking students to clarify ideas or terms used. Thus the interviews were semi-structured. These set of questions remained the basic questions used throughout all the interviews although additional exploratory questions were developed as the research progressed to further explore and clarify students answers.

Analysis of Interviews

The audio-taped interviews were first transcribed. The answers in the transcsripts were then coded according to which questions they answered. These were:

1. What do you mean by understanding?
2. What do you do when you don't understand? How do you try to understand your physics lessons?
3. What is physics?

These answers were then concept-mapped for the students conceptions of "understanding" in the classroom. (See figure 1) The coded answers and concept-map were then analysed for regularities or patterns in how the students were trying to respond to their physics lessons.

Findings

The study found that students' meanings of "understanding" could be conceptualized based on their experiences and knowledge from both inside and outside the physics classroom. The interaction between these two "domains" of their experiences can be synthesized into four conceptions of "understanding" physics by students. It is the interplay of these two spheres of experiences and knowledge that would structure the students understanding of physics. I have called these four meanings of understanding "understanding1", "understanding2", "understanding3" and "understanding4". By using numerical subscripts, does not imply that these meanings exist in hierarhical or even necessarily sequential relationship to each other. Numbers have been chosen instead of names to forestall premature naturalization of terminology. Their chronological appearance in this study had promted the use of this numbering.

Briefly, "Understanding1" comes from relating new physics knowledge to knowledge and experiences from outside the physics classroom; "understanding2" is being able to "work" inside the classroom, eg. carry out experiments or solve problems; "understanding3" comes from interrelating new physics knowledge with knowledge and experiences inside the physics classroom; and "understanding4" is being able to use physics concepts to explain a broad range of everyday experiences outside the physics classroom. These findings suggest another way to conceptualize understanding in high school physics classrooms. They suggest another way of describing the character of what is meaningful to students in the physics classroom. (A more elaborate description of these four meanings of understandings with illustration is given at the end of the article.)

Discussion

Students come into classroom with knowledge and experiences from everyday-life and from other classes in school. This everyday life knowledge and experience enable students first to make sense of their new physics experiences. As students continue learning physics, they interact with other new physics knowledge and experiences, as well as the teacher and develop new sources of knowledge and experience to further learn physics. From these new experiences inside and "old" experiences outside the physics classroom, students appear to develop meanings of understanding physics.

A question that the findings of this study raised is that of how should we include "everyday experiences" into a physics course? Some suggestions that have been made to make physics more relevant to the pupils is through applications of physics to their everyday experiences. The findings of this study suggest that this gives a meaning to the physics concepts, and that there are two ways in which these everyday experiences could be introduced. Introduced as in understandingl, there are limits to the type of conceptual structures developed. Introduced as understanding4, the physics concepts would be enriched by the content of daily experiences. Further, a hierarchy of concepts would be initiated.

Another important aspect of these four meanings of understanding is the how they were formed and what happens after. Further analysis of the data from the interviews suggest that teachers, the curriculum and the administration of the classroom each influences the formation of these meanings of understanding. What however may be more important is how these meanings of understanding then governs the students' future learning. Would these understandings act as "governances" of future learning? This is especially important if students merely attain understandingl and understanding$_2$. Would these two meanings then act to limit the students understanding in the physics classroom? Would different curriculum or teachers bring about changes in these meanings of understanding?

The data from the interviews also suggest that these meanings of understanding do influence the students expectations during physics lessons. They determined for the students what he expects to achieve in terms of understanding for each lesson. They also at times act to determine what particular aspect of a lesson students would focus on. Thus during laboratory lessons, one finds that some students do not think of the concepts and theories involve but rather just what to do and how to do it. Such actions are contradictory to the aims of many new curricular which have introduced laboratory experiments as a means of teaching the science concepts. Students wait for the "lectures" following a laboratory lesson to learn their physics concepts. Thus a curricular should have lessons that are focussed at different aspects of what is happening in the physics class and not just what concepts and theories to teach. The teacher should not merely use ideas because they are interesting and serve to explain. As Dewey suggests, these experiences should have a means to enable students analyse and understand future experiences.

The findings of the study therefore, suggest that teachers in classrooms be more aware of what they use and how they use experiences and knowledge to teach. It raises the following questions for teachers: What kinds of knowledge and experiences are we using to help students build the concepts and understanding of the physics? How are students constructing their meaning? What kinds of understanding are they (students) achieving? How are the students using their physics knowledge to understand events outside the physics classroom? What kind of concepts are we building for our students? Should understanding physics mean achieving all four meanings of understanding? These questions become especially important where the curriculum stresses the "link" between school science and life outside the school.

Thus, students had attempted to reconcile their "two worlds" of experiences. They have not kept their two worlds separate, but seem at every instance to try and combine them as they attempt to reconstruct their understanding of the world they live in. Therefore it is important to recognize the existence of these "spheres" experiences and knowledge from inside and outside the classroom and how it has been used in understanding physics. This could perhaps help teachers to recognize potential conceptual traps. In her study, Laurilland has suggested that students style of learning might be dependent of what they perceive to be the aim of learning, whether it be to explain, to understand, for a project or an examination. It is suggested from this study that the efforts of students may also be determined by what they think is understanding. When they "stop" learning may be suggested by their meaning of "understanding." Thus in addition to the students' previous knowledge and learning styles, students meaning of reaching "understanding" may also be a factor controlling their efforts in the classroom. Hopefully these four meanings of understanding would provide teachers with a means of looking at what is happening in their physics classrooms.

The Meanings of Understanding

In this section I have tried to describe more fully, with some illustrations, the meanings of understanding. However, due to the nature of the data and the length of the paper, I am unable to cite student answers and illustrate every aspect of these meanings.

The claim of understanding1, for students, means a claim to make connections between physics experiences in the classroom and their experiences from outside the classroom, the "everyday experiences". These connections need not be explicit definite relationships, but do provide students with a sense of what they are learning in the classroom. These connections may just involve placing the new material in some relationship to students' known knowledge or experiences. They do not, however help students to conceptualize their physics concepts or their "everyday experiences" in a hierarchy. Thus, for example, when students commented on the lesson of the impact between nuclear particles and their speeds, they said:

> Like he will compare when we are doing alpha and beta particles, compare the speed and the size, like alpha particles acts on the beta particles is like the Greyhound bus coming at a tricycle. (Student 39)

or

> He'll take the technical terms, those scientific jargon and he'll put it into very real examples. He'll use electron flow as the opposite of molasses (flow). (Student 2)

This type of explanation gives students a visual or qualitative sense, and a "feel" for the physics content. The students commented upon teachers, "talking at their level"; using familiar words throughout the whole lesson helped them to understand.

Understanding$_1$ provides students with a sense of "tangibility" as in having some "concrete" picture of what he was learning. This tangibility can come not only from seeing an event but also from working with numbers that are like those found in the "everyday use" of the students, neither very large or small. Applying these numbers to the principles or concepts, and working them out can help give that sense of tangibility to students.

As a consequence of understanding1, students feel that they can use their own words to describe the new knowledge they have. They get a "picture" or a sense of it. Often students said they formed a "visual image" or a representative image of what the new idea was. And, finally, they can place the new material in some known knowledge structure of their own. Student 23 described this "understanding" as having a "general association," like the stage of having some idea, of having many possible meanings or ideas to relate to the new material to give it meaning.

The claim of understanding2 is made when students feel that they can carry out the activities of learning required in each particular physics lesson, such as carrying out experiments and problem-solving, knowing how apparatus work and what they do. Thus understanding2 comes when students have followed the rules and methods of each lesson, so that they can "do" what is required for the lesson.

Understanding2 is not metacognition, where students know separately or have abstracted out the rules and the logic involved. They have not abstracted out the processes and formed a structure of it. Understanding2 is just a matter of understanding, in each individual lesson, how something is being done, and being able to use it or recognize it later in similar lessons. Students achieve understanding2 through using and "seeing" how rules and thought processes operate to link or structure the concepts they have learned. They learn by doing. Thus students learn how to solve problems and carry out experiments with instructions. However, the ability to solve problems using equations is not just "rote" memorization of equations and procedures. Student 17 compared understanding how to use an equations to that of how to use a tool. He said,

> You don't really think of an act or physical event, you know when to use it (the equation) for, you know how to use it. Its just sort of a tool that I could use to manipulate figures. ... to transform physical data into some desired results. (Student 47).

It is not simply plugging in numbers into problems to solve them, or carrying out laboratory procedures that may seem like rote learning. For students there is a need to understand the equations or procedures before using them in a supposedly rote fashion. Student 46 suggested the difference between remembering a formula and rote learning as follows,

> Memorizing formulas seem easier. I mean, like, its making sense and feeling good. It(the formula) fits together and I can see why it would work, but like I cannot think why a word would be spelled like this.

The last reference was to learning words and rules in English class. There, memorizing the spelling of words was "rote" to the student, but memorizing equations or formulas was not rote. After seeking how the equation was derived, from either other formulas or a physical event as in F = Ma, Student 46 commented that, "I never gave it a second thought, I just automatically memorized it."

Understanding3 occurs when students can relate new physics ideas to the physics experiences and knowledge for understanding. This represents a shift in the experiences and knowledge they use for making sense of their physics lessons. "Meaningful learning" or "understanding" still comes from "relating to something one already knows" (Ausubel, 1978). However, the already existing knowledge of the students now comes from what they have learned from the physics class. In laboratory experiments, students expect to be able to link each part of the experiments to the concepts or theories they are studying and to gain a meaning for each of the concepts as well as their relationships. General meanings of "it is similar to" or " it can be found in" some everyday occurrences are no longer satisfactory. Understanding3 means knowing the relationship between the concepts in physics. These relationships are more definite and explicit.

As students advanced in understanding3, acquiring more physics concepts and relationships, they sometimes developed their own systems to understand new concepts; i.e., they tried to relate new ideas to their physics knowledge in their "own way." The simplest of these ways was when students formed a structure of knowledge based directly on relationships from equations, definitions or by grouping them together under areas lkie those found in textbooks, for example kinematices or dynamics.

In the second way, students appeared to have developed their own system for organizing their concepts and would then use this system to try to understand any new physics concepts that they were taught. They would try and assimilate or fit these new concepts into their structure or system of knowledge. These students would say that another meaning of "understanding" for them was being able to explain or relate the new ideas to their systems or relationships of the concepts. These students recognize some sort of order in their ideas. For example, Student 41, used "units" of measurement for each concept to organize his conceptual structure. For him all the concepts that he had learned so far could be related to three concepts - mass, time, and distance. These units were the three basic concepts to him. Thus, he analyzed all the new ideas, looking for the units in which these concepts were measured. He broke down these units into their most basic components. He then related these new concepts by their basic unit components to the structure that he already had. This involved relating the units of these new concepts to his basic concepts of time, mass and distance. Thus he would link force to work because they both had some basic units in common. He had been introduced to heat in his physics class, but not to its measurement by joules. He did not know how to link the idea of temperature to his system and felt that he did not understand heat fully at this stage. At times the system or structure takes a more heirarchical form.

Although the students' conceptions of ideas may not have been correct, the point here is that there was an attempt to organize ideas through analysis and reorganization, using a system that he, the student, had thought out for himself. He was not merely linking ideas as he was told. Thus understanding of any new concept became one of being able to explain it within his "system." This however, does not preclude students from learning new systems. They were systems developed by students based on attending three months of physics class.

Therefore understanding3 comes from the physics classroom. It involves seeing connections or relationships between and among concepts and events from within the physics classroom. The students who work for understanding3 are using physics experiences to understand or make sense of new physics ideas. Understanding3 can lead students to form their own structure of physics and experiences. It leads to a realisation by students that physics concepts from separate lessons are related to each other in some way. This conception also signifies the point where students begin to acknowledge the "related" nature of the different parts of their lessons and that there is a structure with rules governing the relationships. Students may receive a sense of such relationship from their teachers, or they may form it themselves. The outcomes of understanding3 have varied from formation of simple relationships of ideas, formation of their (students') own system of structuring the concepts, to an understanding that there is a structure to all the ideas and events that they are learning in their physics classroom.

Understanding4. involves students' experiences and knowledge from both inside and outside their physics classroom. Students who claim understanding4 try to explain their everyday experiences with physics concepts and principles. Students have attained a certain familarity and ease with physics concepts they have learned. This effort is the reverse of the effort to reach understandingl. Accompanying understanding4 comes a personal idea of what physics is all about. How did we get our physics laws is a question to be answered. This may not be the epistemological meaning desired by the subject-mnatter specialist, but nonetheless, it was the students'. Students are not just concerned with results of experiments. They now recognize the "perfections" of the laws and principles they are learning and want to understand how they were reached. In one sense, students can be considered to have ceased separating their two worlds inside and outside the physics classroom and to have begun to see the experiments in their classroom as something that would occur in the everyday world with all the attendant problems and influences that would make the experimental results inaccurate. One result of understanding4 is that students have started to structure and conceptualize their everyday experiences through physics concepts and principles. For example, Student 44 said,

> I now drive a car, I drive a tractor, I shoot a gun now, things I can feel the action, the recoil, the car not stopping. ... now I can draw a parallel between the two.

This Student 44 had commented that before, when he first learned about the principle of conservation of momentum, he had not thought to associate the recoil of a gun, the effect the bullet had on a target, with a tractor's or a car's not stopping or colliding with something. Thus, diverse events could now be understood and integrated by a physics principle or concept. He now saw these events in terms of mass and velocity and as events where the principle of momentum should be conserved.

Understanding4 differs from understandingl where an event in the daily experiences of the student is used to illustrate or explain an abstract principle or concept. In understanding4, students start to bring together their two "worlds" of experiences, from both inside and outside the physics classroom.

In addition, students become more self-directed, beginning to explore the relationships of ideas on their own, having internalized the subject matter so as to be able to "break it (an idea) apart into its basic components, and reassemble it to your advantage or to your greater knowledge," (Student 44). Thus for understanding4, I see students bringing together their two sources of understanding, the two "worlds" from which they have learned to experiences new ideas, and in doing so, integrating them into their knowledge structure. They have learned new ways of organizing their knowledge and experiences, and thus their meaning of "understanding" has grown to include them. They have learned, in Kuhn's terms, to put new "conceptual goggles" with which to look at the world they live in. In doing so students appeared to have been able to look at the events from a "third person point of view." (Student 44) They have not necessarily discarded the old ones, but have modified them, or have added to them. They have thus become more versatile students, having more ways to understand the world.

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CONCEPTIONS OF FORCE AND MOVEMENT, INTUITIVE IDEAS OF PUPILS IN ZIMBABWE IN COMPARISON WITH FINDINGS FROM OTHER COUNTRIES

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1. Introduction and Summary

A number of investigations have revealed pupils' intuitive ideas and misconceptions about force and movement (Gilbert and Zylbertsztajn 1985, Halloun and Hestenes 1985, Minstrell 1982, Terry and Jones 1986). The studies indicate that students experience conceptual difficulties over a wide range of age and educational levels.

There are, however, only a small number of studies which systematically analyse the nature and frequency of the misconceptions as a function of the amount of instruction received. Also, almost all research has been limited to the study of pupils in Western countries. Research into the occurrence of misconceptions with respect to force and movement in non-Western countries and cross cultural comparisons have hardly been performed.

In this paper we report on the results of a questionnaire on daily life problems involving simple mechanics, which was administered to students in Zimbabwe, Lesotho, Indonesia and The Netherlands.

In Zimbabwe we carried out an in-depth study by investigating misconceptions on force and movement at various levels of education, ranging from Form I Secondary School to a university programme for upgrading teachers to a BEd degree. In the analysis of the test scores an attempt has been made to disentangle the effects of various parameters such as: the amount of instruction received, ability, age, sex and school location (rural or urban). The main variable responsible for the variation of the test scores seems to be the amount of instruction received, the relationship having an approximately linear character. Dominant intuitive ideas are found from the intercepts given by linear extrapolation. The slope of a line gives a measure for the resistance of an intuitive idea to instruction. Preconceptions (Driver and Erickson 1983, Gilbert and Watts 1983) are brought to light as those intuitive ideas which resist tutorial correction.

The test was also administered to pupils and teacher students in Lesotho, Indonesia, and The Netherlands. The total group of students tested shows striking differences in the persistence of particular preconceptions, as identified in the in-depth Zimbabwe study, between the two African and the other two countries.

In the following we first describe the groups tested in Zimbabwe and we report on the results of an in-depth analysis. Thereafter we indicate the groups tested in the other countries and we perform a cross country analysis. Finally, some conclusions will be drawn.

2. Test population in Zimbabwe

Before specifying the particular groups tested, we first briefly describe the secondary education system in Zimbabwe in general. The system consists of 6 school years (Forms) which are structured as follows:

- Forms 1 and 2, resulting in the Zimbabwe Junior Certificate (ZJC). The ZJC examinations are administered by a national examination board in Zimbabwe. The ZJC examination results are often used to stream pupils in Form 3 in classes of more homogeneous ability.

  The transition from Grade 7 of primary education to Form 1 of secondary education has increased after Independence (in 1980) from 60% to almost 100% in 1985. The transition rate from Form 2 to Form 3 is nowadays also near to 100%.
- Forms 3 and 4, resulting in the Cambridge G.C.E. (Ordinary level). The Cambridge Overseas Examination (COSC) results are processed in the U.K. Based on the COSC results the entry into higher forms of secondary education is regulated. The transition rate from Form 4 to Form 5 increased from about 13% before Independence to about 22% in 1984.
- Forms 5 and 6, resulting in the Cambridge G.C.E. (Advanced level). The A-level examination results are processed in the U.K. Pupils take 3 subjects for their A-level. In the physics stream students often take, besides physics, mathematics, and: chemistry or biology. From this it is clear that the amount of physics instruction in the physics stream in Forms 5 and 6 is much more intensive - about twice as much - considering the fewer subjects on which one could concentrate compared to Forms 1-4.

Science teachers in the first Forms of secondary education in Zimbabwe should have at least a Teacher Training College (TTC) Certificate. The TTC programme takes 4 years. Two years are spent in the college in which half of the time is devoted to two science subjects and the other half on educational foundation and professional preparation. The other two years consist of supervised teaching in an approved secondary school. The final academic level of a TTC is sometimes claimed to approximate A-level for the particular science subjects chosen. However, it is doubtful whether this level is actually realised in practice.

The certificate teachers can upgrade their qualification in science teaching by attending the BEd (Science) programme at the University of Zimbabwe. Starting in 1986 this programme is a one-year crash course. For admission into the course a certificate teacher should have a good academic background (in terms of O-level and TTC results) and should have at least two years of teaching experience in a secondary school.

Teachers attending the physics stream of the BEd programme are, after completion of the course, qualified to teach physics in Forms 5 and 6. The final level of the BEd (Science) course is aimed at an academic level half-way Year 2 of the Faculty of Science of the University of Zimbabwe. This objective has been formulated to assure that the BEd graduate can give sound academic guidance to secondary school pupils up to A-level.

We investigated intuitive ideas on force and movement among 162 pupils and 'teacher students' in Zimbabwe. For that purpose we administered a test in May 1986, around the middle of the school year, to students of:

- a rural school, about 30 km from the capital, which has neither electricity nor a running water system. Practicals are performed by making use of the so-called ZimSci kit. The teachers composed a mixed ability group of in total 62 pupils from Forms 1, 2 and 4 to attend the test.
- an urban school in the capital, located near the University of Zimbabwe. The school is well provided with all kinds of practical facilities. In total 85 pupils of the following groups attended the test:
  Form 3^1: rank no.1 out of 8 parallel Forms 3, ordered in terms of school performance;
  Form 4^7: rank no. 7 out of 10 parallel Forms 4, ordered in terms of school performance;
  Form 6^p: out of the 4 parallel Forms 6 the class that takes physics as one of the three A-level subjects.
- the University of Zimbabwe. The test was administered to 15 certificate teachers attending the physics stream of the BEd programme. The participants had on the average 8 years of professional experience after completion of the TTC. Some had an A-level qualification before admission into the TTC.

3. Test instrument

The test booklet contains 22 questions, which are preceded by a brief instruction. All questions refer to problems which require no calculation or application of algorithms. Each problem is illustrated by a diagram.

The questions have either been taken from the literature or been compiled by ourselves. Those questions have been included in the test which expose most explicitly specific misconceptions in the field of force and movement. Out of the 22 questions 16 items were inspired by articles in the science education literature, namely: Lie, Sjøberg and Ekeland 1985, McClosky 1983, Minstrell 1982, Van Genderen 1983.

All questions are set in multiple choice. For some questions, however, a short explanation is also asked for. In administering the test the participants were allowed as much time as they wanted to finish the test paper. In practice the pupils needed about 20 to 30 minutes.

Hereafter the total set of test questions is given briefly, without the accompanying diagrams and without spelling out the alternative answers out of which the students were asked to make a choice.

Q1 Inside a train driving at a constant velocity a person is jumping, either in the same direction in which the train is moving, or in an opposite direction. In comparing the lengths of the jumps as measured on the floor of the train, what result will be found?

Q2 A person cycling at a constant velocity throws a ball straight upwards. Does he manage to catch the ball without moving his hand?

Q3 By continuously applying the brakes a cyclist manages to drive down a very steep hill at a constant velocity. In comparing the breaking force with the force pulling the cyclist downhill, what result will be found?

Q4 Are there forces acting and, if so, which ones, on:
- an apple hanging from a branch of a tree?

Q5 - an apple falling from a branch of a tree towards the ground?

Q6 - an apple lying on the ground after having fallen from a branch of a tree?

Q7 A person is throwing a stone straight up in the air. Which forces act on the stone after it has left the hand on its way upward, and how do they compare?

Q8 - Which forces act on the stone in the highest position which it reaches, and how do they compare?

Q9 - Which forces act on the stone when falling down after it has reached its highest position, and how do they compare?

Q10 Two persons push their cars up a slope just hard enough to prevent the cars from rolling downhill. If the two cars are identical, but one car is at a higher position on the slope than the other, how do the pushing forces required compare with each other?

Q11 Two persons are playing tug-of-war by tightening a rope. If none of the persons move, how do the forces with which they pull compare with each other?

Q12 A person is trying to push a heavy case over a horizontal surface. If the person does not succeed in moving the case, how does his pushing force compare with the force of friction between case and floor?

Q13 A ball thrown straight upwards by a person passes first through point P and then through point Q. Are they upward forces acting upon the balls in points P and Q and, if so, how do the upward forces in P and Q compare with each other?

Q14 - Are there downward forces acting upon the balls in points P and Q and, if so, how do the downward forces in P and Q compare with each other?

Q15 A car is towing a trailer over a level road and is moving at a constant velocity. How does the towing force and the frictional forces acting on the trailer (exerted by the surface of the road) compare with each other?

Q16 How does the towing force and the force exerted by the trailer on the car compare with each other?

Q17 Two magnets, one of them being stronger than the other, are held at a fixed distance with the North poles facing each other. How does the force exerted by the stronger on the weaker magnet compare with the force exerted by the weaker magnet on the stronger?

Q18 A person is standing on a plank which bridges a river. The plank is bent because the man is standing on it. Does the plank experience a force exerted by the man?

Q19 - Does the man experience a force exerted by the plank?

Q20 The moon circles in an orbit around the earth. The earth exerts an attracting force on the moon. Does the moon also exert an attracting force on the earth?

Q21 An airplane is flying at a constant velocity and a constant altitude. If a metal ball is dropped from the plane, what path will it follow as seen from the ground?

Q22 A metal ball is rolling over the top of a cliff at a constant velocity. If the ball goes over the edge of the cliff, what path will it follow as seen from a side view of the cliff?

On the next page some examples of the test questions are given. The test scores are also shown, represented by bar charts which indicate the percentage of pupils who selected the various alternatives. The results of the groups 1 ... 7 (the numbering of the groups is explained in the next section) are followed by the percentages for the total test population in Zimbabwe.

QUESTION 2

A boy is cycling at a constant speed on a straight road.

He throws a ball straight upwards.

Does he manage to catch the ball without moving his hand?

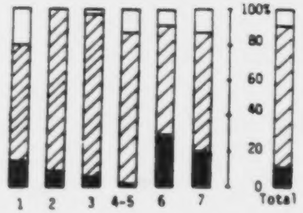
You may ignore wind and air resistance.

a) Yes ■

b) No ▨

Give a short explanation.

____



QUESTION 17:

John has two magnets. He is holding them at a fixed distance with the North poles facing each other.

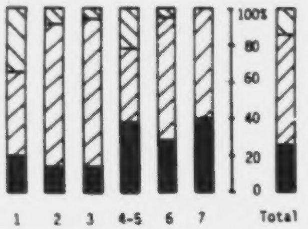
The left magnet is stronger than the right one.

The force exerted by the stronger magnet on the weaker is:

a) greater than the force exerted by the weaker magnet on the stronger ▨

b) equal to the force exerted by the weaker magnet on the stronger ■

c) smaller than the force exerted by the weaker magnet on the stronger ▧



QUESTION 18:

A man is standing on a plank which bridges a small river.

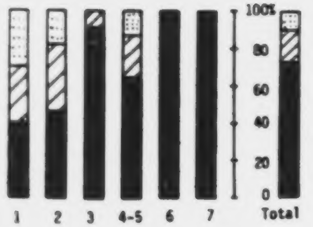
The plank is bent because the man is standing on it.

Does the plank experience a force exerted by the man?

a) No ░

b) Yes ■

c) Yes, only if the man is very heavy ▨



4. Characteristic parameters

A pupil's test performance will of course be influenced by the total amount of physics instruction he or she has received during the years of attending the secondary school. Apart from the amount of instruction other parameters matter, such as: ability, age, sex, school characteristics (e.g. location either in a rural or in an urban area).

In the following subsections we discuss the way in which the values of the various parameters have been fixed. In the last subsection we give some reflections on the interdependence of some parameters.

4.1 A first indication of the amount of physics instruction, the parameter INS, can be found by just taking the corresponding number of school years of formal secondary education. A second indication is derived from the intensity of the instruction as determined by the number of hours per week which are devoted to science subjects, in particular physics.

A pupil of Form 1 can be considered as receiving one unit of instruction in physics; a pupil of Form 2 can be put at two units of instruction; etc. up to Form 4. Then, at the end of Form 4, pupils write the COSC examination. Based on the COSC results about 20% of the Form 4 pupils continue into Form 5 where they are differentiated, e.g. into a particular physics stream where they can spend twice as much time on physics (and two other science subjects) as they did before. So, in the first part of Form 5 pupils can reasonably be put on 5 units of instruction and, by the end of Form 5, they have received about 6 units. Along the same arguments pupils in the first half year of Form 6 can be said to have received about 7 units of physics instruction. The final A-level then corresponds to 8 units of instruction.

The entrance level of the average participant of the BEd (Science) programme can be estimated to be around 7 units. The final level aimed at in this programme (see section 2) corresponds to about 11 units of instruction. This implies that the participants of the BEd (Physics) course, which consists of 4 terms, have to upgrade their physics level from about 7 units to around 11 units. The test was administered in the second term of the programme, so the parameter INS indicating the amount of physics instruction received can be set at 9 units. In relation to this value, it should also be considered that the BEd participants had covered the relevant mechanics part of the syllabus during the first term.

The above values of the parameter INS represent only first estimates of the amounts of instruction received by the various groups. To mention just one complicating factor: it certainly cannot be said that each unit of instruction contains the same amount of physics instruction, certainly not on the particular subject of the test, namely the relationship between force and movement. On the other hand it can also be argued that, even if the science instruction offered refers to different subject matter, the problem solving approach in the field of the test might benefit from the instruction in a more indirect way.

4.2 The parameters AGE, SEX, and the school parameter UR (urban or rural location) can be determined from the data filled in on the front page of the test booklet.

A total of 157 pupils answered the question on age, 5 pupils left the question open. The age histogram of the Zimbabwe test population is shown in the figure below.

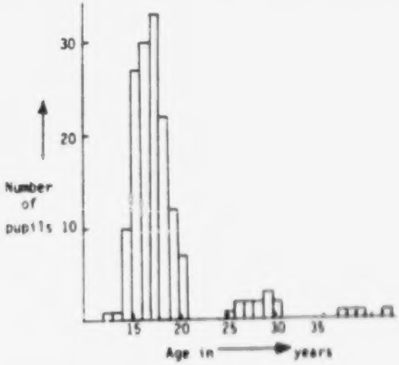


Fig.1: Age histogram of the Zimbabwe test population

The ages range from 12 to 42 years and have an average value of 18.4 years. In order to account in our analysis for age related development stages of the pupils, we have made a division into 5 groups which are labelled by the parameter AGE. The parameter value1 indicates the youngest group of pupils, value 5 indicates the group of students more than 20 years old (in fact the BEd students).

An age group represents, in principle, an interval of two years. Only AGE 5 represents a larger interval, consisting of students from 25 to 42 years of age. It is reasonable to treat this larger interval as one age group for purposes of test score analysis, since it cannot be expected that a 25 year old teacher and a 42 year old teacher differ significantly in their test performance by the very fact of their difference in age. Of course it can be said that the older teacher could be more mature and could have more teaching experience (but not necessarily so). On the other hand the younger teacher has finished his or her teacher training more recently and is perhaps more alert and fresh in tackling new problems. So it cannot be argued that within the (older) BEd group the age of a participant as such is determining a difference in the test performance. However, for the younger pupils (between 12 and 20 years old) age corresponds with a stage of cognitive development and could be a critical parameter in the test performance. For the group of youngsters it is therefore reasonable to differentiate the age groups in small intervals.

The school characteristic parameter UR is to be indicated as:

- rural, for groups 1, 2 and 4 which refer to Forms 1, 2 and 4 respectively of a rural school (with modest facilities);
- urban, for groups 3, 5 and 6 which refer to an urban school (with very good facilities), and for group no.7 which refers to the BEd group at the University of Zimbabwe.

The sex distribution in the various groups is given in Table 1. Female pupils are equally represented (50%) in groups 1-5; they are not equally represented (14%) in groups 6-7.

4.3 The parameter AB indicates the extent to which the pupils have been differentiated into streams of more or less ability, on the basis of results of selective examinations which they have written in the past.

As mentioned in section 2 the streaming into various Forms 3 (high - medium - low ability) is done on the basis of the ZJC examination results. The basis for admission into Forms 5-6 of the High School and into the Teacher Training Colleges is formed by the COSC results. Those students who are admitted into the science stream of a High School or of a Teacher Training College can be considered to have, in principle, top ability for dealing with the test problems (as far as examination results are good indicators of ability in the field of science).

GROUP No.	N	CHARACTERISTIC PARAMETERS UR	INS	AB	SEX % females	AGE years average	TEST SCORE MEAN %	STD.DEV %
1	20	R	1	3	50	15.1	24	11
2	22	R	2	3	46	16.4	27	9
3	35	U	3	4	46	16.1	32	10
4	20	R	4	3	45	18.5	32	11
5	28	U	4	2	64	17.7	25	9
6	22	U	7	5	9	19.4	47	12
7	15	U	9	5	20	30.6	62	16
TOTAL	162				42	18.4	34	16

Table 1: Test results and characteristic parameters of the Zimbabwe groups

Perhaps it is questionable whether entry into the science stream of a TTC could be considered as indicating a top ability. However, it should be realized that in the period before Independence the TTC was one of the few options open for bright black Zimbabweans.

Groups 1, 2 and 4 have been formed by their science teachers who did compose 'mixed ability' groups of pupils out of the Forms 1, 2 and 4 respectively. If we give the parameter AB a range of 1 - 5, the AB value of the above groups can be fixed at 3, assuming that the science teachers have correctly followed up the 'mixed ability' indication.

As indicated in section 2, group 3 and 5 represent a higher ability (Form 3^1) and lower ability (Form 4^7) group respectively. We can therefore fix their AB values at 4 and 2 respectively (a smaller value indicating a lower ability). The AB values of groups 6 (high school) and 7 (BEd group) can be set at the value of 5 as we argued above.

The above approach for obtaining a set of AB values (shown in Table 1) is just a first attempt to account for the fact that we deal with selected groups of various streams in the test population. It remains, however, doubtful whether streaming based on examination results really reflects ability in the context of our test. Anyhow, the inclusion of the parameter AB accounts to some extent for differences to be expected.

4.4 In discussing the values of the characteristic parameters it should be realized that there is a considerable overlap and interdependence of the variables they represent. In the next we give two examples of this situation.

First, if we want to include higher amounts of physics instruction in our study (which we did by also testing A-level students and the BEd group), we have to focus on pupils who are older. Also, as they are older they have passed more selective gates (for example ZJC and COSC exams) and have been streamed into various ability groups. To see the effects of further physics instruction, we have to focus on the high ability (in terms of physics) group. From the above it will be clear that instruction, maturation (age), and ability are rather intertwined, at least for the groups that we study in the test.

Second example. If we include larger amounts of physics instruction in our study we have to include High Schools. These schools, however, are mainly found in the urban areas. Also, it is known that the majority of students who opt for further education (beyond Form 4) in the field of science, are males. So, including higher forms of physics instruction implies an underrepresentation of both rural pupils and of females. This situation is clearly visible in Table 1. In the analysis of the test results we will try to account for these effects of interdependence.

5. In-depth analysis

For subgroups of the total test population in Zimbabwe, referring to particular values of the characteristic parameters INS, AB, SEX, AGE and UR, the mean values and the standard deviations of the test scores are shown in table 2.

Before drawing any premature conclusions, it should be realised that rural and female pupils are underrepresented in higher INS groups. As we stressed in the preceding section, the various parameters are rather interdependent. Observed differences between means of groups may only partially be attributed to the source of variation which is under particular consideration. The apparent differences may also be caused by different contributions to the various groups effectuated by the other sources of variation. In fact we have to correct for intertwining effects, to find as much as possible the separate contributions of the various parameters to the variance of the test scores. This will be discussed next.

5.1. Covariance analysis

We have used the programme New Regression Analysis to trace the various sources of variation. In looking for single responsibilities for separate contributions to the variation observed, the programme indicates that the contributions caused by the parameters AGE and UR are statistically not significant.

The separate contributions caused by the other parameters turn out to be significant and to be able to explain the following parts of the variation observed:
AB = 12.1% , INS = 9.2% , SEX = 1.3%.

Apart from these separate contributions much variation can be explained by a mix of different parameters which have overlapping contributions. In using the programme New Regression Analysis for calculating the part of the total variation that can be explained, it should be realized that the results depend in principle on the order of hierarchy, in which the various parameters AB, AGE, INS, SEX, UR are entered. If one, however, does not require a specific preferential treatment, the programme starts with the variable that explains (partly in combination with others) most of the variance, then it goes to the second best explanatory variable, etc. Following this procedure we have found the following contributions (partially separate and partially overlapping with others) to the variation observed:

INS = 46.3%, AB = 6.6%, SEX = 1.7%,

whereas the parameters AGE and UR do not contribute in a significant way.

INS	MEAN	STD DEV	N
1	24	11	20
2	27	9	22
3	32	10	35
4	28	10	48
7	47	12	22
9	62	16	15
Total	34	16	162

AB	MEAN	STD DEV	N
2	25	9	28
3	28	10	62
4	32	10	35
5	54	16	37

SEX	MEAN	STD DEV	N
M	38	17	95
F	28	11	65

AGE	MEAN	STD DEV	N
1	20	9	11
2	29	11	57
3	31	12	55
4	43	13	19
5	62	16	15

UR	MEAN	STD DEV	N
U	38	17	100
R	28	10	62

Table 2: Mean % scores, standard deviations and numbers of persons for groups tested in Zimbabwe, for particular values of the characteristic parameters

So we can conclude that we can explain in terms of 3 characteristic parameters, with reasonable significance about 55% of the variation of the test scores. Most of the variation can be attributed to the (more or less mixed) parameter INS.

Now we can compare the observed test scores with the scores calculated on basis of the values of the parameters INS, AB and SEX with weights as indicated above. The deviations of the observed test scores from the calculated ones are shown in Figure 2. It can be seen that the standardised residual scores reasonably follow a normal distribution.

In other words, no systematic deviations but mainly random ones are found, if the test scores are described in terms of the parameters INS, AB and SEX, where the first parameter accounts for most of the observed variation.

5.2. Linear regression analysis

As we have seen, much of the variation of the total test scores can already be found by assuming that the scores are linearly dependent on the amount of instruction received by the pupils.

In this section we report on the results of performing a linear regression analysis on the level of the various items and the respective alternative answers (one of them being the correct physics conception and the others incorrect misconceptions).

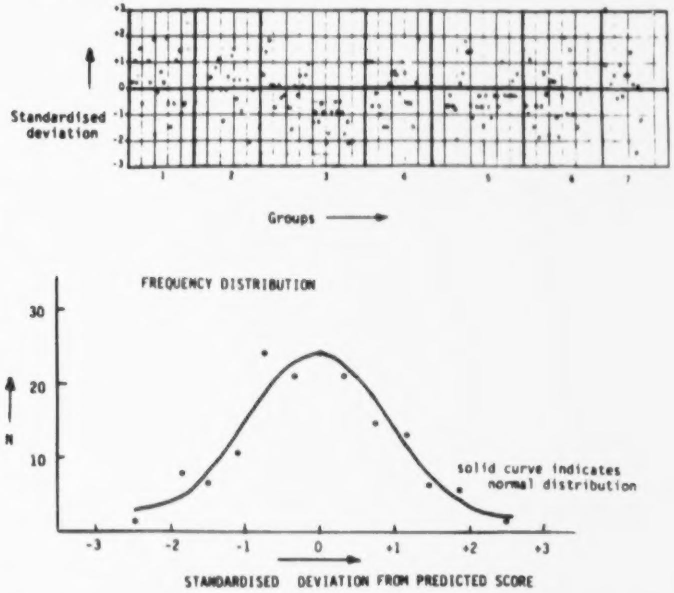


Fig.2: Deviations of the scores (of all testpersons in Zimbabwe) from the scores predicted on basis of the values of INS, AB and SEX

From the analysis we get the intercept, the slope and the value of significance for fitting a straight line to the scores of all alternative answers as a function of INS.

The intercepts can be interpreted as the *intuitive ideas* , the conceptions pupils have before they receive any instruction in physics.

The slope of the line is an indication of the effectiveness of physics instruction. The improvement due to instruction can be called unsatisfactory if the change is less than 4% per unit of instruction (4.6% is the mean value). Improvement means: an increase of the percentage of pupils who choose the correct alternative (physics conception), and a decrease of the percentage of pupils who choose a wrong alternative (misconception). A small slope means that a wrong idea is difficult to correct. A high slope means that a wrong intuitive idea can be easily corrected by instruction.

For the total group of 22 questions, out of the various alternatives on the average 34% of the Zimbabwe test population choose the correct physics conception, and on the average 43% choose particular alternatives corresponding to dominant misconceptions.

The regression analysis indicates whether the line has a slope significantly different from zero. In order to see for small slopes whether the relationship between mean score and instruction has a more or less linear character, we also performed a chi-square analysis. Only in one case (question 1) can the relationship certainly not be indicated as linear. In that item the dependency on the instruction has a parabolic character: the percentage for the correct alternative first decreases as function of instruction, and then increases again. This suggests that the (correct) intuitive idea of the pupils is first confused by the instruction they receive (e.g. by the information they get on how to add velocities), and redressed if they really understand that principle .

After having analysed the relationship between scores and amount of instruction, we identified those questions where:

a. the percentage of pupils selecting the correct option is below average (i.e. smaller than the total test score)
b. the increase of this percentage due to instruction is below average (i.e. less than mean improvement due to instruction)
c. the decrease of the percentage of pupils selecting the dominantly chosen wrong option is also below average.

These questions are relatively difficult and concern wrong intuitive ideas, to be called *preconceptions,* which resist tutorial correction.

These questions consist of items 2, 7, 8, 10, 12, 13 and 17. For some of these items the mean scores as a function of the amount of instruction are plotted in Figure 3.

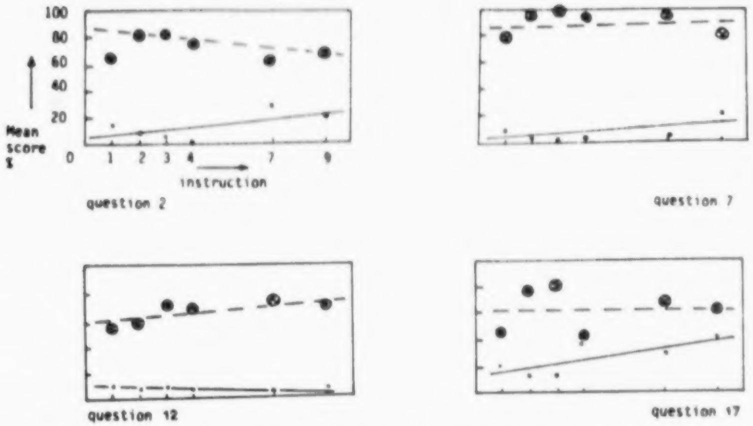


Fig. 3: Examples of questions where mean % scores for the correct alternative answer . and for the dominant wrong alternative ø do not change drastically as function of instruction; linear interpolation is indicated by —— and ------ resp.

The striking difference between the increase of the correct response to these 7 questions and to the 15 others, is shown in Figure 4.Before discussing these results we would like to emphasize their tentative character. The dominant contribution of the INS variable is found for the distributions of the *total* test scores. On the level of one particular *item* and its alternatives the desciption in terms of linear dependency on one variable INS is certainly less reliable (compare e.g. the small numbers of pupils responding to the alternative answers for each item).

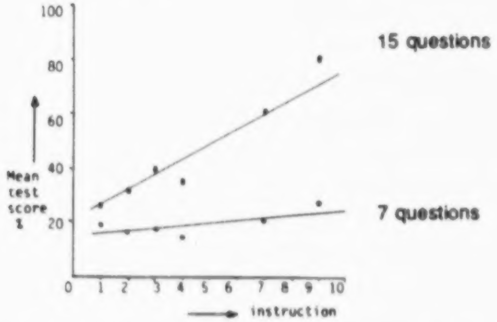


Fig. 4: Mean % scores for the correct alternative answers to the 15 easily corrigible questions and the 7 persistently difficult questions, as function of instruction.

5.3. Preconceptions

We can categorize the main preconceptions under the following headings:

FRAMES OF REFERENCE (item 2)

Pupils have a tendency to define motion with respect to an absolute reference frame (such as the ground), rather than with respect to other reference frames (such as the moving bicycle). A typical explanation given by pupils in item 2 was: *the boy does not manage to catch the ball, because he is moving relative to the ball.*

IMPETUS (items 7, 8, 13)

A stone thrown upwards receives a force from the thrower. This force is gradually 'used up' in due course, because of air resistance or gravity. This conception is the medieval 'impetus idea': the thrower transmits an impressed force to the projectile (Dijksterhuis 1961, p.179). Typical comments given by the pupils: *the force imparted to the stone when it was thrown / the force of the boy's power / the force will expire and be reduced due to wind.*

INDUCED FORCES (item 12)

Frictional forces are seen as having fixed values (the limiting case), rather than being considered as induced forces which equal (if there is not an accelerated movement) the forces that induce them.

INTERACTION (item 17)

Force is seen as produced by a strong agent of force , rather than being considered in terms of an interaction between two objects.

5.4. Coherency

As to the cognitive status of preconceptions, there is still much being debated. McClelland (1984), for example, expresses reservation about the interpretations of the evidence of so-called alternative frameworks of pupils. In his view the teacher has to clarify and to merge very nebulous concepts, rather than having to overthrow 'preconceptions'. Indeed, we also noticed a lack of differentiation and articulation between various concepts of physics e.g.:

- velocity and acceleration (In item 7 a pupil's comment: *the stone moves upward, so there is an acceleration upward - because F = m x a there is an upward force*)
- velocity and force (In item 13 a pupil indicates the force as being the *kinetic force*)
- force and energy, both potential and kinetic (In item 10 a pupil comments: *car B has greater potential energy which has to be overcome to prevent the car from rolling down*; in item 7: *an upward force due to the kinetic energy of the stone*)
- gravity and air pressure of wind (In item 5: *the atmospheric force of gravity*, or: *the force of the wind through which a stone has to come down*)

- force not being associated with inanimate matter (In item 19 a pupil comments: *the plank does not give force but the man, because the man is alive*)
- the vector character of forces (In item 12 a pupil explains the case not being moved because: *the weight of the case may be greater than the force applied*, so he does not discriminate between the vertical direction of the force called weight, and the horizontal direction of the force applied on the case).

So there really is a lack of differentiation between various concepts of physics in situations of force and movement. However, in spite of the articulating efforts which are made during years of physics instruction, still a number of misconceptions survive.

These misconceptions can certainly not fully be attributed to lack of differentiation between concepts. For example: in item 8 there is a remaining dominant misconception of the existence of an upward force in the highest position of a stone thrown upward. In this case there cannot be a confusion between force and velocity, or: kinetic energy, because the latter variables are clearly equal to zero in the highest position.

Therefore, apart from the need to clarify nebulous concepts, our data also suggest the need to account for the existence of alternative preconceptions, such as the impetus idea.

We analysed the answer patterns of the pupils, to see whether we could trace crucial questions or groups of (mis)conceptions which determine to a great extent the total test performance.

First, we did a Principal Components Analysis to see which items can be considered as statistically belonging together. If we group the questions which load on the same factor with a value ≥ 0.40, we can explain 44% of the variation of the test scores by 5 factors; 7 factors explain 54% of the variation. However, the items which according to this analysis belong together, can not be interpreted as all corresponding to the same physics conceptions.

Then, having in mind that the pupils perhaps reason consistently following conceptions which are alternative to physics, we did the same analysis by now entering the dominantly chosen wrong (in terms of physicis) alternative answers as the best options. Also in this case, however, did we not find groups of questions, which can fully be associated in terms of referring to the same conceptions (either right or alternative).

Second, we performed an item-rest analysis. Questions with a value higher than 0.20 can be regarded as representative for the test as a whole. The analysis resulted in indicating 10 such questions, which are just the items where the correct answer scores improve considerably as consequence of instruction. In other words, this analysis does not provide us with additional information. The correlation item-rest is presumably caused by the simultaneous effects of instruction on both of them.

Finally, we calculated the Cronbach alpha for an indication of the homogeneity of the test. We did find an alpha value of 0.6. Maybe the low value is due to the fact that the test covers a large number of conceptions in a relatively small number of questions. It could also be that the test is simply too difficult, in particular for pupils at low levels of education, and that the low test scores do not allow a sophisticated analysis of the answer patterns. However, if we distinguish the results for the subgroups of the test population separately, we notice that the r_{it} values and the alpha values increase as a function of instruction. For example, for the group with most instruction the value of alpha is 0.74 and the average r_{it} values for the 7 preconception items and the 15 other questions are 0.41 and 0.23 respectively. Although, also for this group, the scores for the preconception items are still very unsatisfactory, it turns out that these questions are highly indicative for the total test performance.

It seems as if conceptual coherency and consistency comes in as result of a process of instruction.

6. Test population in other countries

In the preceding section we reported on the analysis of a test administered to in total 162 students in Zimbabwe. For comparison the test has also been administered to, in total 259, students attending several science programmes in Lesotho, Indonesia and The Netherlands. In the following we indicate the groups tested.

LESOTHO

The test was administered to all 118 students attending the so-called LESPEC course in 1986. This programme is an intensive six-months course for upgrading students in the field of science, after they have written the COSC 0-level examination and before they enter into further science based studies (either at the National University of Lesotho, or at professional institutions in the field of science teaching, agriculture or technology). The LESPEC students had been selected from all 3580 Lesotho students who wrote the COSC examination in 1985. Criteria for selection are scores for science aptitude tests and the pupils' science teachers recommendations. So, in principle, the group tested represents the country's top-pupils in terms of science ability.

INDONESIA

The test was administered to, in total 77, students of the Universitas Kristen Satya Wacana in Salatiga, Java. The population consists of the following groups:

- Students in the first year of the 2 year diploma programme (D2 students) which qualifies them to teach mathematics and biology in the junior secondary school
- Students in the first year of the 2 year diploma programme (D2[P] students) which qualifies them to teach mathematics and physics in the junior secondary school
- Students in the first and the second year of the 3 year degree programme (S1 students), which qualifies them to teach physics in the senior secondary school.

THE NETHERLANDS

The test was administered to, in total 64, students of the following groups:

- Forms 4 and 5 of the pre-university stream (VWO) of a secondary school
- Students of a 3 year diploma programme for certificate teachers in physics at the teacher training college Hogeschool Holland (HH) in Amsterdam.

The average test results for the above groups are given in Table 3 below.

	SUBGROUP	MEAN	STD DEV	N	MEAN AGE
Lesotho	LESPEC	40	12	118	20
Indonesia	D2	30	11	17	21
	D2(P)	43	17	26	21
	S1	58	18	34	21
Netherlands	Form 4	46	19	20	15
	Form 5	74	14	25	16
	HH	74	20	19	21

Table 3: For groups of the test population in Lesotho, Indonesia and The Netherlands the test results and the mean age (in years)

7. Cross country comparison

In this section we will compare the results obtained in the Zimbabwe study with the findings of administering the same test in other countries (Lesotho, Indonesia, The Netherlands), as well as with indications from literature.

The first observation to be made in comparing the results of the 4 countries is that the series of item mean scores are rather similar. Although the various series of mean scores have different absolute values, the structures of the series are reasonably the same. The value of Tucker's phi is greater than 0.90 for all cases of comparing two sets of series, which indicates that the structures are similar. That is to say that, more or less, the same questions are relatively easy or relatively difficult to be answered in all 4 countries.

In order to perform a cross-country comparison of test results, we have to take care that the subgroups of the test populations which we compare have about the same instructional background and motivational attitude towards physics.

For that reason we have taken those subgroups which have taken physics to the highest degree of specialization in the secondary school or which are being trained as physics teachers. This results in the choice of the following subgroups:

- Zimbabwe	: Form 6P and BEd(physics) 'teacher students'	(N=37)
- Lesotho	: the LESPEC students with top scores in physics	(N=50)
- Indonesia	: the subgroups D2(P) and S1	(N=60)
- Netherlands	: the subgroups VWO Form 5 and HH	(N=44)

So, out of the total number of 421 students we have tested, we now are going to compare a total of 191 students, who have opted for a special training in physics.

In Table 4 the results are given separately for the group of 7 questions (items no. 2, 7, 8, 10, 12, 13, 17) which according to the Zimbabwe study apparently test preconceptions being most resistant to instruction, and for the 15 other questions which are more corrigible by instruction.

The table also shows the average values taken from the literature which is available for 5 questions of the first group and 10 questions of the second group, namely:

- Lie et al 1985, a Norway group of pupils in grade 3 of the upper secondary school, age 19
- McCloskey 1983, a USA group of undergraduate students
- Van Genderen 1983, a Netherlands group of pupils in Form 4 of VWO (the pre-university stream of secondary education), age 16

From the table we can see that a cross-country comparison of average scores indicates much more similarity for the 15 questions referring to 'other misconceptions', than found in the items related to 'strong preconceptions'. The mean score for the correct alternatives of these 7 items is 22% in Zimbabwe compared to about 56% in the Netherlands/literature. The (small) difference between the average value for the Lesotho group and the Zimbabwe group could be accounted for by the difference in level of instruction (which is about 5 units for the LESPEC group and 7 to 9 units for the Zimbabwe top group in physics).

Mean % scores for correct answers to questions testing	Persistent Preconceptions	Other Misconceptions
Zimbabwe	22	67
Lesotho	16	53
Indonesia	34	59
Netherlands	58	81
Literature	54	70

Table 4: Mean % scores of the correct alternatives for the 7 items which, according to the Zimbabwe study, test preconceptions resisting tutorial correction, and for the other items, for comparable subgroups in the 4 countries and the literature .

So we get the following picture from a cross-country comparison. The mean test scores for most items of the test that we administered are reasonably similar in the various countries. However, for the subgroup of 7 items, which we traced in the Zimbabwe study as being very resistant to change by instruction, we find a different picture. These preconceptions are more strongly present in Zimbabwe/Lesotho than in The Netherlands/USA/Norway. The Indonesian results have values which are somewhere between the two groups.

The most remarkable differences concern preconceptions relating to: frames of reference (item 1), the 'impetus' idea (items 7, 8, 13,) induced forces (item 12).

Now the question arises as to how cross-country differences in the persistence of preconceptions could be explained. There are, in principle, two possibilities.

a) Preconceptions are present to the same extent and depth everywhere, and equally difficult to be changed by instruction.
   Cross-country differences arise, because of the fact that the teaching strategy and/ or the school facilities do not allow for addressing these preconceptions in a similar way, so that surviving chances of preconceptions are different.
b) Preconceptions are generated and maintained by sources and mechanisms, which are outside the school environment. The persistence of preconception can be different, if these sources and mechanisms are not the same in various countries.This possibility we would like to elaborate somewhat in the following.

The question can be raised whether preconceptions have some connections with more general pictures and images of reality which are prevalent in a culture. On such a cultural-anthropological level differences between western and non-western countries might be expected. One could think of the mechanization of the world picture which took place in the seventeenth century in the West (Dijksterhuis 1961), and the total different symbolic loading of concepts such as force (Tempels 1969) and time (Mbiti) in a traditional African world. The question, however, is whether these deeper symbolic layers will be actualized and mobilized by students when they have to answer concrete questions on situations involving mechanics in formal education.

The mechanization of the world picture, as mentioned above, has been a radical process. When in the seventeenth century Aristotelian science was replaced by Newtonian physics "substantial thinking which inquired about the true nature of things, was exchanged for 'functional' thinking, which wants to ascertain the behaviour of things in their interdependence, with an essentially mathematical mode of expression" (Dijksterhuis 1961, p.501).

"The thinkers of the Enlightenment abandoned the earlier concerns of philosophers, the question of *why* (and the search for a final cause) and moved to the question of *how* (and the method of causation)." (Heisenberg 1973, p.38).

The above pictures of reality are quite in contrast with explanations in a traditional African setting, where e.g. force is seen as the essence (inner nature) of an object (whether animate or inanimate) and not as an accidental quality. In studying Shona cosmology Bucher considers 'power' as the root concept, and states: "Even when power is thought to be immanent in inanimate matter it bears personal traits" (Bucher 1980, p.15).

We, however, should be careful in drawing conclusions, since, as Horton warns us: "Even those familiar with theoretical thinking in their own (western) culture have failed to recognize its African equivalents, simply because they have been blinded by a difference of idiom" (Horton 1967, p.50). Therefore, it is very much questionable whether pupils from a western world (with a tradition of mechanizing the world picture) are, for example, less prone to a preconception like the 'impetus' idea, than pupils from a setting with a more or less traditional African world view.

We had interviews with Zimbabwean teachers and also raised the question as to how they incorporated physics in a traditional setting. One teacher told us that he often relates his examples in the physics lessons to traditional experience. He e.g. compared the exerting of a force with 'the casting of a spell over somebody'. By such an example, however, pupils are tempted to see force as an inherent quality of an object, instead of an external agent only.

In review articles on preconceptions it is argued that pupils construe meaning from their direct sensory experiences occurring in the *life world domain*. Preconceptions are then considered as alternative frameworks constructed in this way into a *symbolic* domain, where they compete with models and theoretical entities constructed by the natural sciences.

However, in science education research literature it is often neglected that the construing of the symbolic domain is also a cultural activity. In terms of physics an alternative framework has legitimacy, if it can answer the *how* question of natural events. However, preconceptions are part and parcel of a broader cultural outlook where also the *why* question is addressed. This could explain why in a non-western setting preconceptions are different or more resistant to instruction than in the west.

To put it in other terms, the developments that have taken place in the west have resulted in a two world model of symbolic domains: the world of the natural sciences and the world of the spiritual values. Difficulties which students in non-western countries have with the particular thing called 'science', could originate from this dichotomy. As Ladrière puts it "The direct impact of science on culture does seem to exist in setting the cognitive system apart from other systems, particularly the axiological systems, and hence introduce into a culture a dualism or pluralism which runs counter to its integrating ability" (Ladrière 1977, p.78).

The above considerations are somewhat speculative and cannot be tested easily. However, they should make it clear that cultural-anthropological data have to be taken into account, before one could attempt to draw conclusions with respect to preconceptions, on the basis of cross-country test results. Therefore, apart from straightforwardly administering tests, efforts should be made to trace the character of the arguments pupils use in explaining their choice of a particular alternative answer in a test item. For many test items we asked for a written explanation. However, to really probe the motives and arguments pupils have in their mind, the administering of the test should be followed by more in depth interviews.

8. Conclusions

Our investigation has the character of a pilot study to give us methodological experience as to how questions on intuitive ideas of pupils, and preconceptions in particular, could be handled in further research. In the following we will summarize our findings.

a) In analysing the test results of the Zimbabwe test population, it turns out to be possible to explain most of the variation of the test scores in terms of three variables: the amount of instruction, 'ability', and sex. The first of these parameters, the amount of instruction, accounts for most of the observed variation. The residual variation, which is left over after the effects of the three variables have been accounted for, is randomly spread following the normal distribution. This suggests that no systematic effects have been overlooked, in analysing the performance of the various subgroups in terms of the characteristic parameters.

b) The main variable for explaining the observed variation of the mean scores, is the amount of instruction received. Now the relationship between total test score and amount of instruction turns out to be linear. Also on the level of one test item and its various alternative answers the relationship between mean score and amount of instruction has a more or less linear character.

This finding is most important, because it allows us to trace the intuitive ideas of the pupils, i.e. the conceptions they have before receiving any instruction in physics. If there is a linear relationship of mean score and level of instruction, we can find the dominant intuitive ideas from the intercepts as determined by linear extrapolation. Also, the slope of the line provides us with a measure of the effectiveness of instruction. A high slope means that the intuitive idea can easily be changed by instruction. A small slope means that an idea resists tutorial correction. In this way we can bring preconceptions to light, i.e. those intuitive ideas which resist change.

So, by cutting across a range of levels of instruction, we have obtained a fruitful methodological instrument to identify intuitive ideas and preconceptions of pupils.

c) Following the method indicated above we have identified 7 test items referring to serious preconceptions, in particular: the 'impetus' idea, the idea of relating movements to fixed frames of reference, and the problem of dealing with a force of an induced character. The other 15 items refer to conceptions which are more corrigible by instruction. For a cross-country comparison of test results those subgroups of the various test population have been examined which are comparable in terms of instructional background and of motivational attitude towards physics.

The 7 items which we traced in the Zimbabwe study as referring to strong preconceptions, turn out to be also most difficult in the other countries as well as in the literature. However, the difficulties noted in Zimbabwe and also in Lesotho, are much more serious than those noted in The Netherlands and in the literature (concerning western groups of pupils). On the other hand, for the other 15 items across-country comparison of mean scores indicates a reasonably great similarity in testperformances, also when compared with the literature.

Now there are two possible explanations for the cross-country differences in the persistance of preconceptions. Either: the teaching strategies in the various countries are different in the way they emphasize conceptual thinking. Or: the sources and mechanisms which generate preconceptions outside the school environment are different in character and/or degree.

In order to shed light on these most difficult questions, information should be obtained on the class room situation, the teaching strategy and the teaching materials. Also, the problems met by the pupils should be probed more in depth by interviews. And, finally,data on the level of cultural anthropology should be collected, in order to see whether preconceptions have some connections with more general pictures and images of reality which are prevalent in a culture.

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In Class Methods for Getting at Student Misconceptions

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In recent years, there have been a number of important efforts made to get at student misconceptions in science and mathematics[1]. The research has indicated that students bring with them models of the universe that, because they are so deep-seated and so necessary to their feeling comfortable with their world view, remain entrenced even after new information and new conceptual models are offered.

My work builds on these observations but extends the notion of "misconceptions" to more mundane confusions that emerge in the classroom setting and in the reading of science and mathematics textbooks. Moreover, in addition to contributing to the growing body of recognizable "error patterns", I seek to empower students to find out for themselves what is making the subject difficult for them to grasp -- personally and immediately; also how they can take control, i.e., make the subject more accessible. In this brief resume of my efforts, I shall detail four techniques that are currently being tested for their utility in ferreting out difficulties in learning.

1. The Comfort/Non-comfort Zones

My work in "mathematics anxiety" initially began in an effort to identify the reasons that "otherwise intelligent and school-successful students" had a task-specific disability in mathematics.[2] Hence, from the outset, we were dealing with students who had at least one area -- a "comfort zone" -- in which they studied efficiently and well. It was our task to dissuade them of the prevailing view that they were "dumb in math" (or science) congenitally and to get them to think of their problem as partially self-imposed. In various settings of "math anxiety reduction" interventions, students were instructed in how to monitor themselves while working at mathematics in contrast to how they approached and dealt with problems in their "comfort zone." Math, we hypothesized, was for them a "non-comfort zone," and needed to be treated as such.

The technique involved nothing more complicated than keeping a journal of the time, the circumstances and the internalized emotions they experienced in doing mathematics in contrast to, say, writing a paper (which most of these students did very well and with pleasure). Eventually, they would return to the math-anxiety discussion group with details as to how they were dis-serving themselves in handling mathematics (and science); and in time, comparing their strategies in the one area to their lack of strategies in the other, they could devise for themselves new ways of working. This entailed noticing, for example, that a mathematics or science text needs to be approached differently, and in my new book, Suceed with Math,[3] I devote an entire chapter to the subject of "Reading Math." The process also revealed that students who have particular difficulties in a certain discipline will set higher standards for themselves than elsewhere. Getting a pretty decent "first draft," in three hours, for example, would give them a feeling of satisfaction. "Making a little headway" in math in the same amount of time, would not. Reviewing, revising, learning to be critical of their own thinking and work was part of the process. We consider it to have been a success.

2. The Divided-Page Exercise

Another device, more focused on the math or science problem at hand, we called "the divided- page exercise" and by means of this attempted to get students to "tune into their own negative feelings, negative self-statements, and confusions" and tried to make them independent of their teachers and more precise in posing questions when they did need help in class. The left side of the page was reserved for "thinking out loud," including (never excluding), what might seem "irrelevant" to the teacher. The right side of the page was reserved for straight forward laying out of problems, sketches, and calculations. While the initial "divided-pages" were filled with emotional detritus, in time, students began to focus, rather, on posing and answering two questions of themselves: 1) What is making this problem difficult for me? 2) What could I do to make it easier for myself? We believe that this learned self-monitoring contributed substantially to their learning to recognize their own error-patterns and to correct these.

3. Peer Perspectives

In time, I came to realize, being myself an outsider to the field of mathematics and science, that some of the perspectives of an adult-observer could be of use in ferreting out student misconceptions. In turning to the physical sciences I set out to answer the question: What Makes Science Hard?[4] And to operationalize that question, selected groups of highly educated, very confident newcomers to science, namely professors in the humanities, the arts, social science and even law, to attend artificially designed learning experiences in physics, mathematics and, soon to come, bio- and physical chemistry. Again, the divided-page format was used to encourage these "Peers", as I called them (since in every way except newness to the fields, they were peers of their instructors, to note as they attempted to learn what was being presented, what -- in as much detail as was possible -- was getting in their way. From these reports, detailed elsewhere, came observations having to do with use of language, ambiguities in demonstrations, relations between abstract and concrete examples, the uses of mathematics in science, and so on. I am now attempting to use Peers at lower levels of education (graduate and advanced undergraduate students) and to extend the experiment to more subjects. I also believe that Peers can be of some use in critiquing existing textbooks since texts for those who are not majoring in science very often represent a barrier to understanding rather than illumination.

4. The "Muddiest Issue" Technique

Another approach, using students themselves, has been attempted at Harvard University by Prof. Fred Mosteller, in his introductory statistics course. About once a week, Mosteller asks his students to describe -- anonymously -- the "muddiest issue" covered in that week of lessons. From these "muddy issues," he selects those frequently mentioned and prepares specific handouts to deal precisely with the problems raised by students. In addition to getting invaluable feedback as to how students are faring in the course (between examinations), Mosteller's assignment gets them to think constructively about their own misconceptions. A collection of his "muddiest issues" and those of instructors in other subjects would, I believe, provide a very valuable addition to any instructor's resource library. For, while no two students are exactly alike, "muddiness" most likely resides not entirely in the learner but in the subject matter itself.

None of these experiments is systematic. Nor are the designs sufficiently controlled so that we who do them can bring any more than anecdotal evidence to bear on the subjects we observe. However, they have a valuable dual function (to summarize): They provide classroom-based specific feedback. They give students the opportunity and the experience in identifying and then trying to unravel their own misconceptions.

I offer them to this conference for your consideration.

1. A good bibliography of research into misconceptions is appended to an article, "Inquiry in the Mathematics Classroom" by Jeremy Kilpatrich, published in Academic Connections, by the Office of Academic Affairs, The College Board, 45 Columbus Avenue, N.Y., 10023-6917.

2. See Overcoming Math Anxiety. New York: W.W. Norton, 1978; in paperback, Boston: Houghton Mifflin, 1980.

3. Succeed with Math will be published by The College Board in September, 1987.

4. See "Math Anxiety and Physics: Some Thoughts on Learning 'Difficult' Subject," in Physics Today, Vol. 38, No. 6, pp. 61-66. Also "Peer Perspectives on the Teaching of Science," in Change Magazine, March-April, 1986, pp. 36-41. Also, "Colleges Ask: What Makes Science Hard?", N.Y. Times, May 13, 1986, p. 17. Also, "The Frustrations of a College Physics Class: Some Professors Get to Experience It, Again," in The Chronicle of Higher Education, October 15, 1986, p. 18.

HIERARCHICAL ORGANIZATION OF KNOWLEDGE AND COHERENT EXPLANATION IN THE DOMAIN OF ELEMENTARY MECHANICS

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I. Introduction

This paper reports on a study of the ability of novices to produce written explanations of physical situations. The study was part of a multi-faceted project conducted to explore differences between expert and novice problem-solvers in physics, and to test whether a particular computer-based regimen can help nurture more expert-like behavior in novices. The major features and results of the broader project are described in a companion paper to this one (Dufresne et al., 1987, in these Proceedings). Included are descriptions of the computer-based problem analysis environment being tested (the Hierarchical Analysis Tool, referred to below as HAT) and a control analysis environment (the Equation Sorting Tool, or EST). Related results are reported in a second companion paper (Hardiman et al., 1987, in these Proceedings). Here we report the outcomes and implications of a written explanation task administered to novices before and after an extensive period of interaction with the problem analysis environments. Some preliminary results of this work have been reported previously (Touger et al., 1986, 1987).

A. Knowledge Structures Underlying Problem Solving and Explanation

Before considering how written explanations might change, it would be helpful to consider those characteristics widely thought to distinguish expert from novice physics problem-solvers:

1) Structure of memory: Experts store physics knowledge hierarchically, with fundamental concepts at the top of the hierarchy and specific equations and facts at the bottom. Novices, in contrast, tend to store knowledge homogeneously (see, for example, Reif and Heller, 1982). Some researchers (Ferguson-Hessler and de Jong, 1987) argue

that, at a stage where mastery of abstract concepts at the top of a hierarchy might still be limited, organization of knowledge by more concrete problem types or problem schemata may characterize successful novice problem solvers.

2) Relationships among concepts: The expert knowledge store tends to be dense and rich in interrelations, that of the novice sparse and poor in interrelations (Chi and Glaser, 1981).

3) Categorization: Experts tend to categorize problems by fundamental concepts, whereas novices tend to categorize problems by surface features (Chi et al., 1981).

4) Solution strategies: Experts tend to solve problems by strategies that begin with principles at the top of the hierarchical knowledge structure and proceed through levels of increasing specificity in order to select the equations needed for solution. Novices, in contrast, typically hunt for equations by a "house of mirrors" approach.

5) Procedural knowledge: The factual knowledge of experts is rendered useful by extensive procedural knowledge which novices tend to lack. Hestenes (1987) views "mastery of various modes of external knowledge representations" (e.g. equations, diagrams, graphs), and presumably the development of interrelations among them, as "essential to the development of physical intuition." Other recent work (Larkin and Simon, 1987; McDermott et al., 1987) has also stressed the importance of establishing a clear connectedness for particular representations.

6) Explanations: Experts tend to give coherent explanations of physical situations, whereas novices, as a rule, are notoriously poor explainers.

The HAT is an environment intended to bring to bear on a given problem the hierarchically structured approach typical of the expert. Using the HAT, the subject attempts to delimit or categorize the problem to be solved by selecting the governing concept(s) and specific conditions that apply from a hierarchically ordered sequence of menus. A suitable sequence of menus and choices for a problem involving conservation of energy is shown in Figure 1. The EST, in contrast, simulates the hunt-and-peck approach presumed typical of novices. Presented with a problem, the EST user selects variable names, physics terms, or problem types, is then presented with a list of equations associated with the selection made, and must in turn decide which equations are applicable for solution.

While the bulk of recent research on expert-novice differences in the domain of physics has focused on the solution of numerical problems, the ability to explain physical situations is clearly an important aspect of expertise. This suggests it might be profitable to look more closely at novices' written explanations of physical situations and how they might change if the novice's knowledge-store becomes more structured as a consequence of using the HAT.

Specifically, it seems reasonable to expect that the physics expert draws on the same hierarchical organization of knowledge elements in formulating strategies both for generating coherent explanations and for correct and efficient problem-solving. This view is supported by a formal analysis of learning strategies proposed by Brown et al., (1978), which indicated that a kind of means-end analysis is common to both problem solving and understanding stories or narrative. The hierarchical structure used by the expert problem-solver, in this picture, would be a particularly efficient way of proceeding from governing concepts down through intermediate goals to a desired end, namely, a correct solution. What is common to problem-solving and understanding narrative should also be an aspect of strategies for constructing or organizing narrative which in the domain of physics typically means explanation.

In less formal work consistent with this view, Kirkpatrick and Pittendrigh (1984) taught undergraduates a general hierarchical structure for physics explanations in order to improve the explanations that they wrote, and Grumbacher (1985) reported that high school students who write regularly about physics also perform better by more usual measures. Viewed in this context, the HAT study appeared to afford a unique opportunity to consider the active use of a well-defined hierarchical organization of a specific domain (classical mechanics) by novice subjects, and to explore its relationship to or impact on the explanatory ability of these subjects in the same domain.

B. Categories of Explanation

A further objective of the explanation task was to categorize novice physics explanations. The experience of a pilot study led us to believe that the explanations written by student subjects would fall into four broad categories, with some hybridization among them:

1) Formula-driven: Explanations of this type typically begin with an array of equations that apparently come to mind in response to a given question. The subsequent narrative may or may not make use of these equations.

2) Qualitative/Intuitive: These explanations discuss the given set-up in real world terms. Phrases like "impact", "slowing down", and "not going as far" predominate over more formal physics language.

3) Qualitative/Physics-ese: Explanations of this type use formal physics terms in ways that suggest the user has not fully mastered the concepts represented by the terms. At worst, the terms may be embedded in inappropriate locutions (e.g. "hits with a force") or may be inappropriate to the given situation.

4) Hierarchical: These explanations clearly identify relevant governing concepts and use them in relatively structured ways, such as identifying initial and final states when conservation of energy is involved. At best, they involve integration of mathematical and visual representations with related narrative.

We expected to find a high frequency of formula-driven explanations. This assumption about how novices deal with explanations is related to the assumption, underlying the design of the EST, that novices usually solve problems by a kind of hunt-and-peck approach that is surface-feature and formula-driven. In contrast, one might expect that after using the HAT, subjects might produce a greater number of hierarchical explanations.

C. Context Dependence

Explanation tasks can also be of interest insofar as they afford the subject freedom in the choice of variables to be considered and concepts to be applied. Whether or not a subject invokes a particular variable or concept in various contexts provides insight into the degree of connectedness or generalizability that the concept or term may have in the subject's knowledge-store. In a broad sense, the range of applicability of a term is an essential part of its meaning to the individual. The work of Rosch (1977) on categorization provided cross-cultural evidence that common semantic noun categories (such as "bird" or "furniture") are organized around perceptually salient prototypes for the categories, and that membership in a category is determined by a kind of semantic distance from that prototype in terms of co-occurrence of attributes. (For instance, if the prototype bird were robin-like, a kiwi or a penguin might be a weaker candidate for inclusion in bird-dom than a sparrow because it has fewer salient features in common with a robin.) We suggest that for the more abstract noun categories, i.e. concepts such as "force" and "energy", which form the working vocabulary of physics, the contexts in which a concept is applicable are important attributes and are part and parcel of its semantic content. In the explanation task, we expected that novices would apply concepts in a more restricted range of contexts than would experts, making those concepts less rich in meaning for the novice and thus less appropriate for superordinate placement in a hierarchical structure.

D. Misconceptions

Much has been written elsewhere about students' misconceptions in the domain of mechanics (see, for example, such bibliographies as Maloney, 1985, Pfundt and Duit, 1985, and Dykstra and Schroeder, 1987). The explanation task provides a further opportunity to seek evidence of misconceptions, and also to see what effect those misconceptions may have on whether students consider particular variables or invoke particular concepts.

II. Method

A. Subjects

Participating subjects were University of Massachusetts undergraduates who had taken either of two first semester physics courses for majors or for engineers (both mainly mechanics) in the previous semester and received a grade of B or above. A total of 42 participants were divided into three equal groups: HAT users, the control group of EST users, and a second control group of textbook users. All participants did both the pre and post explanation tasks. Ten of the subjects were selected to be interviewed about their explanations after the completion of all testing. Those interviewed were chosen because their explanations exhibited features that were recurrent in the total population of subjects. (An additional 3 HAT users and 4 EST users completed a pilot study the previous summer. In the pilot study, all subjects were interviewed about each explanation immediately upon its completion, but the larger size of the subject population for the main study prohibited more comprehensive interviewing. Data from the pilot study will be included only when correlations with effects of the problem analysis environments are not at question, since exposure to those environments was less extensive in the pilot study.)

B. Procedures

Subjects were recruited from the previous semester's physics classes on the basis of grade, and were paid $50 for ten hours of participation over roughly a five week period. Five hours of this time was spent solving problems using one of the problem analysis environments (HAT, EST, text). The same 25 problems were done by all subjects. Explanation tasks (see Section C, below) were administered to all subjects before and after this problem-solving phase. Pre and post problem-solving exams and categorization tasks (see Dufresne et al.,in these Proceedings) were also administered to all subjects. Subjects agreeing to a final interview were paid $5 for an additional half hour of their time.

C. The Explanation Task

The explanation task given to the novice subjects involved three questions requiring explanations of physical situations. Each question presented the subject with a physical set-up and asked the subject to explain what happens when a particular change is made in the set-up. The questions are shown in Figure 2. (The governing principles, not presented to the subjects, are indicated in the figure for the convenience of the reader.) In each question, the subject must decide which variable or variables to address, and what concepts to apply. The general instructions directed the subjects to write out explicitly the reasoning by which they reached their conclusions. Since subjects were not directed to solve for a particular variable, the inclusion and use of equations in the explanations was entirely at the subject's own discretion. The subject was allowed up to a half hour to complete three such written explanations, and the time for each response was recorded.

As Figure 2 shows, two sets of questions were administered. Each subject responded to one set of questions before using the problem analysis environment and the other one after. Half of the subjects in each of the three groups did Set A first, while the others did Set B first. Two questions in Set A were paired with related questions in Set B. In each pair,the same set-up was considered in both questions, but the variable affected by the change differed.

D. Analysis of Data

To avoid bias, all analysis of the written explanations was done before consulting project records to see which subject had used which problem analysis environment. For each written explanation, a detailed evaluation form (Figure 3) was completed. The evaluation criteria addressed in the form became the basis of much of the subsequent data analysis.

Prior to considering these forms in detail, an attempt was made, based partly on the information in the forms and partly on a prima facie reading of the explanations, to assess the degree of pre- to post-task improvement of each subject on a coarse-grained 0 (none) to 3 (a great deal) scale along each of several dimensions. These included use of higher order concepts, overall organization, use of equations, use of physics vocabulary, presence of misconceptions, and ratio of volume written to time spent on task.

A more refined measure was developed for the level of use of one particular higher order concept, energy, which was addressed frequently enough by subjects to afford an adequate basis for statistical analysis. The measure (hereafter designated as Level of Concept Use, or LCU) consisted essentially of the sum of the numerical entries in the Energy column of Figure 3, modified by the elimination of the first two items in the column and the inclusion, where relevant, of questions about an intermediate state parallel to those about initial and final states.

A group-by-time analysis of variance was carried out to see whether the pre- to post-test change in LCU differed significantly among the three groups of subjects. In addition, a correlation analysis was performed to see whether there was any significant correlation between pre-to-post changes in the LCU and changes in subjects' scores on problem-solving exams administered before and after use of the problem analysis environments. Also, subjects were divided into quadrants by amount of pre-to-post change in LCU, and the average pre-to-post change on the problem-solving exam was compared for the different quadrants.

Other facets of the explanation task were considered as follows. Using items in the evaluation form as a guide, an attempt was made to classify each written explanation according to the classification scheme proposed in the introduction (formula-driven, qualitative/intuitive, qualitative/physics-ese, or hierarchical). In addition, compilations were made of the frequencies with which different variables were addressed and different concepts applied in response to each question. Finally, evidence of misconceptions was also compiled.

III. Results and Discussion

The results of the explanation task fall into four broad areas, which are addressed below in the same sequence as in the introduction, i.e. (A) relationships between hierarchically structured problem solving and structured explanation, (B) categories of explanation, (C) context dependence of variables considered and concepts applied, and (D) evidence of misconceptions and their effect on the aforesaid context dependence.

A. Hierarchical Organization and Explanation

We first consider the matter of whether extensive active involvement with a hierarchical structure (the HAT) of a domain has any effect on the novice's explanations in that domain. Our prima facie examination of whether subjects' explanations showed pre-to-post improvement in each of several dimensions indicated that differences in improvement among subject groups undergoing the different treatments (HAT, EST, and text) were greatest for the dimension of "use of higher order concepts." Of 14 students in each group, 9 HAT users, 6 text users, and 3 EST users showed some improvement, but the differences were not statistically significant and the criteria admittedly vague.

This led us to focus on energy as the higher order concept most widely applicable and most widely applied to the questions in the explanation task, and to develop the LCU (see Section II: Methods) for that concept. We then applied it to those questions (A1, A3, B2, B3) for which significant numbers of students considered energy.

The data presented in Figure 4 show that overall, the HAT users show the best pre-to-post change in LCU, the EST users the worst. A group-by-time analysis of variance showed that the differences in a three-way comparison are significant at the p = .05 level. Differences in a two-way comparison between HA and EST users are significant at the P = .035 level. In other words, the explanations of HAT users improved in their structured use of energy considerations to a significantly greater degree than did the explanations of the other two groups. The EST users, in fact, showed a general tendency to decline. Textbook users, who were at liberty to use the textbook in either a structured or a house-of-mirrors fashion, not surprisingly fell in between.

Some further comments on these data are necessary. Because subjects could approach A3 fully and A1 and B2 partially correctly without considering energy, it was possible for a subject with some grasp of energy ideas to apply energy to a pre-test question and not to the paired post-test question, thus showing a negative change in LCU, whereas a subject who did not consider energy before or after would show zero change. This accounts in part for the negative values in the data of Figure 4. This effect is greater than one would have hoped because A3 and B3 were not intended as parallel questions, and because the deep-structure parallel between A1 and B2 was broken for some students by surface-feature-dependent considerations (discussed in Section IIIC below). One might expect that with test questions more carefully paired to account for those considerations, an even greater level of statistical significance could have been achieved.

We suspect that these considerations are also the major reason why we found no significant overall correlation between improvement in the LCU for energy and pre-to-post improvement on the problem-solving exam. Nevertheless, as the data of Figure 5 show, students in the quadrant showing the greatest increase in LCU also showed much greater than average improvement on the problem-solving exam. This suggests there may be some connection between improved problem-solving ability and more extensive and structured use of at least one far-reaching higher order concept, energy.

B. Categories of Explanation

In attempting to categorize subjects' written explanations according to our proposed classification scheme, we found that many explanations could not be readily classified, and that other explanations fell into more than one category. Nevertheless, most explanations could be categorized by this scheme, and some patterns are evident in the frequency data (Figure 6). Of 753 explanations, 31% were formula-driven to a significant degree. Exit interviews indicated that this figure would have been higher but for subjects' inability to recall formulas. Only quantitative/intuitive explanations were as frequent. This was consistent with our expectations, and corroborates the assumption inherent in the design of the EST, that novices tend to be formula-driven in their approach to problem solving.

Subjects were most hierarchical in their explanations on questions (A1,B3) where they were most readily able to apply energy considerations. They were least hierarchical and most formula-driven on those questions (A2, B1) that they tended to treat kinematically.

Frequently, subjects wrote down equations and then wrote narrative that did not refer back to the equations. Follow-up interviews indicated that the equations represented a kind of initial memory search. In some instances the narrative was loosely motivated by the equations, while in other cases, the subject simply could not apply the equations to the question asked. Diagrams were sometimes treated in a similar manner: force diagrams or sketches of trajectories were set down at the beginning and then not integrated into the subsequent narrative. This seems to be evidence of the lack of interconnectedness of the novice knowledge store, and in particular of the lack of interconnectedness among different representations.

Subjects who failed to integrate equations with narrative were sometimes uncomfortable with the question format because it did not ask for specific variables. One subject who began by "sort of doodling, just trying to remmber different formulas... that had variables that I was looking for" was puzzled by the fact that no values were provided for the variables so that she could plug into an equation to generate an answer, as had been the accustomed situation in her course exams. At one point she comments, "You have to think about the time also, and those are things that weren't given... I knew what affected it but they weren't given so I didn't know how to correlate what was given. On that one [a projectile motion situation] I thought about the velocity but they didn't say anything about time... and the equation there [she had written $s = v_o t + 1/2 at^2$] had a variable t in it."

For this student, as for others, the way to deal with a physics problem was to find suitable equations to connect the value of an unknown variable with given values of other variables. One could then work wholly within the mathematical representation, without thinking of it as a model for or connecting it wholly with the actual physical situation. What we see here is an attempt to carry over this approach from problem-solving to explanation, clearly without success.

C.Context Dependence

As noted in the introduction, the assumption that novices' knowledge stores are less dense and interconnected, and their noun categories (in Rosch's sense) less inclusive than those of physicists, led us to expect that novice subjects would consider variables and apply concepts in a range of contexts that would seem restricted to a physicist. The open-ended aspect of the explanation task questions did indeed elicit considerable evidence of such context dependence. Several of the questions asked how changing a feature of the set-up "affected the motion of an object," leaving the respondent to select the specific variables to address. As figure 7 indicates, the choice of variables was clearly situation-dependent.

For the first situation depicted in Figure 7, there was a greater tendency on the pre-task to focus on the velocity of the block in the rough region than on the distance it travels (the post-task results are addressed below). In contrast, on both pre- and post-tasks, subjects showed a much greater tendency to focus on the distance traveled by a projectile fired from a cliff than on its velocity at any point, such as where it strikes the ground. This tendency was reinforced, in the case where the cannon's angle of elevation was varied, by the fact that many subjects recalled the well-known problem of finding the angle that maximizes the range, but the contrast persists when height rather than angle is the variable. Despite the fact that both set-ups can be addressed by energy considerations which interrelate distance and velocity, subjects tended to focus on one variable or the other, depending on the situation, and as the data show, only very infrequently did they address both.

This focus on a particular variable can be changed by exposure. One of the problems assigned for solution using the problem analysis environments (main study only) specifically asked subjects to calculate the distance a block travels over a rough surface before stopping. This accounts for the striking change in variable focus from pre- to post-test (main study) on the first situation in Figure 7.

It is also interesting to note the high percentage of students who focused on the "impact" (a word several of them in fact used) and thus addressed velocity in the last of the situations shown in Figure 7. From an energy point of view this is an intermediate state, but as the interviews corroborated, for most novice subjects the "impact" was the most salient feature. It seems clear from these instances that subjects' variable choices are dependent on the surface features of the problem, or perhaps from the Ferguson-Hessler and deJong point of view, on the problem type, although it is not clear to what extent, if at all, these are distinct or separable dependences. In any event, these observations seem to indicate that the assumption that novice approaches are surface-feature driven, made with regard to problem-solving in the design of the EST, seems applicable to explanation as well. This fits with the inference from Brown et al., that explanation and problem-solving performance will draw on the same knowledge structure.

What concepts subjects apply in their explanations was also found to be situation-dependent. Consider the last situation depicted in Figure 7. On paired pre- and post-task questions, subjects were asked in one case to explain how the block's motion is changed if h is changed to .5h, and in the other case if the set-up is switched from the earth to the moon. From an energy point of view, the two questions are identical except that in the gravitational potential energy mgh, h is being varied in one case and g in the other. However, of 20 subjects who used energy concepts in explaining the h-dependent situation on the pre-test, fully 30% (6 subjects) abandoned energy concepts completely in explaining the g-dependent situation on the post-test, preferring instead to focus on considerations related to Newton's Second Law. A few others invoked energy concepts on the post-test only after treating the situation dynamically, and the energy treatment in these cases tended to be less full than on the pre-test. There were recurrent indications in subjects' written and oral comments that a specific reference to h prompted thinking about potential energy. A reference to g seemed rather to prompt thinking about acceleration.

Even students' use of language was situation-dependent. Many subjects talked explicitly about "acceleration" in reference to the gravitational situation. But of the subjects who considered as a variable the rate at which the block slows down on the rough surface, only 3 out of 15 (2 of 8 pre, 1 of 7 post) used the word "acceleration" (or even "deceleration"). The rest chose to use language like "slows down faster" despite the fact that many of them were perfectly capable of using the word "acceleration" correctly in other contexts.

Rosch's ideas about categorization appear relevant here. Gravitation seems closer to a prototype acceleration situation (at least for descending objects) than does slowing down on a rough surface (and very likely, closer than any negative acceleration). Situations in which h varies seem closer to prototype situations for consideration of gravitational potential energy than do situations in which g varies. Thinking of concepts like "energy" or "acceleration" as categories, it would seem that students will most readily come to grasp the full extent of a category by repeated exposure to sufficiently diverse instances that press the bounds of the category. Initial or early exposure to the gravitational potential energy mgh should involve instances where each of m, g, and h is varied. That exposure can affect a student's view of when a concept is relevant is evidenced by the pre-to-post-task change we saw in students' consideration of distance traveled in the first situation of Figure 7.

D. Misconceptions

Several recurrent misconceptions held by subjects about mechanics were disclosed by the explanation task. Those discussed below are possible addenda to the ever-growing catalog of student misconceptions documented in the literature. We will also note how each misconception bears on subjects' choices of what variables to consider or concepts to apply.

1) For the projectile question in which the firing angle is changed, many subjects, either in their written answers or the closing interviews, expressed the belief that for greater angles the cannonball strikes the ground with greater velocity. Several subjects reasoned that because the ball rises higher, it acquires more potential energy, and thus would have greater kinetic energy upon striking the ground (neglecting the fact that when the angle is increased, the horizontal component of velocity and its contribution to the kinetic energy are reduced). Others reasoned that since the ball rose higher, it would have more time to accelerate downward and would achieve a greater speed. In both lines of reasoning the horizontal situation is neglected in the end, even where component reasoning is used to obtain the initial vertical velocity component. Thus we cannot assume the invariability of impact velocity with angle to be the reason why very few subjects addressed impact velocity for this question. Subjects' variable choices for the parallel case in which h is changed also belie this. Rather, we seem safer in assuming that students do not focus on the impact velocity because they do not see the collision with the ground as a salient feature of the given situation.

2) The existence of another rather surprising misconception, this one involving the block sliding onto a rough surface, was verified by following up in the closing interviews on such ambiguous written statements as "the block slows down in the rough region." Although this could mean that the block slows down monotonically to zero velocity, as a physicist reader would likely assume, it could also mean that it drops stepwise from a greater constant velocity to a smaller constant velocity. Rather surprisingly, two interviewees asserted unambiguously that the latter was the case, and that the block did "not necessarily" stop. Recall that this was after subjects had been exposed to a problem specifically asking for the distance traveled in such a set-up. Clearly, subjects harboring such a misconception would fall into the group addressing velocity but not distance. Also, this occurrence serves as a strong reminder that language used in exchanges between novices and experts does not necessarily have the same meaning for both parties.

3) The concept of "impact", also going by the names of "force" and "momentum", but without the physicist's meaning, was noted previously. In several cases it was clear that this was an instantaneous concept, as distinct from the physicist's concept of impulse over an interval.

Interestingly, in questions involving a block sliding down a ramp onto a spring, this impact was frequently a central focus, but as we have noted, in questions involving a projectile fired from a cliff the impact of the projectile on the ground is largely ignored. The surface aspects of the situations predominate over the fact that both are situations involving an object losing gravitational potential energy and gaining kinetic energy.

Numerical frequencies are not presented for these misconceptions because often their existence was only verified unambiguously either by follow-up questions (in the pilot study) or in selective interviewing (in the main study).

IV. Summary

The administration of a written explanation task to novice physics problem solvers before and after use of a hierarchically structured computer-based problem solving environment has yielded a diversity of findings about novices' explanations of physics situations. These findings have been considered in the context of a theoretical overview in which (1) in comparison to experts, the novice knowledge-store for a domain is viewed as sparse, with a paucity of interconnections, including interconnections among different representations, (2) the novice knowledge store lacks the hierarchical structuring of the expert's, (3) novice noun categories in the domain have a smaller experiential base and are generalizable to fewer instances, and (4) an individual draws upon the same knowledge store and structure both in problem-solving and in explanation in the domain.

Consistent with this overview, we found:

1) that exposure to a hierarchical structure does lead novices to use at least one higher order concept, energy, in a significantly fuller and more organized fashion.

2) that while there was no overall correlation, those students showing greatest improvement on the explanation task also averaged well above the mean improvement on the problem-solving exam.

3) that student explanations frequently reflect the formula-driven and surface-feature-driven approaches which we (and the design of the EST) assumed characterizes novice problem solving. This assumption is based both on experience and a view of the novice knowledge-store as lacking in hierarchical structure. The tendency to call up equations and diagrams from memory and then not use them is consistent with a lack of interconnectedness among representations.

4) that context, shaped by the surface features of a situation, strongly affect what variables novices consider and what concepts they apply in their explanations when given the latitude to do so. This is consistent with the notion that novices' noun categories are less inclusive and cannot as readily assume superordinate positions in a hierarchy of concepts.

5) and that novices hold misconceptions which affect whether they will consider a variable or concept in a given context. Insofar as applicable contexts are attributes of a noun, and thus part of its meaning, the misconception may give the variable or concept a different or more restricted meaning than it would have for the expert.

We feel that consideration of novice explanations is a fruitful approach to the broader task of seeking understanding of the novice and expert states and the potential ways of generating transitions between them. In further studies, we intend to examine expert performance on similar tasks, and to compare novice responses to prepared explanations in the different categories that we identified.

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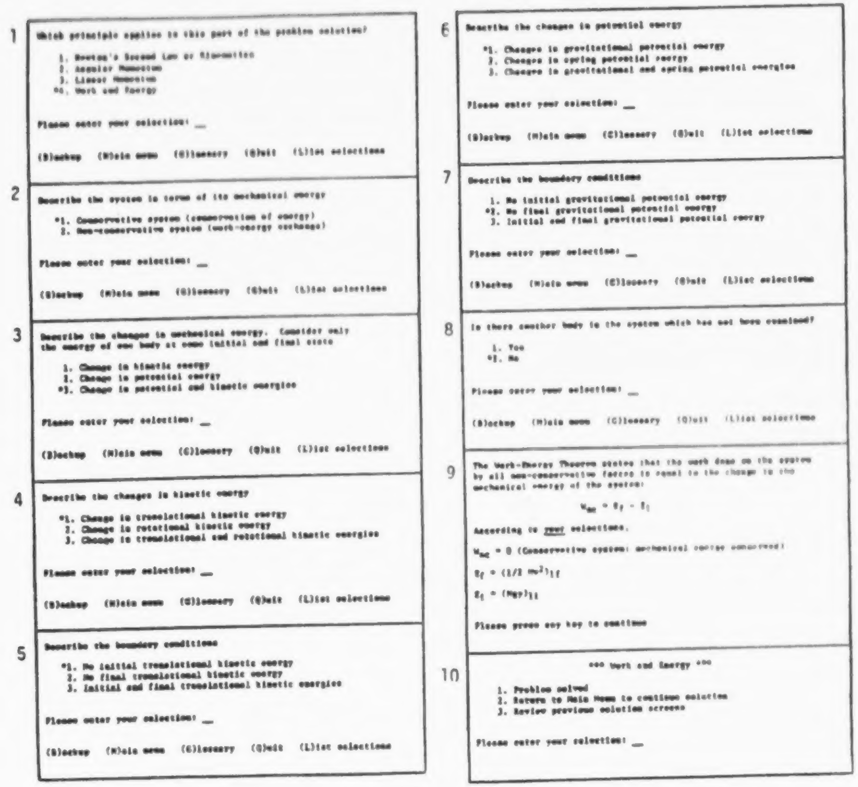
FIGURE 1



1

```
Which principle applies to this part of the problem solution?

     1. Newton's Second Law or Kinematics
     2. Angular Momentum
     3. Linear Momentum
    *4. Work and Energy

Please enter your selection: _

(B)ackup   (M)ain menu   (G)lossary   (Q)uit   (L)ist selections
```

2

```
Describe the system in terms of its mechanical energy

    *1. Conservative system (conservation of energy)
     2. Non-conservative system (work-energy exchange)

Please enter your selection: _

(B)ackup   (M)ain menu   (G)lossary   (Q)uit   (L)ist selections
```

3

```
Describe the changes in mechanical energy.  Consider only
the energy of one body at some initial and final state

     1. Change in kinetic energy
     2. Change in potential energy
    *3. Change in potential and kinetic energies

Please enter your selection: _

(B)ackup   (M)ain menu   (G)lossary   (Q)uit   (L)ist selections
```

4

```
Describe the changes in kinetic energy

    *1. Change in translational kinetic energy
     2. Change in rotational kinetic energy
     3. Change in translational and rotational kinetic energies

Please enter your selection: _

(B)ackup   (M)ain menu   (G)lossary   (Q)uit   (L)ist selections
```

5

```
Describe the boundary conditions

    *1. No initial translational kinetic energy
     2. No final translational kinetic energy
     3. Initial and final translational kinetic energies

Please enter your selection: _

(B)ackup   (M)ain menu   (G)lossary   (Q)uit   (L)ist selections
```

6

```
Describe the changes in potential energy

    *1. Changes in gravitational potential energy
     2. Changes in spring potential energy
     3. Changes in gravitational and spring potential energies

Please enter your selection: _

(B)ackup   (M)ain menu   (G)lossary   (Q)uit   (L)ist selections
```

7

```
Describe the boundary conditions

     1. No initial gravitational potential energy
    *2. No final gravitational potential energy
     3. Initial and final gravitational potential energy

Please enter your selection: _

(B)ackup   (M)ain menu   (G)lossary   (Q)uit   (L)ist selections
```

8

```
Is there another body in the system which has not been examined?

     1. Yes
    *2. No

Please enter your selection: _

(B)ackup   (M)ain menu   (G)lossary   (Q)uit   (L)ist selections
```

9

```
The Work-Energy Theorem states that the work done on the system
by all non-conservative forces is equal to the change in the
mechanical energy of the system:

                    W_nc = E_f - E_i

According to your selections,

W_nc = 0 (Conservative system: mechanical energy conserved)

E_f = (1/2 mv^2)_1f

E_i = (Mgy)_1i

Please press any key to continue
```

10

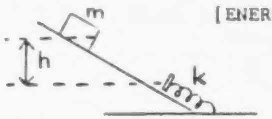
```
                    *** Work and Energy ***

     1. Problem solved
     2. Return to Main Menu to continue solution
     3. Review previous solution screens

Please enter your selection: _
```

SET A | SET B

- A block of mass m, initially at rest, slides down a frictionless ramp from a vertical height h onto a light spring of force constant k.



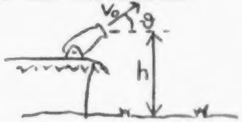
[ENERGY (and Dynamics in Part)]

Explain any changes in the behavior of this setup...

A1... when the block is released from a vertical height of .5h rather than h.

B2... if it takes place on on the moon rather than on earth.

- A cannon sitting on a cliff of height h fires a cannonball of mass m with a muzzle speed v_0 at an angle above the horizontal.



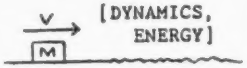
[KINEMATICS, ENERGY]

Explain as precisely as you can how increasing...

A2... the angle ϑ will affect the subsequent motion of the cannonball.

B1... the height from h to 2h will affect the subsequent motion of the ...ball.

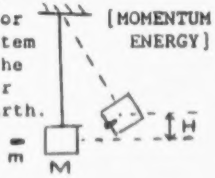
- A3 A block of mass M across a smooth floor at velocity v, then enters a rough region



[DYNAMICS, ENERGY]

Explain how increasing coefficient of friction affects the motion of the block in the rough region.

B3 A bullet of mass m strikes and embeds itself in a block of mass M hanging from a string. Explain any changes in the behavior of the system if... on the moon rather than on earth.



[MOMENTUM ENERGY]

FIGURE 2: THE EXAMINATION TASK QUESTIONS

FIGURE 3
GENERAL CRITERIA FOR EVALUATING EXPLANATIONS

What governing concept gets used? ________________

Is it explicitly identified by the subject? ☐Yes ☐No

If yes, is it ☐at the outset of the argument or ☐later on?

	Kin.	Dyn.	Energy	Mom.
Applicable? (yes=1, no=0)	☐	☐	☐	☐
Is it mentioned? (yes=1, no=0)	☐	☐	☐	☐
(by name=2, by eq'n.=1, no=0)	☐	☐	☐	☐
Is it used in reasoning? (great deal=3 to not at all=0)	☐	☐	☐	☐
Does reasoning justify conclusion? (totally=2 partially=1 no=0)	☐	☐	☐	☐
Is it addressed conceptually=2, fair mix=3, mathematically=1, not=0?	☐	☐	☐	☐
Is conservation(or work energy) cited as gov. princ.? (explicitly=2, implicitly or eq'n.=1 not at all=0)			☐	☐
Is init. state identified?(fully=2, partly=1, no=0)			☐	☐
(correctly=2, partly=1, incorrectly=0)			☐	☐
Is final state identified?(fully=2, partly=1, no=0)			☐	☐
(correctly=2, partly=1, incorrectly=0)			☐	☐
Is notion of conservative force mentioned? (Y=1,N=0)			☐	

Choice of variables:

How many variables get addressed? ________________

Which variables get addressed? ________________________________

Are the variables interrelated conceptually? ☐ fair mix? ☐ mathematically? ☐ not at all? ☐

Mathematics:	3 great deal	2 fair amt.	1 min. amt.	0 never
Are combinations of symbols/equations used__?	☐	☐	☐	☐
Is there actual mathematical manipulation?	☐	☐	☐	☐

Do one or more equations begin the explanation?
☐Begin
☐Nearly begin
☐No

What diagrams are used?
☐None ☐Vector ☐Trajectory ☐Other ____________________

	Always 3	2	1	Never 0
Is there description of "real-world action" as well as changing of variables?	☐	☐	☐	☐
Are variables represented by name?	☐	☐	☐	☐
Are variables represented by symbol?	☐	☐	☐	☐

Are any "pre-physics instruction" concepts (e.g. impact) used? Specify________________________________

Are any misconceptions in evidence? Specify.________________
____

Group	Question Sequence	Average Change (Post LCU-Pre LCU)
HAT	A1→B2 (n = 8)	-5.1
	B2→A1 (n = 7)	+8.6
	A3→B3 (n = 6)	+0.2
	B3→A3 (n = 5)	+2.2
EST	A1→B2 (n = 8)	-8.5
	B2→A1 (n = 6)	+6.2
	A3→B3 (n = 7)	-0.6
	B3→A3 (n = 5)	-11.2
TEXT	A1→B2 (n = 8)	-1.6
	B2→A1 (n = 7)	+1.2
	A3→B3 (n = 7)	+5.8
	B3→A3 (n = 7)	-6.4

Note 1: Max LCU is 25 for each of A1 and B2, 21 for each of A3 and B3.

Note 2: n's vary because explanations not completed were excluded from analysis

FIGURE 4
Average Change in Level of Concept Use (LCU) for Energy, Arranged by Group

	Number of Subjects In Quadrant	Pre-to-Post Change in LCU (Out of 25 for A1,B2. Out of 21 for A3,B3)	Average Pre-to-Post Score Increase on Problem-Solving Exam
Questions A1 and B2	5	$\Delta > 8$	21.4
	11	$0 < \Delta < 8$	13.0
	20	$-8 < \Delta < 0$	8.2
	7	$\Delta < -8$	8.2
Questions A3 and B3	6	$\Delta > 8$	16.33
	4	$0 < \Delta < 8$	6.00
	18	$-8 < \Delta < 0$	7.60
	9	$\Delta < -8$	10.55

FIGURE 5

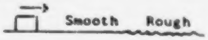
Score increase on problem solving exam, compared to change in LCU for two pre-post pairs of questions.

		A1	A2	A3	B1	B2	B3	Total
PRE-TEST	F	7	7	6	9	7	4	40
	QI	7	5	4	5	7	7	35
	QP	1	1	1	0	2	2	7
	H	6	0	5	0	5	7	23
POST-TEST	F	4	9	5	9	5	5	37
	QI	2	7	4	10	10	9	42
	QP	1	1	1	4	3	2	12
	H	10	1	6	2	5	7	31

FIGURE 6

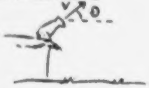
Frequency of occurrence of Categories of Explanation: A1 - B3 are the question numbers. Categories are formula-driven (F), Qualitative/Intuitive (QI), Quanitative/Physics-ese (QP), and Hierarchical (H). The diagonal arrows show that subjects who did set A on the pre-test did set B on the past, and vice-versa.

Question A3 (...if coefficient of friction increased?)



	Distance traveled		Velocity (or change in velocity)		Both distance and velocity	
	Pilot	Main	Pilot	Main	Pilot	Main
PRE	$^0/_4$	$^7/_{22}$	$^4/_4$	$^{14}/_{22}$	$^0/_4$	$^2/_{22}$
POST	$^1/_4$	14 or $^{15}/_{19}$	$^3/_4$	$^{13}/_{19}$	$^0/_4$	$^{10}/_{19}$

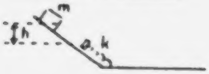
Questions A1 and B2



(A1... if θ is increased;
B2... if fired from 2h)

	(Horizontal) distance travelled		velocity	
	Pilot	Main	Pilot	Main
PRE (A1)	3/4	21/24	1/4	3/24
POST (A1)	-	18/19	-	3/19
PRE (B2)	3/4	14/21	2/4	5 or 6/21
POST (B2)	-	18/24	-	8/24

Questions A2 and B1



(A2... if .5h rather than h;
B1... on moon rather than on earth)

	(Impact*) velocity
	Main
PRE (A2)	10/19
POST (A2)	17/24
PRE (B1)	(7) 11/21
POST (B1)	(14) 17/24

* lower value in parentheses is number who unambiguously specify *impact* velocity

FIGURE 7

Frequency with which variables are addressed

Investigation of Student Difficulties with Graphical Representations in Physics[*]

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The Physics Education Group at the University of Washington has been investigating student difficulties with graphical representations.[1] This work is part of an ongoing investigation of student understanding in physics.[2] We have used the results of this research to guide the development of curriculum to address the specific conceptual and reasoning difficulties identified.[3] In this paper, we report on an investigation of the reasoning that underlies common errors in drawing and interpreting graphs of moving objects.

As physics instructors, we have been interested in how well students who have studied the relevant content in lectures, texts, and homework problems can apply their knowledge in drawing graphs of a motion they observe in the laboratory. In what ways does the approach of students who can draw qualitatively correct graphs differ from those who cannot? Can such differences be generalized in a way that distinguishes the behavior of an "expert" from that of a "novice"?

Previous studies of student difficulties with graphical representations have been conducted primarily with precollege students.[4] The students involved in this study were drawn from two groups. The first consisted of nine undergraduates who had almost completed the first quarter of calculus-based introductory physics. The course was taught by a faculty member not involved in the research. The second group consisted of ten junior high school physical science and high school physics teachers. All were enrolled as students in physics courses taught by members of our group as part of a summer program for in-service teachers.

Individual Demonstration Interviews

Our primary data source was the individual demonstration interview, in which an investigator presents a series of tasks based on demonstrations that the student observes. In each task used in this study, the student is asked to make a prediction, observe a motion demonstrated in the laboratory, and sketch an appropriate graph while thinking out loud. In addition to asking a series of structured questions, the investigator can expand the questioning to clarify a student's response or to pursue an interesting point that may arise during the interview. Each interview usually lasts about an hour, is recorded on audiotape, and then transcribed for detailed analysis.

The four tasks in this study are based on demonstrations in which a ball rolls along a combination of level and inclined tracks, as shown in Figures 1-4. Before the demonstration of each motion, the student is asked to describe the type of motion expected. The investigator explicitly defines x to be the position of the ball along the track and asks the student to sketch three graphs: position versus time (x vs t), velocity versus time (v vs t), and acceleration versus time (a vs t). No measurements are made with clocks or meter sticks. Instead of plotting data, the student must sketch the general shape of the graph strictly on the basis of observation. For each track, the student is given a response sheet with three sets of axes. The horizontal axis in each set is labeled t; the vertical axis is labeled x, v or a. The sheets also include scales on which the student indicates degree of confidence in each response, ranging from 0 (virtually impossible I'm right) to 10 (virtually certain I'm right). The correct graphs for each motion are shown in Figures 1-4.

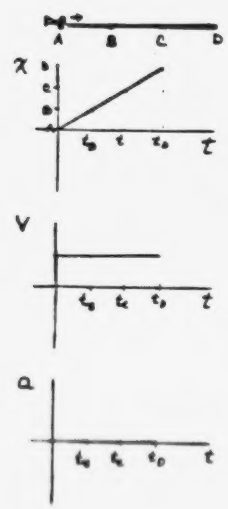


Figure 1: Interview task #1

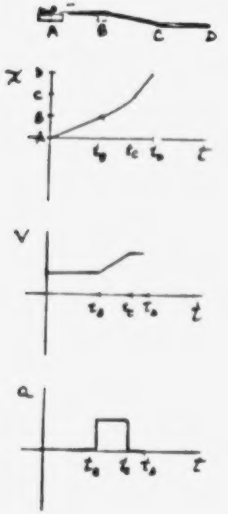


Figure 2: Interview task #2

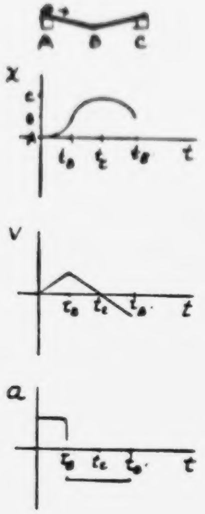


Figure 3: Interview task #3

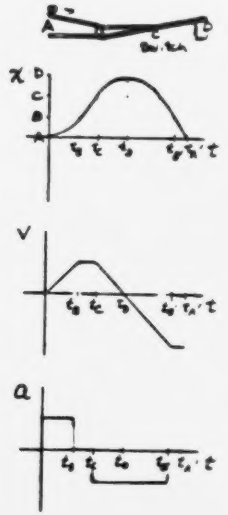


Figure 4: Interview task #4

Supplementary Data Sources

Supplementary information was obtained from several sources. To assess the frequency of specific graphing errors in various populations, we devised several written questions which were administered to ninth grade physical science students, high school physics students, calculus-level introductory physics students, and high school teachers. One of these questions was a written version of the second task used in the interviews. To gain further insight into the development of graphing skills, we monitored the progress of undergraduates in a special physics course in which graphing is emphasized in the study of kinematics.

Analysis of Data

All of the students interviewed were able to draw correct graphs for the first track, on which the ball moves with constant velocity. About 60% drew adequate graphs for the second track, on which the ball first moves with constant velocity, speeds up along an incline, and then moves with a higher constant velocity. About 40% drew acceptable graphs for the third track, on which the ball speeds up from rest, slows down, turns around, and speeds up in the opposite direction. All who succeeded on this task were able to complete the fourth task satisfactorily, in which the ball moves along additional level and inclined segments.

The students were divided into two groups on the basis of their performance on the interview tasks. Those who could draw appropriate graphs for all the tracks were classified as "experts." Those who made substantial errors on one or more graphs were classified as "novices." The third interview task, in which the ball turns around, proved to be the dividing line between "novice" and "expert." There were novices and experts among both the undergraduates and in-service teachers interviewed, with more experts in the latter group.

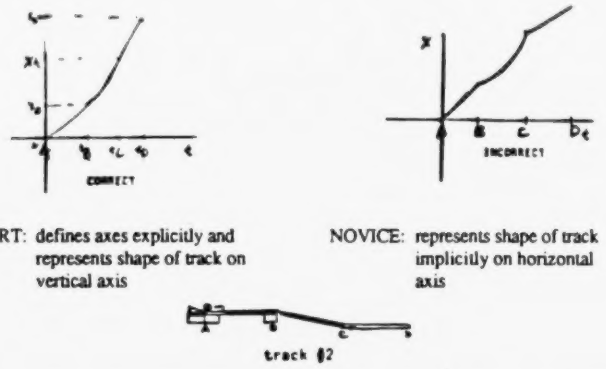
We were particularly interested in how experts and novices began drawing their graphs, how they represented the path of the motion, and how they tried to relate features of the graph to various aspects of the motion. By analyzing the transcripts from the interviews, we were able to identify several differences which are illustrated with specific examples below. In the discussion, we have also included an estimate of the prevalence of certain errors. This was obtained by administering a written version of the second task to 116 students enrolled in a calculus-based physics course.

Initial Procedures

Experts and novices differed in their initial approach to the tasks. Experts usually started by labeling the vertical axis with lettered points that corresponded to the lettered positions along the track. Novices typically began by drawing a line without explicit reference to either axis.

After observing the three equal lengths of track, AB, BC, CD, on the second task, experts typically would mark off three equal intervals along the vertical axis as shown in Figure 5. An expert who drew a correct x vs t graph said:

> We can put A at the origin of the graph. And put D at top so that the graph will be as large as it can get ... And then divide it into three equal intervals...so we have the vertical axis, A B, C and D. And that's the...position.



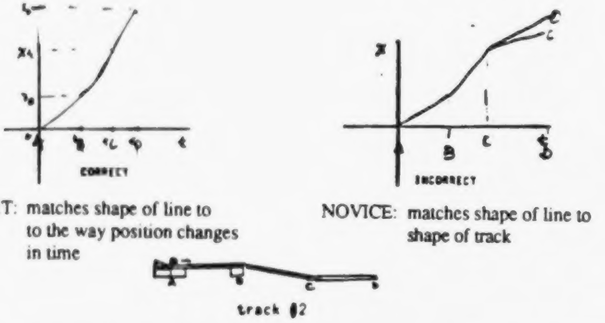
EXPERT: defines axes explicitly and represents shape of track on vertical axis

NOVICE: represents shape of track implicitly on horizontal axis

Figure 5: Expert and novice approaches to representing the shape of the track on the axes of an x vs t graph

The correct graph in Figure 5 also shows the difference in time intervals for the different segments of the motion. The ball speeds up along BC and takes a shorter time to travel BC than AB, and an even shorter time to cover CD, when it is moving with a constant but higher velocity.

Novices generally began by drawing a line that implicitly represented the shape of the track on the horizontal axis. For example, when asked to label his graph, the student who drew the incorrect graph in Figure 5 labeled the horizontal axis with approximately equal spacing between AB, BC, and CD, reflecting the equal lengths of track segments AB, BC, and CD.



EXPERT: matches shape of line to to the way position changes in time

NOVICE: matches shape of line to shape of track

Figure 6: Expert and novice approaches to sketching the shape of the line on an x vs t graph

Representation in Space or Time

Experts tried to match the shape of the line to the way the variable (x, v, or a) changed during the time interval of the motion. For example, the student who drew the correct x vs t graph in Figure 6 consciously considered the distance the ball moved in a unit of time along various segments of the track.

> From A to B...it's covering a distance in a certain amount of time...it's not changing, it would be the same over any small increment...and then along B and C if you divided the length from B to C in ten little segments, the distance it covered over the time would increase along the ten segments.

He correctly drew a line for BC curved up, indicating that the ball is moving an increasingly larger distance during each time interval. The steadily increasing slope on the correct graph for BC indicates the steadily increasing speed as the ball rolls down the inclined segment of track.

Unlike the experts who attempted to draw a representation in time, novices seemed to be representing the motion in space. Two frequent errors seemed to reflect attempts by novices to replicate the shape of the track on the line they drew for the x vs t graph. As shown in Figure 6, for example, the student drew a straight segment for the middle portion of the graph, BC, and thus mirrored the straight middle section of track, BC. On the written version of this task, almost 50% of the calculus-level introductory physics students drew a straight middle section on the graph, mimicking the straight middle section of the track.

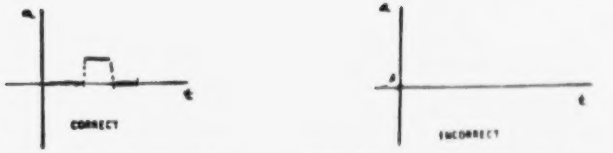
A second error occurred in the incorrect x vs t graph shown in Figure 6. In representing the way the position changes during the third segment of the motion, CD, the student attempted to draw the line CD with the same slope as AB. Not being satisfied with his first attempt, he corrected it so that the line lay more nearly parallel with his line for AB. In making the correction, he said:

> Ideally the slope of the line CD would be the same as the slope from A to B...because at segment C to D, it is a horizontal track...

On the written version of this task, about 40% of the calculus-level physics students drew a third segment on the x vs t graph parallel to the first segment, thus mimicking the structure of the track rather than the way the position of the ball was changing during the time interval of the motion.

Representation of Constant Values of the Variables

Failure to focus on representing the motion in time led to errors in graphing constant values. Students who focused on the constant value of a variable along a given segment of track, rather than on the constancy of this variable during an interval of time, sometimes used a single point, rather than a horizontal line, to represent the value of the variable along that segment of track. For example, the novice who placed a single point at the origin on the incorrect graph in Figure 7 intended the point to represent an acceleration of zero all along the AB segment of track.



EXPERT: represents a zero value during a time interval with a line on the horizontal axis

NOVICE: represents a zero value during a time interval with a point

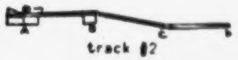


Figure 7: Expert and novice approaches to representing a constant acceleration of zero on an a vs t graph

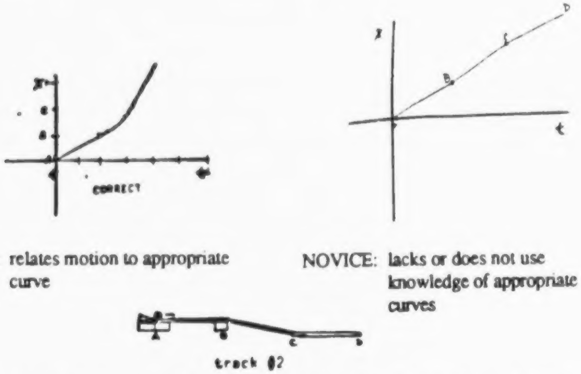
> The acceleration is zero so it starts at zero...from A to B can I represent it as a point?

Relating Motion to Shape of Graph

Both experts and novices drew the lines on their graphs by thinking about what was happening during each segment of the motion: AB, then BC, then CD. They differed, however, in the extent to which they could invoke knowledge relating various types of motion (uniform or accelerated) to the shapes of various types of motion graphs (x vs t, v vs t, a vs t).

Experts tried to match the type of motion they observed on a segment of the track with the appropriate curve for a particular motion graph. The experts seemed to have readily accessible a repertoire of knowledge that helped them connect the motion they were seeing on the track with the shape of the line they needed to draw on the graph. They seemed to know, for example, that a constantly increasing velocity is represented by a concave up curve (constantly increasing slope) on an x vs t graph, by a straight upward slanting line (constantly increasing height) on a v vs t, and a straight horizontal line (constant height) on an a vs t graph.

The quote below, from the student who drew the correct graph in Figure 8, also illustrates an awareness that the curve for one segment must connect appropriately with the curve for the next segment.



EXPERT: relates motion to appropriate curve

NOVICE: lacks or does not use knowledge of appropriate curves

Figure 8: Expert and novice use of knowledge relating motion on a segment of track to appropriate curve for graph

> The position will go up from A to B in a straight line...then it will not take the same amount of time...so it's going to be a parabolic curve (on BC)...and from C to D, it will continue to go up but it will be going with the same slope (on CD) that we have here (i.e., line CD is tangent to the curve at C).

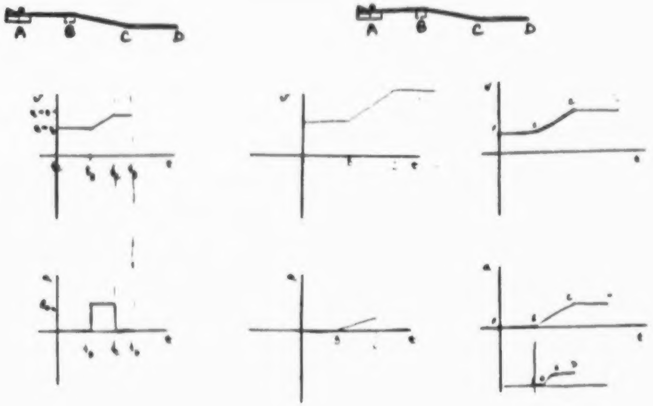
Novices either did not possess, or did not use, knowledge of how uniform or accelerated motion is represented by a particular type of curve on the various motion graphs. For uniform motion, all of the novices correctly drew a straight inclined line to represent a constant velocity on an x vs t graph and some described it as a line of constant slope. However, as discussed above, some failed to recognize that a bigger constant velocity is represented by a steeper line than a smaller constant velocity. For accelerated motion, many students did not seem to make the connection between a velocity that is changing and a slope that is changing on an x vs t graph. For example, the student who drew the incorrect x vs t graph in Figure 8, said:

> From A to B is a straight line, from B to C is a straight line with more slope, and from C to D is the same as it started.

Conceptual Clarity

In addition to a lack of explicit knowledge about the proper shapes of curves to represent different types of motion, novices frequently exhibited confusion between variables that were "constant" and "constantly increasing" such as a constant acceleration and a constantly increasing velocity.

Experts could distinguish clearly between velocity and acceleration and drew different graphs to represent these concepts. In describing what they were drawing, experts generally used the terms velocity and acceleration correctly. They also recognized that one motion can be represented by very different graphs, depending upon which variable one chooses to consider. A set of correct - and very different - v vs t, and a vs t graphs for the second task are shown in Figure 9.



EXPERT: draws different v vs t and a vs t graph

NOVICES: draw nearly identical v vs t and a vs t graphs

Figure 9: Expert and novice approaches to representing velocity and acceleration on different graphs

Novices frequently used velocity and acceleration interchangeably and drew nearly identical graphs to represent them. A frequent error on the second task was to represent the velocity and acceleration on the middle segment of the track (BC) in exactly the same way by drawing a straight slanted line for BC on both the v vs t and the a vs t graphs. For example, the student who drew the first set of novice graphs in Figure 9 said:

> From B to C, the acceleration will increase linearly up to point C...in my mind I always knew that acceleration was that an object was speeding up and I used to race cars a lot so I know what that feeling is.

He has not understood that acceleration involves not only a changing velocity but also the time interval during which this change takes place. On the written version of this task, about 20% of the calculus-level introductory physics students drew a slanted middle section on their a vs t graphs.

The student who drew the second set of novice graphs shown in Figure 9 generated the same incorrect a vs t graph twice. In both cases, she drew BC and CD segments for her a vs t graph that looked nearly identical to her essentially correct v vs t graph. In drawing the slanted middle segment BC on the a vs t graph, she said:

> From B to C there is an increase...the velocity is increasing, so there is an acceleration...the acceleration is increasing, so it is not constant.

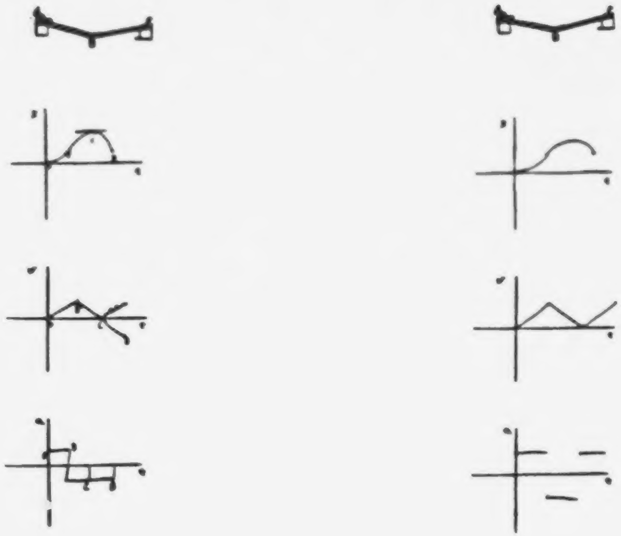
Throughout the interview, this student seemed to focus on something "increasing" during the middle segment and switched back and forth between the terms velocity and acceleration. In neither her speech, nor her drawing of graphs, did she separate the two concepts. She was aware, however, that her a vs t graph should not look like her v vs t graph and tried again, generating the same graph a second time. Saying that she knew she was wrong, she marked her confidence scale at 0. Other students who drew slanted BC segments on their a vs t graphs (mimicking the BC segments on their v vs t graphs) were not as aware of their mistakes and generally marked their confidence scales between 7 and 10.

The student who drew the second set of incorrect graphs in Figure 9 also mimicked her v vs t graph for the third segment CD, on both attempts to draw the a vs t graph. Here she failed to distinguish between velocity with a constant high value and acceleration with a constant zero value and said:

> But then (C to D) it's constant...so it would be a straight line.

Representation of Changes in Direction

Differences in both conceptual knowledge and graphical facility were most apparent on the third and fourth tasks which involved motions in which the ball turned around. The third task involved two inclined tracks on which the ball speeded up, slowed down, turned around, and speeded



EXPERT: draws straight line on v vs t graph that crosses the axis

NOVICE: draws V-shaped line on v vs t graph that mirrors path of motion

Figure 10: Expert and novice approaches to representing motion in which the object turns around

up again in the opposite direction. Experts invoked the concept of negative velocity and were able to represent a change in direction of motion correctly on their v vs t graphs by drawing a straight line that crossed the t=0 axis, as shown in Figure 10.

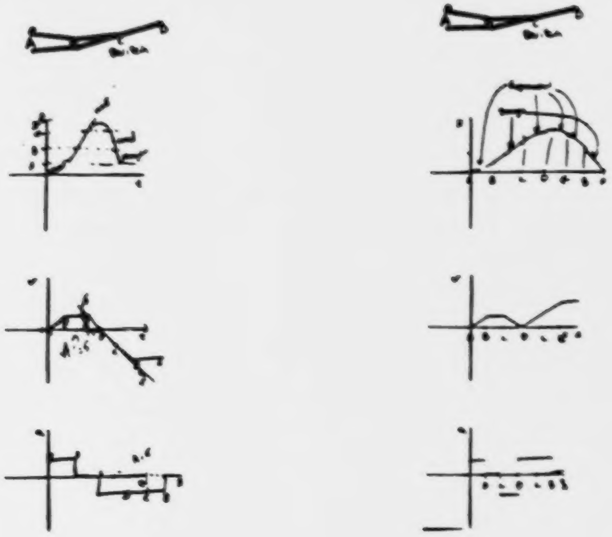
We found that novices generally had not assimilated the physicist's distinction between speed and velocity or interpretation of a negative velocity or acceleration. For example, on the CB portion of the third motion, the ball has turned around and is speeding up back down the track. Instead of representing this as a changing negative velocity, novices drew a line indicating an increasing positive velocity on their v vs t graphs, as shown in Figure 10. Thus their v vs t graphs reflected the shape of the motion. They also associated the increasing speed with a positive acceleration and drew a vs t graphs that showed different accelerations for an incline, depending upon which way the ball was traveling on the track. The student who drew the graphs in Figure 10 said:

> At C, the velocity is zero...and then it's traveling back down to point B, so therefore the velocity is picking up and the acceleration must be positive.

Consistency between Slopes and Heights among Graphs

Checking back and forth among the graphs for mathematical consistency was characteristic of the experts' approach to graphing the more complicated motions demonstrated in the third and fourth interview tasks. The student who drew the correct set of graphs in Figure 10 initially drew a positive velocity for the third segment for the third task, but corrected himself by noticing that the slope of the third segment of the x vs t graph was negative and therefore the velocity should be negative for that segment. He said:

> I know that at a point of change on this graph, there's got to be a definite point of change on this graph...any change on the velocity graph goes with a change in slope on the position graph.



EXPERT: checks for consistency in slopes among graphs

NOVICE: ignores or rejects relationships involving slopes of graphs

Figure 11: Expert and novice approaches to checking for mathematical consistency among graphs

As shown in Figure 11, he then went on to sketch the negative velocity and acceleration correctly for the fourth task, which consisted of a similar track with additional level and inclined segments.

Novices frequently ignored or rejected relationships among graphs. The student who drew the incorrect set of graphs in Figure 11 did notice the inconsistency in the segments of his x vs t and v vs t graphs for the segments of the motions in the third and fourth tasks in which the ball was speeding up in the negative direction. However, he rejected this information because of his conviction that speeding up must mean a positive acceleration. In discussing his v vs t graph for the fourth task, he said:

> How come, with positive and negative slopes in x vs t I don't get positive and negative velocities?...say I did make [DB] a negative velocity [the correct choice], well, that's a negative slope which on acceleration from D to [B] would be negative acceleration. But I KNOW that's positive acceleration. I know this velocity is increasing.

He has failed to understand the physicist's distinction between "speed" and "velocity" and that an object that is speeding up in a negative direction has a negative acceleration. Instead, he associates negative acceleration only with "slowing down."

Conclusion

In this paper, we have examined difficulties encountered by students in drawing qualitative graphs to represent motions observed in the laboratory. There were substantial differences in approach by "experts" and "novices." These are summarized below:

1) experts generally began by explicitly defining the axes while novices typically started by drawing a line;

2) experts tried to match the shape of the graph to the way the variable was changing in time while novices often tried to match the shape of the graph to the shape of the path of the motion;

3) experts represented a constant value of x, v, or a during a time interval with a line while novices often represented a constant value with a single point;

4) experts tried to match the type of motion they observed with the appropriate curve for a particular type of graph while novices seemed to lack, or did not invoke, this knowledge;

5) experts distinguished clearly between velocity and acceleration while novices often used these terms interchangeably and drew nearly identical graphs to represent them.

6) experts used the concept of negative velocity in considering motions that changed direction and graphed them appropriately while novices lacked this understanding and drew v vs t graphs that mirrored the turnarounds in the motions.

7) experts checked for consistency in slopes and heights among graphs while novices seemed to ignore or reject such relationships.

The success of the experts depended upon more than knowledge of correct definitions of position, velocity, and acceleration and the ability to calculate and plot these quantities. Underlying such skills was a conceptual clarity that enabled the experts to distinguish clearly between velocity and acceleration. In addition, these students were explicitly aware of how the features of a graph correspond to various aspects of a real motion and to its graphical representation. It has been our experience that for many students development of this type of qualitative understanding requires direct instruction. It is important that there be an emphasis on both the development of concepts and of graphing skills. An example of the type of instruction that we have found effective is described elsewhere.[5]

Examination of the strategies used by the experts suggests that the transition from novice to expert can be facilitated by teaching specific procedures for drawing qualitative motion graphs. These include carefully marking the axes with attention to the points at which the motion changes, visualizing the way the given variable is changing in time, matching the type of motion observed between two points with a line of the appropriate shape, and checking for consistency in the relationships among the heights, slopes, and areas for the various graphs.

Such detailed teaching of graphing requires more time than is typically provided in an introductory physics course. It also requires active involvement by the students in many graphing experiences. Although extra time and effort must be invested, careful development of graphing skills will provide the students with a powerful tool that will be useful to them in many other contexts besides physics. The ability to interpret what a point, a line, or an area represents on a graph is a valuable skill for obtaining and representing information in many different fields.

Footnotes

+Supported in part by NSF Grant No. DPE 84-70081.

1. McDermott, L.C., Rosenquist, M.L. and van Zee, E.H. (1987). Student difficulties in connecting graphs and physics. Am. J. Phys. **55**, 503-513.

2. See, for example, the following articles that report on the work of the Physics Education Group relevant to the topic of this paper: Trowbridge, D.E and McDermott, L.C. (1979). Investigation of student understanding of the concept of velocity in one dimension. Am. J. Phys. **48**, 1020-1028; Trowbridge, D.E. and McDermott, L.C. (1980). Investigation of student understanding of the concept of acceleration in one dimension. Am. J. Phys. **49**, 242-253; Lawson, R.A. and McDermott, L.C. (1987). Student understanding of the work-energy and impulse-momentum theorems. Am. J. Phys., **55**, 811-817.

3. See, for example, Rosenquist, M.L. and McDermott, L.C. (1983), Kinematics. Seattle: ASUW, University of Washington.

4. See, for example, Hitch, G.J., Beveridge, Avons, S.E., and Hickman, A.T. (1983). Effects of Reference Domain in Children's Comprehension of Coordinate Graphs. In D. Rogers and J.A. Sloboda (Eds.), Acquisition of Symbolic Skills. New York: Plenum Press; Janvier, C. (1978). The Interpretation of Complex Cartesian Graphs Representing Situations - Studies and Teaching Experiments. Ph.D. dissertation (unpublished) University of Nottingham; Clement, J. (1985). Misconceptions in Graphing. Proceedings, Ninth Conference of the International Group for the Psychology of Mathematics Education, Noordwijkerhout, the Netherlands; Brasell, H.M., (1987). Effectiveness of a Microcomputer-based Laboratory in Learning Distance and Velocity Graphs. Ph.D. dissertation (unpublished) University of Florida; Linn, M.C., Layman, J., and Nachmias, R. (in press). Cognitive consequences of microcomputer-based laboratories: Graphing skills development. J. Cont. Ed. Psych.; Mokros, J.R. and Tinker, R.F. (1987). The impact of microcomputer-based science labs on children's ability to interpret graphs. J. Res. Sci. Teach. **24**, 369-383.

5. Rosenquist, M.L. and McDermott, L.C. (1987). A conceptual approach to teaching kinematics. Am. J. Phys. **55**, 407-415.

A Variable for Use in the Study of the Categorization of Physics Problems and Associated Expert-like Behavior

by

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and

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Introduction

A variable that measures the degree of expert-like behavior of subjects who have sorted a set of physics problems and solved one of these problems is described.

The DEGREE variable was used in Differences in the Categorization of Physics Problems by Novices and Experts (Veldhuis, 1986), a study that investigates categorization of physics problems. Expectations were that novices use surface structures (explicitly-stated features in the text of physics problems) and experts use deep structures (physics principles that determine and control solutions to physics problems) in the formation of representations.

The perspective is obtained from information-processing theory: The representations are viewed as organized knowledge structures within short-term memory, constructed by problem solvers, that describe the environment. Problems are solved by operations on such descriptions. The knowledge within long-term memory used in the formation of a problem representation is accessed when a problem solver categorizes a problem. Choosing a problem category, i.e., the categorization process occurring in short-term memory, allows for the inference of structures that exist in the domain-dependent knowledge base in long-term memory.

One of four sets of physics problems was sorted and one physics problem was solved by each of 94 novices (students who had finished the mechanics portion of a first-year, calculus-based physics course), five intermediates (students who had completed an advanced undergraduate mechanics course), and 20 experts (professors holding the Ph.D. degree in physics who had taught an introductory calculus-based physics course).

Cluster analysis shows that a) experts categorize according to deep structures; novices use both surface features and deep structures in the categorization process and b) the categorization by novices is less consistent than the categorization by experts.

The variation in the categorization by the novices, first shown through the cluster analysis, was quantified further by the DEGREE variable. Specifically, values of the DEGREE variable obtained by the subjects, showing differences in expert-like behavior in the sorting and solving tasks among the novices, allow for the analysis of variance. This analysis, on the alpha = .05 level, does not show these differences to be related to the ACT science score, the final grade in Physics 221 (a calculus-based physics course at Iowa State University), and the high school class rank. However, expert-like behavior correlates with the final grade in Physics 221 at the significance level of 0.0338.

The use of the DEGREE variable may be of use toward the design and increased reproducibility of studies dealing with novice and expert behavior in problem solving.

Although this article has as its subject the DEGREE variable, some of the substance in Differences in the Categorization of Physics Problems by Novices and Experts (Veldhuis, 1986), hereafter referred to as the study, has been included in order to provide for the context in which the DEGREE variable was constructed and used. It is to this end that part of the problem-solving background is traced next.

The Perspective of the Study

In the context of the study a physics problem is viewed in a manner similar to that of Newell and Simon (1972) in that a problem makes available information about the initial state, what is desired, the conditions, the available tools, and access to resources. This problem-solving process includes, as one of the initial steps, the formation of an internal representation of the external environment. This internal representation provides the framework within which the problem is to be solved. It follows directly that the representation formed by the problem determines whether and how the problem can be solved. The problem solver operates on the representation rather than the statement of the problem. Chi, Feltovich, and Glaser (1981) define a problem representation as a cognitive structure, corresponding to a problem, that is constructed by a solver on the basis of domain-related knowledge and its organization. The presented information in the problem is thus transformed.

Previous learning affects later learning (Ausubel, 1968; Gagne, 1977). Much problem-solving research has been done in physics, and more particularly in mechanics since it, while being sufficiently complex, is based on a relatively small number of principles and has a mathematical structure. Prior knowledge affects the comprehension of physics principles (Champagne, Klopfer, & Anderson, 1980; Champagne, Klopfer, & Gunstone, 1982; diSessa, 1982). Heller and Reif (1984), Larkin (1980), and Chi, Feltovich, and Glaser (1981), have found that the representation formed by the problem solver, based on domain-dependent knowledge, is a crucial step in the problem-solving process. Chi et al. (1981) believe that the categories that problem solvers impose on physics problems represent organized knowledge structures in memory (schemata) that determine the quality of the representation process.

The Research Question

The study is designed to answer the question: "Do novices and experts differ in the categorization of physics problems?" Operationally the study investigates differences in the categorization, important to the representation of physics problems, that are believed to exist between novices and experts. Chi, Feltovich, and Glaser (1981) claim that the categorization imposed on physics text problems by problem solvers and concomitant representations formed by them reveal differences between novice and expert physics problem solvers. The study, as does the Chi et al. research (1981), requests subjects to categorize sets of mechanics problems using a sorting procedure. The categories are based on similarities of solutions that would occur if the subjects were to solve the problems. The subjects do not actually solve the problems in order to form the categories but express the reasons for their selection of the categories in written form. Chi et al. (1981) found that subjects with greater amounts of physics knowledge categorize primarily according to deep structures, i.e., physics laws and concepts. Subjects with lesser knowledge key on surface structures, i.e., objects such as springs, pulleys, and levers, specific physics terms such as friction, and spatial arrangements.

In order to answer the research question the following hypotheses were tested:

1. Experts will categorize physics (mechanics) problems on the basis of deep structures and novices will categorize these problems on the basis of surface features.
2. Experts will categorize a different set of physics (mechanics) problems on the basis of deep structures and novices will categorize this set on the basis of surface features.
3. Experts will categorize physics (mechanics) problems according to deep structures regardless of surface features and novices will categorize these problems according to surface features regardless of deep structures. Intermediates will reveal a categorizing pattern that is characterized by a mixture of deep structures and surface features.

4E. *Experts* (E) will categorize a set of physics (mechanics) problems according to deep structures regardless of surface features with the number of established categories approximately equal to the number of deep structures contained within the set.

4N. *Novices* (N) will categorize a set of physics (mechanics) problems according to surface features regardless of deep structures with the number of established categories approximately equal to the number of surface features contained within the set.

Description of DEGREE

The dependent DEGREE variable was designed for investigating possible relationships between expert-like behavior by the novices and ACT science scores, the final grades in Physics 221, and the percentile ranks in the high school class.

Each subject in the study solved one of the problems in the particular problem set which he/she sorted. Each solved problem was checked against the category imposed on the problem.

The value of DEGREE is obtained according to the model:

A. Does the solution fit the imposed category?

yes = 1.0
partly = 0.5
no = 0

B. Is the imposed category an "expert" category and does this category lead to a correct solution?

yes (and correct solution) = 1.0
yes (but incorrect solution = 0.5
no = 0.0

The score on the DEGREE variable = DEGREE = Score A + Score B.

The independent variables and their levels are:

ACT science score

greater than or equal to 33 3
less than or equal to 32 to
greater than or equal to 28 2
less than or equal to 27 1

Final grade in Physics 221

(scale: A = 11, A- = 10 ...
D- = 1, F = 0)

7 - 11 3
4 - 6 2
0 - 3 1

High school class rank (expressed as a percentile)

Upper third of novice sample 3
Middle third of novice sample ...2
Lower third of novice sample 1

The question, addressed by post hoc analysis, is:

"Are there differences in average DEGREE scores attributable to the ACT science score, the final grade in Physics 221, and the high school class rank?" The ANOVA procedure (SAS, 1985), with alpha = .05, was used in the testing of the two null hypotheses:

HO(1).

There are no significant differences in average DEGREE scores among the students when categorized on the basis of the ACT science score, the final grade in Physics 221, and the high school class rank.

HO(2).

There is no significant interaction in average DEGREE scores among the students when categorized on the basis of the ACT science score, the final grade in Physics 221, and the high school class rank.

Results

Table 1 shows the values for DEGREE (the dependent variable), the numbers of subjects in each group, and the levels of the ACT science scores, the final grades in Physics 221, and the high school class ranks (the independent variables). None of the F-values for the main and interaction effects are significant (see Table 2), resulting in the statistical conclusion of failure to reject Hypotheses HO(1) and HO(2) with $p < 0.05$. The research is unable to show that there are differences in average DEGREE scores attributable to the ACT science score, the final grade in Physics 221, and the high school class rank.

It is of some interest, however, that the Pearson Correlational Coefficient between the average DEGREE score and the final grade in

DEPENDENT VARIABLE		INDEPENDENT VARIABLES			
DEGREE	N	LEVELS	ACT	GRADE	RANK
2.0	11	3	20	55	69
1.5	25	2	60	30	18
1.0	9	1	14	9	7
0.5	17				
0	32				
	--		--	--	--
	94		94	94	94

Figure 1. Table: Numbers of subjects and levels of the dependent and independent variables for the analysis of variance

SOURCES OF VARIATION	df	SUM OF SQURES	F-VALUE	PR F
EXPLAINED	19	12.8395	1.33	0.1912
RESIDUAL	74	37.5861		
TOTAL	93	50.4255		
ACT	2	2.2136	2.18	0.1203
GRADE	2	3.0233	2.98	0.0571
ACT X GRADE	4	0.6494	0.32	0.8640
RANK	2	0.0134	0.01	0.9869
GRADE X RANK	4	1.9381	0.95	0.4380
ACT X GRADE X	2	0.9638	0.95	0.3919

Figure 2. Analysis of variance of DEGREE scores by ACT science scores, final grades in physics 221, and high school class rank

Physics 221 is 0.21913 at a significance level of 0.0338.

Dendograms resulting from the cluster analysis show cause for accepting all four of the hypotheses involving the experts and Hypothesis 1 involving the novices. Hypotheses 2, 3, and 4N, all involving the novices, were rejected: The novices demonstrate expert-like behavior as well as the expected novice-like behavior. The reader may want to refer to the study (Veldhuis, 1986) for details and discussion of the cluster analysis.

Schemata and DEGREE

Schemata are organized knowledge structures within memory that contain knowledge about concepts. According to Gagne (1985), schemata include static qualities (structures), active qualities (expectancy toward information), conscious use (for example, retrieval guidance), and automatic use (for example, recognition of a new instance of a concept). Operationally, as used by Andre (1986), schemata are representations of concepts (categories), principles/rules (relationships between concepts), and skills (activities requiring several steps).

Chi et al. (1981) hold that the selected categories constitute the schemata that determine the quality of the representation process. Hinsley, Hayes, and Simon (1978) claim that such schemata exist as they show that college students can classify algebra problems in types that are functions of underlying principles. Silver (1981) asked seventh-grade students to sort 16 word problems and to solve 12 of these problems. Analysis of the data showed that the good problem solvers categorized problems primarily by the processes that they intended to use in the subsequent solutions and that the poor problem solvers tended to use the content of the problem statements. It appears that categories are fundamental to problem representations. Representations determine the nature of problem solutions (Newell and Simon, 1972; deKleer, 1977; Novak, 1977; Simon and Simon, 1978; Larkin, 1980, Mayer, 1983).

Two problems used in this study are now considered:

1. A block of mass M starts up an
   incline of angle theta with respect
   to the horizontal, with an initial
   velocity v. How far will it slide up
   the plane if the coefficient of

friction is mu? (Halliday and Resnick, 1974, p. 133).

2. A child of mass M descends a slide of height h and reaches the bottom with a speed of v. Calculate the amount of heat generated.

A group of 17 novices solved Problem 1 and a group of 16 novices solved Problem 2 with both groups randomly assigned to the problems.

Recall that the DEGREE variable measures the degree to which the solution to a problem fits the imposed category and the degree to which the imposed category is an expert category leading to a correct solution. The scores of the novices were distributed among the possible values of DEGREE (0, 0.5, 1.0, 1.5, and 2.0) with approximately equal frequencies for both groups. Each group of novices categorized and solved a problem that differs in text from that solved by the other group. The numerical differences in the attained DEGREE scores in each group show approximately the same pattern.

Across the study the expert subjects demonstrated a good match between the choice of category and solution in terms of that category; 87.5% of them showed a perfect match. The experts who sorted and solved the two problems

presently under consideration attained, without exception, a value of 2.0 (maximum value) for DEGREE (the operational meaning of expert behavior in the study).

A problem solver describes the environment, in this case the statement of a physics problem, and attempts to solve the problem by mental operations on this description, i.e., the reresentation.

Representations are viewed as organized knowledge structures in short-term memory. Knowledge in long-term memory is used in the formation of a problem representation. This knowledge is accessed when a problem solver categorizes a problem. Part of the nature of the schemata in long-term memory may thus be inferred from categorization patterns.

The DEGREE variable, being an operational measure of expert-like behavior, describes the type of categorization and the match between categorization and the subsequent solution.

It is inferred that differences in DEGREE values across problems (different in text but alike in surface features and deep structures) having approximately the same distributions among the possible values of the variable, indicate differences in the schemata in the long-term cognitive structure of the subjects.

The rejection of the three hypotheses involving novices is more likely due to differences among the schemata of the novice subjects rather than being caused by differences in the problems in the first set and those in the other three sets. In view of the attained DEGREE scores, the differences among novice schemata are of a lesser degree than the marked differences between novice and expert schemata.

Novice Differences

Eleven novices (12 %) attained a score of 2.0 on the DEGREE variable, i.e., they imposed an expert category on the problem solved by them and subsequently solved the problem correctly within the imposed category: They functioned like the experts in this study. Seventeen novices (18 %), not including the 11 aforementioned subjects, attained a score of 1.0 on the A part of the DEGREE variable, i.e., the solution fits the imposed category: They functioned like the subjects in the Silver (1981) study.

The theoretical model designed and tested by Heller and Reif (1984), with the knowledge

and procedures necessary for human problem solvers to generate good representations of scientific problems, allowed subjects to construct improved representations. These investigators hold that problem-solving deficiencies exist in students who understand basic physics concepts but do not have the more strategic knowledge specified in the formulated model. This kind of knowledge, possessed by experts, according to Heller and Reif (1984), is seldom taught explicitly in physics courses.

The DEGREE variable involves categorization which is linked to representation. If knowledge possessed by experts (including the ability to form good representations) is seldom taught explicitly, a failure in finding a more pronounced relationship between the final grades in Physics 221 and the attained scores on the DEGREE variable seems reasonable. The novice sample, however, includes 12 % who functioned like experts and 18 % who functioned like the subjects in the Silver (1981) study. It seems equally reasonable to assume that a given amount of expert behavior is taught (implicitly or explicitly) in Physics 221. The use of multiple-choice examinations in Physics 221, with their limitations in testing strategic knowledge, is a more prosaic explanation of the absence of a grade-DEGREE relationship.

Summary

Do novices and experts differ in the categorization of physics (mechanics) problems?

The findings of this research confirm the Chi et al. (1981) results regarding experts: Experts categorize according to deep structures.

The behavior of novices is more complex. Novices use both surface features and deep structures in the categorization process. Novices demonstrate a lesser degree of consistency in the categorization process. Approximately one third of the novices demonstrate expert-like behavior.

The DEGREE variable was used to discern and quantify differences among novices. It can serve in the replication and extension of experimental results. A few examples of such studies are now given.

The differences in DEGREE scores and the final grades in Physics 221 (r = 0.21913 at a significance level of 0.0338) can be investigated by the replication of the study with the accompanying use of tests in which subjects solve problems of the types used in

this study. These tests may be constructed in order to serve the purposes of such an investigation and the evaluative program in a calculus-based physics course.

The study found marked differences in DEGREE scores attained by the novices. It was inferred that such differences are indicative of differences in the schemata of the novices. A longitudinal study investigating when and how such differences originate may clarify the categorization process in ways that would allow for classroom testing and use of theoretical problem-solving models.

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Some Aspects of Geometry as a Deductive System in High School Students

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The general impression of the weak and average students about Mathematics is that it is a collection of techniques to be applied either in mathematical situations (simplify, solve, etc.) or to some word problems which are meaningless, unimportant or irrelevant to real life. It is hard to tell what percentage of the students share this view, but it is impossible to ignore it. This impression is created mostly in the Algebra courses. Although there are some attempts to teach Algebra in a more meaningful way the impact of these attempts has not yet been seen. The only topic in high school Mathematics which might create a different impression than the above one is Geometry. It is the only topic in high school Mathematics where some typical features of Mathematics can be seen: the idea of deductive systems, the role of mathematical definition and its nature, the "definition-theorem scheme" which is the basic scheme of mathematical representation, analysis of statements which decomposes them into "given" and "prove", drawing figures which illustrate statements, translating from everyday language to mathematical symbols, the role of proof and its nature and so on. All this can be illustrated, for instance, by a quotation from a Geometry textbook which describes the demonstration of a theorem process as having the following stages:

1. Statement. 2. Figure. 3. Given. 4. To prove.
5. Analysis. 6. Proof. (Jurgensen et al., 1965, p. 143).

In spite of the importance of Geometry to the development of mathematical thinking, there are very few studies about learning Geometry as a deductive system. Some studies on learning Geometry have been done in the theoretical framework of van Hiele (Geddes et al., 1985, Hoffer, 1983, Usiskin, 1982, van Hiele, 1986). The studies usually suppport van Hiele's main claim that since many students are in level 2 or 3 and teaching deductive Geometry (Euclidean or Transformational) is in level 4, meaningful learning cannot occur.

The starting point of our study is not the van Hiele theory, but some aspects of Geometry as a deductive system, chosen from the list above. We tried to examine whether Geometry students had become aware of these aspects and whether they had acquired some of the specific abilities, required for learning Geometry as a deductive system. This information seems to us crucially important, since it tells us about our chance to convey to our students some general mathematical ideas beyond the basic algebraic skills. Of course, prior to Geometry as a deductive system, one has to know the geometrical figures. He or she has to be able to identify and to draw them and also to explain what he or she does. So we will deal with that first and only later will we get to the other aspects mentioned above.

METHOD

Sample

We included in our sample students of grades 9 to 12. 69 students in grade 9, 53 students in grade 10, 64 students in grade 11 and 15 students in grade 12. The

11th grade group consisted of 2 different subgroups. One of them consisted of good students (N = 42) and the second one consisted of weak students (N = 22).

The schools from which the students were selected are considered to be either average or good schools in Jerusalem. Only the students in grades 9 and 10 studied Geometry in the year they filled out the questionnaire. All the others had studied it in previous years, when they were in grades 9 or 10. We do not have any particular information about the teachers who taught Geometry to those students. We believe we had a typical selection of teachers. This is because the schools and the classes of our sample were selected accidentally. So we believe that our results reflect the outcome of typical teaching. One might claim that if the aspects we dealt with had been emphasized appropriately by competent teachers, the outcome would be entirely different. We are not sure that this is true, but even so, the purpose of this study was different. Namely, to characterize the geometrical knowledge of the typical student as an outcome of typical teaching.

The Questionnaire

Our questionnaire had 5 parts. Each of them was supposed to examine a particular aspect of learning Geometry.

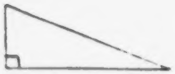
Part I: The ability to identify figures and to explain it analytically.

A. Name the following figures:

B. How can you explain your answer?

C. Did you rely on a theorem or a definition?

(1)



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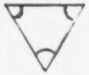

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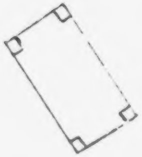

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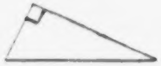

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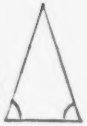

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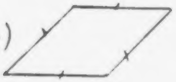

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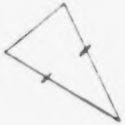


The items in this part examine the student's ability to identify basic figures in different orientations. They also examine the student's ability to justify analytically their answers. One should distinguish between the identification stage and the justification stage. The identification can be a result of a global impression of which the respondent is not fully aware. At the justification stage he or she is asked to relate to the impression in an analytical way; namely, to relate to geometrical properties of the figures. Also "structural knowledge" is required here. By this we mean the knowledge whether the properties one relies on are implied by the definition of the figure or by a theorem about the figure. An additional question which is examined here is whether the students are conditioned to follow "notational conventions". Namely, if in a given triangle two angles are marked, it comes to tell you that these two angles are congruent and the same about the sides of a given triangle. Thus, when explaining why a given triangle, for instance, is an isosceles triangle, you are expected to relate to the elements which were marked. If you do not do it, it is quite possible that the notational convention is not dominant enough in your cognition and therefore your perception focused on other elements instead.

Part II: The ability to draw figures and to define them.

Please draw and define each of the following figures:
1. Adjacent angles. 2. Isosceles triangle.
3. Parallelogram. 4. Trapezoid. 5. Square. 6. Rectangle.

In this part we examined the mental pictures formed in the respondents' mind when the figures' names were presented to them. We also examined the students' ability to define figures. We are going to relate to the question whether the respondents give a mathematically correct definition or just characterize the figures, sometimes providing us with redundant information, sometimes omitting necessary information. We believe that this is an important point, since the understanding of the nature and structure of mathematical definition is one of the goals of Geometry courses. This point is examined systematically by the following part of the questionnaire.

Part III: The nature of mathematical definition.
A teacher asked his students to define an equilateral triangle. Student A wrote: An equilateral triangle is a triangle that has 3 congruent sides. Student B wrote: An equilateral triangle is a triangle which has 3 congruent sides, 3 congruent angles and each of its altitudes is also a median and an angle bisector. Student C wrote: An equilateral triangle is a triangle which has 3 congruent sides and 3 congruent angles.
Which student has the correct answer? If you think that there is more than one correct answer, please, say it and state your preference if you have any. Please, explain!

From the systematical point of view a mathematical definition replaces a long term by a short term (the term which is defined). In most cases, the mathematical definition is minimal. The reason for that is that otherwise we will have to prove the consistency of our definitions. For example, if you define an isosceles triangle as a triangle with at least two congruent sides and at least two congruent angles, you will have to prove that there exists such a triangle. The proof will rely on the theorem that if a triangle has 2 congruent sides then the angles opposite to these sides are also congruent. Hence, it is much simpler to define an isosceles triangle as a triangle with at leat 2 congruent sides and to prove that it also has 2 congruent angles opposite to these sides. The student, on the other hand, brings to the Geometry class his own views about definitions, stemming from his experience with lexical definitions (see Vinner, 1976). In these definitions, very often, the more you say the better it is for the definition. Of course, one should not exaggerate. The definition should not be too long, but within a reasonable size, the longer the better. The above item was supposed to examine whether the respondents were closer to to the concept of mathematical definitions or to the concept of lexical definitions.

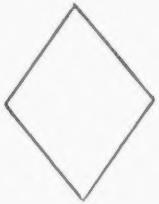
Part IV: Definitions and theorems and which of them have to be proved.

Which of the following should be proved? Explain!
1. An isosceles triangle has 2 congruent sides.
2. An isosceles triangle has 2 congruent angles.
3. In an isosceles triangle the median to the basis is an angle bisector.
4. A parallelogram is a quadrilateral in which each two opposite sides are congruent.
5. A parallelogram is a quadrilateral in which each two opposite sides are parallel.
6. A parallelogram is a quadrilateral in which each two opposite angles are congruent.
7. Parallel lines are lines which do not intersect.

In this part we wanted to examine whether students know that in Mathematics only theorems have to be proved, not definitions (we have not related to axioms in this study). The way we formulated it, there are no identifying elements in the definitions (like, a triangle with 2 congruent sides is called an isosceles triangle). Thus, the distinction between definitions and theorems became a matter of specific knowledge. Unfortunately, there is not a simple way to handle this problem. We will relate to it in the following sections. Therefore, part of the answers to the above items can be considered as specific knowledge. On the other hand, the explanation part to the above questions will reflect general knowledge about what has or has not to be proved. Namely, theorems have to be proved and definitions do not. Thus, a student who is aware of this principle should say, for instance, that (1) should not be proved because it is a definition, whereas (2) should be proved because it is a theorem.

Part V: The basic scheme of mathematical presentation - a definition of a new concept and a theorem about it.

Here is a geometrical figure:



1. What is its name?
2. What is its definition?
3. Write a theorem about this figure?

In this question we expected the respondents to define the figure (what we had in mind was a rhombus, but we were ready to accept anything reasonable, like a parallelogram or even a quadrilateral). The crucial point was that the theorem the students were supposed to give had to be specific to the figure defined. This was expected since it is typical to what we call the basic scheme of mathematical presentation. We could not state it explicitly in the questionnaire that the required theorem should be specific to the figure. We had to examine it implicitly. It is an implicit knowledge, supposed to express itself when a certain stimulus (like this question) is presented. Thus, an answer which defines the figure as rhombus and later claims that in a rhombus the sum of the angles is 360° would indicate to us that the basic scheme of mathematical presentation has not been appropriately internalized. Of course, one can claim that also this is a question of specific knowledge. A student might believe that the sum of angles being equal to 360° is specific to the rhombus. We will relate to this point in the following sections.

Part VI: Decomposition of statements into "given" and "prove", picture drawing and translation to mathematical symbols.

Decompose, in words, the following statements into "given" and "prove".

A1: In a rectangle the diagonals bisect each other.
A2: The diagonals of a rectangle are congruent.
A3: A rectangle is a parallelogram whose diagonals are congruent.
A4: A parallelogram whose diagonals are congruent is a rectangle.

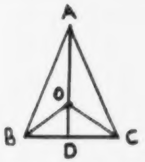
The second group of questions in this part of the questionnaire had the form:

In each of the following questions there is a geometrical statement and a figure which corresponds to it.

(I) Write in words the "given" and the "prove" of that statement.
(II) Write in mathematical symbols the "given" and the "prove" you wrote in (I).

From this group we will bring 2 out of the 3 questions in the questionnaire.

B1: ABC is an isosceles triangle. AD bisects its basis BC. O is an arbitrary point on AD. Prove that BOC is an isosceles triangle.



B2: Prove that the bisectors of the four angles of a parallelogram (which is not a rhombus) form a rectangle.

The last group of questions in this part of the questionnaire differed from the previous group only in the opening, which was:

In each part of the following questions there is a geometrical statement. Draw a suitable figure.

From this point on it continued exactly as in the second group. We will bring here only one typical example out of the 3 in the questionnaire.

C1: A segment drawn from the middle of one side in a triangle which is parallel to the other side bisects the third side.

Administration

The questionnaire was administered to the students in their regular Geometry classes by their Mathematics teachers. They had to write their names on it. It took them up to 40 minutes to complete it.

RESULTS

There were some differences between the 4 grade levels of our sample. Usually, the upper graders did better than the lower graders. (This is not true about the 22 weak students of the 11th grade who did much worse than all the other groups.) However, the differences were gradual and not drastic. Hence, in order not to overwhelm the reader with unnecessary information, our tables will relate to the entire sample. Cases of drastic differences between the grade levels will be informed.

Table 1

The ability to identify figures and to explain it analytically

Distribution of answers. N = 201

Question / Answer	Correct identification	Correct explanation	Correct characterization of explanation
1	99%	96%	def: 82%
2	99%	91%	def: 10% th.: 52%
3	98%	84%	def: 44% th.: 7%
4	98%	95%	def: 81%
5	98%	97%	def: 4% th.: 70%
6	74%	64%	def: 40%
7	96%	93%	def: 78%

The table shows us that the students in our sample can identify the simple geometric figures and also know to explain it analytically. This indicates that they are at least in the van Hiele 2nd level. As to their own characterizations of their explanations, the correct answers go from 40% in (6) up to 82% in (1). In (3), for instance, it is not so clear what the definition of a rectangle is. If it is a quatrilateral with 4 right angles then those who noted it relied on a definition. If it is a parallelogram with a right angle then those who mentiomed 4 right angles relied on a theorem. We accepted both answers as correct. Yet only 51% had a correct characterization of their explanation, while 98% identified the figure (rectangle) correctly. We also see that in cases where perception could focus either on angels or on sides (items 2, 5 and 7) the notational convention was dominant. For instance, only 10% in (2) related to the sides instead of the angles. Among the correct explanations we found some (up to 26% in (3)) which related both to definitions and theorems. For instance, in (3), students claimed: it is a rectangle because it has 4 right angles and its opposite sides are parallel and congruent. This, of course, was accepted as a correct explanation. However, if the student claimed later that he had been relying merely on a definition or merely on a theorem we did not consider it as a correct characterization of the explanation. This partially explains why the percentages of the correct characterizations are lower than the percentages of correct explanation. The hardest item in this part of the questionnaire was the rhombus. Note that students who named it as a parallelogram or even as a quadrilateral were not wrong from the mathematical point of view. They failed, however, to see the more specific properties of the figure. There is, perhaps, a perceptual difficulty. The moment you identify one property of the figure (parallel sides or 4 sides) it prevents you from relating to additional properties. There is, of course, another explanation to it: some students are not familiar with the rhombus. Needless to say that these two explanations do not exclude each other.

Table 2

Draw and define. Distribution of answers. N = 201

Answer Question	Correct drawing	Correct mathematical definition	Definitions and theorems	Partial definition
Adjacent angles	93%	4%	59%	22%
Isosceles triangle	100%	70%	29%	--
Parallelogram	100%	49%	49%	2%
Trapezoid	96%	84%	1%	3%
Square	98%	64%	14%	20%
Rectangle	98%	48%	31%	15%

In the case of the adjacent angles 60% included in their definitions the fact that the sum of the angles is 180°. The tendency seems typical: to include in the definition the most well known information about the defined concept. Thus, in a similar way, 30% of the respondents defined an isosceles triangle as a triangle with 2 congruent angels and 2 congruent sides. 50% defined the parallelogram as a quadrilateral with opposite sides which were parallel and also congruent or, in addition, they said that the opposite angles are congruent. With the adjacent angles, there was a drastic difference between the 12th graders and the others. 27% of them gave a correct definition.

Note that 63% (4% + 59%) of the entire sample characterized adjacent angles correctly. But only 4% gave the correct mathematical definition. Additional 22% gave only a partial definition, saying that the two angles should have a common ray but failing to mention that the other two rays should be on the same line, in opposite directions.

Table 3

The nature of mathematical definition. Distribution of answers. N = 201.

Accepting A as the only correct answer	31%
Preferring A but also accepting B or C	34%
Preferring B and accepting A or C	17.5%
Prefering C and accepting A or B	15.5%

In addition to the table we would like to bring here some answers to illustrate the students' views.

Students accepting A as the only correct answer:

Answer A gives the basic condition according to which the equilateral triangle is defined. Answers B and C point at properties of the equilateral triangle that should be proved.

In answers B and C there are some details which we do not contribute to the definition. They are implied by the definition in A.

A is the shortest definition by means of which one can understand what an equilateral triangle is.

A is the definition of an equilateral triangle.

Students preferring A but also accepting B or C:

A is the correct definition. C uses it and adds a theorem. B uses A and C, and adds another theorem.

A is the correct definition. B and C added to it some details which follow from A. B added the most, therefore I considered it the worst.

Students preferring B:

B is the complete definition, C is partial to B and also A is partial to B.

All the answers are correct, but B has more correct data, C has less and A the least.

B wrote all the properties of an equilaterial triangle, C wrote only part of them, while A mentioned only one property.

Students preferring C but also accepting B or A:

In A it is possible that the triangle will have non-congruent angles and this is incorrect. In B the definition includes theorems which do not have to be included in it. C is a sufficient definition.

I chose C because it is the most accurate answer. A is insufficient and the last part of B is unnecessary.

In a triangle, if all its sides are congruent then also its angles are congruent, but this does not imply that every altitude is also a median and an angle bissector.

If we accept as correct also answers which prefer A but do not reject B or C, it can be claimed, with caution, that between 1/3 to 2/3 of our sample understand the minimality aspect of mathematical definitions. If this is true, it can be considered as a real achievement. Nevertheless, it is important also to point at the one third which does not understand it. It demonstrates a typical approach to definitions, one which should be eliminated in the context of Mathematics learning.

Table 4

Definitions and theorems and which of them have to be proved. Distribution of answers. N = 201.

Questions	Answers	Not to be proved	To be proved	No answer
Defi-	1	84%	7%	9%
ni-	5	75%	10%	15%
tions	7	75.5%	11%	13.5%
Th-	2	19.5%	70.5%	10%
eo-	3	5	86%	9%
re-	4	35%	55%	10%
ms	6	17%	68%	15%

Table 5

Distribution of explanations to the answers "not to be proved" in Table 4. The percentages are out of the entire sample.

Question	Answer	Definitions you do not prove	Axioms you do not prove	Other Explanations or no explanations
Defi-	1	70.5%	1 %	12.5%
ni-	5	66%	--	9%
tions	7	38.5%	23.5%	13.5%
Th-	2	6	--	13
eo-	3	--	--	5%
re-	4	26.5%	--	8.5%
ms	6	8%	--	9%

Table 6

Distributions of explanations to the answers "to be proved" in Table 4. The percentages are out of the entire sample.

		A theorem or a proposition you should prove	It is not given or not implied, therefore you should prove it	It is a property, therefore it should be proved	Other explanations or no explanations
De-	1	--	--	--	7%
fini-	5	--	0.5%	0.5%	9%
tions	7	0.5%	1%	0.5%	9%
Th-	2	10.5%	39.5%	3%	17%
eo-	3	12.5%	43.5%	3%	27%
rems	4	8.5%	26.5%	2%	18.5%
	6	10.5%	33.5%	2.5%	21.5%

Table 4 shows us that between 55% and 84% know which statement has to be proved and which one does not. As to the explanations, the majority of the answers in the cases which do not have to be proved uses a correct argument: definition or axiom (axiom, however, was a mistake in this questionnaire). Note that 23.5% consider the definition of parallel lines as an axiom (a common misconception). On the other hand, only a minority uses "theorem" as an argument in the cases that have to be proved. The preferred argument is a vague one: "not given" or "not implied". Also, one should not ignore the percentages of students who give no explanation or irrelevant explanations. They go from 18% in (5) up to 32% in (3). Again, we see the importance of specific knowledge when trying to determine general knowledge. 26.5%, for instance, claimed that (4) should not be proved because it is a definition.

Table 7

The basic scheme of mathematical presentation.
Distribution of answers. N = 201.

Correct identification of the figure (rhombus, parallelogram, quadrilateral)	Correct definition	Correct theorem	A theorem specific to the figure defined
97%	60.5%	72.5%	34.5%

Some additional information to Table 7:

(1) 20% gave a theorem instead of a definition or in addition to the definition. This also includes incomplete definitions together with redundant information.

(2) 3% gave a definition or an axiom instead of a theorem.

(3) When we say "a theorem specific to the definition" it is, of course, the "expert's view". We cannot tell whether a student, giving a theorem not specific to the defined figure, knows it or not. He or she might believe that it is specific, but in fact it is not. In the 65.5% which are not included in the last column of Table 7 we find a variety of examples. Some of them are really silly. For instance:

(1) The figure is identified as a parallelogram. The definition is: A quadrilateral whose opposite sides are congruent and parallel and its opposite angles are congruent. The theorem is: In a parallelogram each pair of opposite sides are parallel and congruent.

(2) The figure is identified as a rhombus. The definition is: A quadrilateral whose diagnals are congruent and perpendicular to each other and also bisect the angles. The theorem is: In a rhombus the diagnals are congruent to each other, perpendicular to each other and also bisect the angles.

It seems that the view behind these answers is that both the definition of the figure and the theorem about the figure should mention some properties of the figure. The definition should mention as many properties as possible. The properties in the theorem can form a subset of the properties in the definition.

On the other hand, we did not find an answer in which the theorem related to a different concept than the definition. Namely, nothing like a definition of a rhombus and a theorem about a parallelogram. However, many students in their theorems, related to the figure they had defined, properties which were not specific to these figures. It is impossible to tell whether this is lack of specific knowledge or lack of understanding of the basic scheme above. For instance:

(3) The figure is identified as a rhombus. The definition is: A quatrilateral whose opposite angles are congruent and all its sides are congruent. The theorem is: In a rhombus the sum of the angles is 360°.

(4) The figure is identified as a rhombus. The definition is: A quadrilateral all of whose sides are congruent to each other, whose opposite sides are parallel and whose opposite angles are congruent. The theorem is: The diagonals of a rhombus bisect each other.

Table 8

Decomposition of statements into "given" and "prove" in words. Distribution of answers to questions A1-A4.
N = 201.

	Answer		
Question	Correct	Incorrect	No answer
A1	88%	10.5%	1.5%
A2	89%	9.5%	1.5%
A3	34%	59.5%	6.5%
A4	76.5%	17.5%	6%

Table 8 shows us that when familiar statements are concerned the students' performance is satisfactory (items A1, A2 and A4). However, the moment they are confronted with unfamiliar, perhaps strange statements their level of performance goes down to 34%. What is this ability to decompose statements? Is it content-dependent or content-free? It seems to us that it should be content-free. Otherwise, it is alost mere memorization. In items A1, A2 and A4 the students got statements with which they dealt in the past. Hence, the decomposition of the given statements could be supported by memory. In A3, on the other hand, the students got a statement they had never seen. In a way, it is not a "correct theorem". Roughly speaking, when you decompose a statement, the subject is the "given" and the predicate is the "prove". In A3 it is claimed that a rectangle is a parallelogram. This does not look right. Either because it is strange or because it is trivial. Hence, the decomposition should be different. Thus, we found that 36% included in the "given" part of the "prove" (the parallelogram element) and 23% confused the "given" with the "prove". Even some Mathematics teachers, with whom we discussed the questionnaire, claimed that A3 and A4 express the same theorem. But if so, how come they have different subjects and different predicates? Our conclusion is that the decomposition ability that the students demonstrated in A1, A2 and A4 is content-dependent. It is not content-free or formal as one might expect.

As to the last group of questions of this part (B1, B2 and C1), firstly, it has been found that almost everybody knew how to make suitable drawings in the cases where drawings were not given. On the other hand, the translation into mathematical symbols was a major problem for many students as Table 9 shows us, especially in the more complex case (B2).

Table 9

Decomposition of statements into "given" and "prove" in words and in symbols. Distribution of answers to questions B1, B2 and C1. N = 201.

Answer Question	Success in words and in symbols	Success in words and failure in symbols	Success in symbols and failure in words
B1	48%	30%	3%
B2	13.5%	40.5%	4.5%
C1	44%	3%	6%

In the above items, the success of the 12th graders was drastically better than that of the other grades.

As we said in the beginning of this section, because of space problems we did not relate to the differences between the grade levels. The common pattern was that the higher levels did better than the lower levels, except the 22 weak students of the 11th grade. Thus, to demonstrate this pattern we graded the students' questionnaires, giving 2 points for every full correct answer and 1 point for partially correct answers. The results are in Table 10. The numbers indicate percentages out of the total score.

Table 10

Mean success in the 4 grade levels and the entire sample.

Grade level	Mean and standard deviation
9 (N = 69)	59.1 20
10 (N = 53)	66.7 16.4
11 weak students (N = 22)	43.6 22.9
11 good students (N = 42)	73.1 12.4
12 (N = 15)	83.9 8
The entire sample (N = 201)	64.2 20

From the fact that the upper graders did better, one cannot conclude that there is a maturation factor here. This is because of the selection factor. The mathematics students of the upper grades, except the 22 weak students, might be better than those in the lower grades.

DISCUSSION

It is only reasonable to believe that structural knowledge is harder to acquire than factual knowledge. This is because structural knowledge is usually taught and acquired implicitly, contrary to factual knowledge. On the other hand, as it was already shown in the results, it is impossible to separate structural knowledge from factual knowledge. Structural knowledge stems from factual knowledge or, if you wish, meta-knowledge stems from knowledge.

So, let us relate first to the factual knowledge factor in this study. We examined the students' ability to identify and to draw simple geometric figures. In this particular aspect it can be claimed that mastery has been achieved (Tables 1 and 2). This is contrary to other studies about different populations (Hershkowitz and Vinner, 1982; Usiskin, 1982; Vinner and Heshkowitz, 1980). The reason for that difference might be that the population we exmined had studied Euclidean Geometry for at least one year. Nevertheless, it should be noted that only very simple tasks were given to the students. It is hard to predict the success percentage if harder tasks were given to them (like drawing altitudes, drawing altitudes in complex cases as an obtuse angle triangle or a right angle triangle, drawing the distance of a point from a straight line, etc.). What we said about the success in identifying and drawing is also true about the justification level. Students can analytically explain why a certain figure is what it is. From the van Hiele theory point of view, this means that most students are at least at the 2nd van Hiele level. On the other hand, when talking about stuctural knowledge (characterize your explanation!), the percentage of success immediately goes down (Talbe 1). They are between 51% and 82% if we ignore the rhombus item. When it comes to defining tasks, they go further down to 48%, if we ignore the hard case of the adjacent angles in which only 4% fully succeeded. Here we start to see the common confusion between definitions and theorems. A particular aspect of the mathematical definition was examined in Part III. A liberal interpretation of Table 3 can lead to the claim that between 1/3 to 2/3 of our sample is aware of the minimality aspects of mathematical definitions. Again, from the van Hiele theory point of view, these students are at least in the beginning of the 3rd van Hiele level.

As to the distinction between definitions and theorems and which of them have to be proved, once again it was shown how factual knowledge and structural knowledge depend on each other. It was found that if a statement is identified by a student as a definition, he or she knows that it does not have to be proved. The reason for that -- definitions you do not prove. Unfortunately, in some cases up to 2/3 of the students cannot identify definitions (Table 5, item 7, the case of the parallel lines). When it comes to theorems the situation is similar, but worse. Contrary to the case of definitions, only about 10% of the students know explicitly that theorems have to be proved because theorems are what you prove in Geometry. Of course, a liberal interpretation of Table 6 will consider other types of explanations like "not given" or "not implied" as an attempt to justify it beyond the tautological stipulation: theorems should be proved because they are what you prove in Geometry. We are not inclined to accept this here because our question was: "which statement has to be proved?" and not "why does a theorem have to be proved." The fact that we have not found attempts to explain why definitions do not have to be proved only supports our non-liberal approach in this case.

As to the basic scheme of mathematical presentation, it can be claimed that a majority of the students has a rough idea what it is. They have acquired its surface structure. But when it comes to the deep structure and accurate knowledge, only 1/3 of the students has it (Table 7). This is also true about the formal ability to decompose theorems into "given" and "prove". When the formal ability is needed (item A3, Table 8) the success percentage goes down to 34. Therefore, we conclude that the decomposition ability that students acquire is content-dependent. We do not claim that they memorize the decompositions of the theorems they have learned. But the fact that they did these decompositions once in the past, with the same theorems or with similar theorems, helps them to construct (or reconstruct) the decomposition in a given task.

Finally -- the ability to use mathematical symbols to express mathematical statements formulated in an ordinary language. We know it is a major problem in Algebra. It is, as well, in Geometry. In the hard case (item B2 in Table 9), 54% succeeded in decomposing the theorem in words, but only 13.5% succeeded in doing so also in symbols.

We have drawn a partial picture of some deductive aspects of Geometry in high school students. (It is quite partial since we have not dealt at all with proofs.) The rather poor situation in these aspects cannot be separated from what we called factual knowledge. If you want to examine the students' conception about definitions, they should know particular definitions first. Since the factual knowledge is quite poor (see for instance Carpenter et al., 1983), we can expect very little structural knowledge. Only a minority of the students has some of it. Whether this is a fact that we wish to put up with or not -- it is a matter of goals in Mathematics education. This question is beyond the framewrk of this study which was merely factual.

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Representation and Problem-solving in Basic Electricity. Predictors for Successful Learning.

1. Introduction

In the domain of basic electricity two different research approaches are predominant. The first approach analyzes the problem-solving strategies of the students and provides information about students' misapprehensions about concepts and rules (Cohen et al., 1983, Caillot, 1985). The second approach emphasizes the importance of the student's representation of the processes in an electric circuit. In this view, the misconceptions derive from an integrated representation. This representation originates in past experience and structures knowledge that determines perception and comprehension processes.

The student's representation of basic electricity may be described in different ways. Tiberghien and Delacôte, 1976, describe students' conceptualisation in terms of a linear causal effect between batteries and bulbs, i.e. the battery is seen as acting on the bulb. Andersson, 1980, points to a source-consumer model underlying students' understanding of electric circuits. Fredette and Lochhead, 1980, use a sink model to describe the student's representation. Maichle, 1982, utilizes a give-schema to analyze the processes in a circuit. Finally, Rhöneck, 1983, interprets the semantic structures on the basis of students' 'energy view' where current and energy flow are intermixed. Common to all of these studies is an integrated representation which comprises the students' differing concepts on the one hand and the different elements of an electric circuit on the other.

This paper examines the question whether the learning difficulties may be interpreted as isolated or as interrelated aspects of an integrating representation. For this purpose the formation of different concepts and rules was studied in a small group of gifted students (grade 9). Ten students with an I.Q. between 107 and 140 were taught, tested and interviewed over a period of one year. The most striking result was that one half of the students developed a correct representation whereas the other half formed an integrated representation incorporating several incorrect reasonings which are frequently found in students. In a second study the development of the student's representation was examined in a normal class situation. Here the positive and negative results were less stable and only a small percentage of the students began to construct the physical representation. In search of an explanation for the differences in learning progress of the students some predictors for successful learning were investigated.

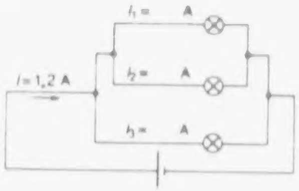
Before the two studies are described in more detail a survey will be given of the most prominent learning difficulties in basic electricity. For this purpose some results of a European test are presented (Shipstone et al., 1987). The participating countries were Sweden, France, the Netherlands, England and the Federal Republic of Germany. Results from Germany stem from two states, Baden-Württemberg and Hesse, which have different school systems. The total of 1250 students were tested in grade 10 after instruction (age range 15 - 17 years). In Sweden, France and Baden-Württemberg the sample consisted of students who attended a 'gymnasium' or grammar school. In the remaining countries only part of the students were preparing for university education.

Despite differences in teaching methods the test results were surprisingly similar. Only in the domains of current flow and voltage did the results show significant differences. It is not possible to comment upon the results of all 13 test items here. In the following only five test items will be discussed (Fig. 1). The first item is related to the consumption of current. The idea that current is consumed is predominant even after instruction. The second test item is related to 'local' reasoning. Here local reasoning means that the current divides into two equal parts at every junction regardless of what is happening elsewhere. This item was the most difficult of all the items in the test. The third test item focuses on 'sequential' reasoning. In sequential reasoning a circuit is viewed in terms of 'before' and 'after', the student believing that the current in a certain bulb is influenced by changes 'before' the bulb whereas a change 'after' will leave the current in the bulb unchanged. The fourth test item is related to the differentiation of voltage and current. In all countries except France, where

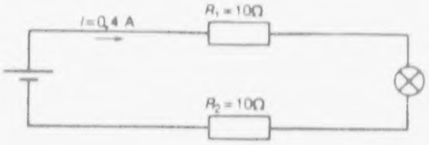
① The bulb is connected with the battery. The bulb is lit up. Read each of the sentences below and tick off the correct box.

		true	false	I don't know
a.	The bulb uses all of the electric current.	☐	☐	☐
b.	The bulb uses up a little of the electric current.	☐	☐	☐
c.	All of the electric current from the battery to the bulb goes back to the battery.	☐	☐	☐

② In the circuit shown below all the bulbs are of the same type. Complete the currents I_1, I_2 and I_3 in the wires.

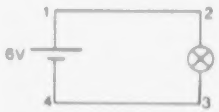


③ In the circuit shown below the current is 0.4A.

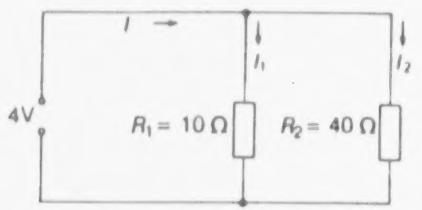


Now the resistor R_1 and afterwards the resistor R_2 are replaced by a 20Ω resistor. The battery and the bulb remain the same. Compare the current after the first change and after the second change with the current in the original circuit...

④ Look at the following circuit: Insert the values of the voltages across the points 1 and 2: ....V, 2 and 3: .... V, 3 and 4: .... V.



⑤ Look at the following circuit:



The resistor R_2 = 40Ω is replaced by a resistor of 50Ω. Put a tick in the box with the correct answer.

a) The electric current I_2 increases. ☐
The electric current I_2 stays the same. ☐
The electric current I_2 decreases. ☐

b) The electric current I_1 increases. ☐
The electric current I_1 stays the same. ☐
The electric current I_1 decreases. ☐

c) The electric current I increases. ☐
The electric current I stays the same. ☐
The electric current I decreases. ☐

- -

Results: (*: correct)		S	F	NL	BW	H	E
1.	current not consumed *	39	11	30	28	15	40
	current consumed (b)	55	49	56	50	49	58
2.	values 0.4/0.4/0.4 *	14	21	17	18	12	15
	values 0.6/0.3/0.3	59	60	57	53	31	49
3.	completely correct*	30	32	34	33	20	27
	sequential reasoning	43	30	32	38	45	37
4.	values 0/6/0 *	14	52	25	34	08	04
	values 6/6/6	61	00	45	42	60	55
5.	compensation, I =constant	32	15	34	21	12	05
	I_i = constant	14	05	15	08	13	16
	completely correct*	16	20	15	30	28	03

Figure 1: International test: 5 test items and results.

this problem had been specially emphasized in the teaching sequence, the two concepts were not properly differentiated.

The fifth item shows a circuit of two resistors connected in parallel. Some students argued that a larger resistor leads to an increase in current. Even more interesting is the reasoning called 'compensation': Here a decrease in current in R_2 is compensated for by an increase in current in R_1 on condition that the total current does not change. Another group of students extended I = constant to all currents. In both cases the battery is regarded as supplying a constant current, independent of the circuit to which it is connected. Regarding the battery as a supplier of constant current is another example of the more general tendency of students to engage in 'local' reasoning.

The test results do not affect the question whether or not the different learning difficulties form an integrated representation. An answer to this question can only be found when the development of the concepts and rules used is carefully observed during instruction.

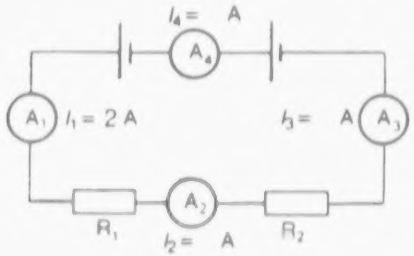
2. The development of the representations in a group of gifted students

2.a The study and the tests

The formation of physical concepts and rules was studied in a small group of ten gifted students with an I.Q. between 107 and 140. These students were taught outside compulsory teaching over a period of one year according to the following principles: Every student had at his disposal his own experimental equipment with an ammeter and a voltmeter to carry out the various experiments. A lot of experimental work was integrated into the course in order to improve motivation. During the course the students were regularly tested and interviewed. This information enabled the development of concepts and rules to be followed. In the following the results of the first six tests are reported. The tests relate to several well-known learning difficulties.

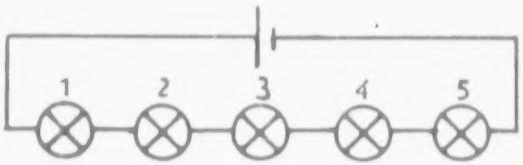
Test 1: The conservation of current in a closed circuit. Test 1 is concerned with the constraint I = constant in a closed circuit. Only two of the original three test items are presented in Fig. 2. All ten students solved these tasks correctly. This good result may be explained by the emphasis placed on the bicycle chain analogy and experimental evidence.

① In the circuit drawn below the resistances R_1 and R_2 are different.



There are four ammeters in the circuit. A_1 shows a reading of I_1 = 2A. Write on each of the other meters what you think they will read.

② Five identical bulbs are connected in a series circuit with a battery.



Put a tick in the box with the correct answer:

Bulb 5 is brighter than bulb 1. ☐

Bulb 5 is just as bright as bulb 1. ☐

Bulb 5 is dimmer than bulb 1. ☐

Figure 2: Tasks related to I = constant in a closed circuit (test 1).

Test 2: Voltages in simple connection diagrams. This test checks the ability of the students to classify simple circuits with the connections in series and in parallel (Fig. 3). The correct classification is a precondition for solving the second question of the test item. The students did not meet with difficulties in this part of the test. The second part of this test is related to the question of whether or not the students have at their disposal two different rules for the voltages in these circuits. The results show that five students have available two independent rules whereas the other five students have only one rule for the voltage: The voltage is divided up in every circuit. For these students the concept of voltage has not developed as it should.

① Arrange the circuits drawn below in groups. (A classification in terms of circuits with bulbs and circuits with resistors is not intended.) The number of groups may be more or less than the number given here:

group A: group C:

group B: group D:

② All the bulbs in the circuits drawn below are of the same type. Insert the values of the voltage across the given points.

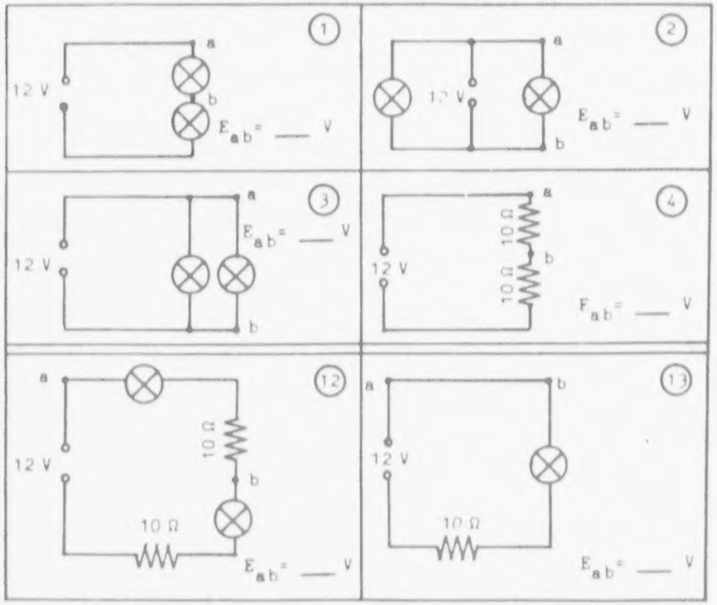


Figure 3: Voltages in different connection diagrams (test 2).

Test 3: Currents in simple connection diagrams. The third test (Fig. 4) combines items which are equivalent to the items of test 2 and which are now related to the concept of current. All students know two independent rules for the currents but one student confuses the two rules. Generally, the rules for currents are learned more easily than the rules for voltages.

① Text as in Figure 3.

② All the bulbs in the circuits drawn below are of the same type. Insert the values of the current at the given points.

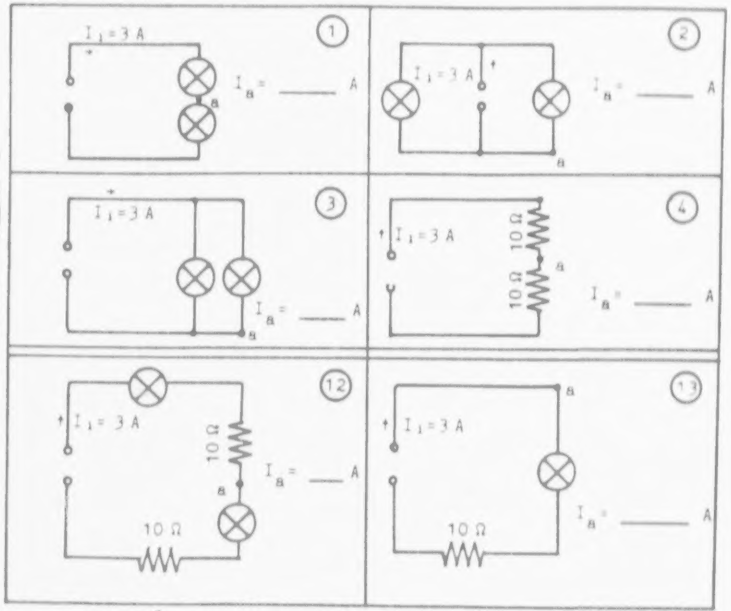


Figure 4: Currents in different connection diagrams (test 3).

Test 4: 'Local' reasoning, 'sequential' reasoning and circuits with measuring instruments. Test 4 (Fig. 5) comprises items related to 'local' reasoning, 'sequential' reasoning and circuits where the measuring instruments are integrated. The type of error called 'local' reasoning fails to consider what 'happens elsewhere in the circuit'. The currents in the different wires leading to a junction do not derive from the voltage and the resistors. In the difficult item 1.b four students inserted $I_1 = 0.6A$, $I_2 = I_3 = 0.3A$ regarding the current as dividing into two equal parts at each successive junction encountered. Four other students solved item 1.b correctly and the remaining two students met with difficulties in classifying the wiring diagram in 1.a as a parallel connection. The items 1.c/d were less difficult. The percentage of correct solutions is 70%, one student used 'local' reasoning and split the voltage as if it were current.

'Sequential' reasoning was employed by students in item 2 who analyzed the circuits in terms of 'before' and 'after'. The increase in the resistor 'before' the bulb led to less current in the bulb whereas the increase in the resistor 'after' the bulb left the current in the bulb unchanged. Two students employed 'sequential' reasoning and violated the constraint that current is conserved (see test 1).

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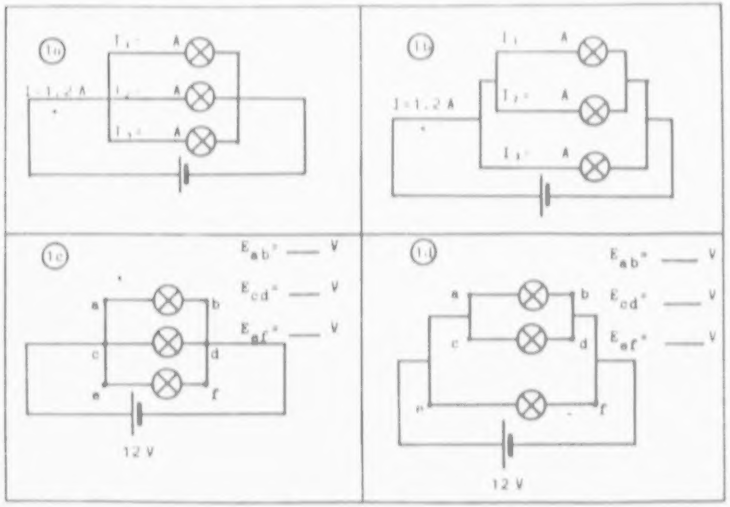
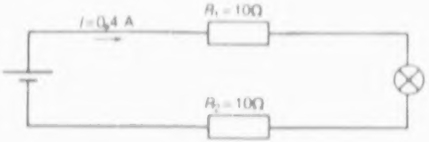
① Insert the values of the current and the voltage:

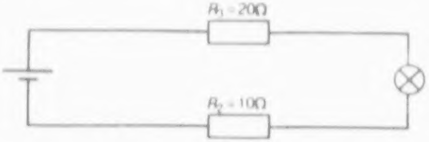


② In the circuit drawn below the current is 0.4A.



First the resistor R_1 and then the resistor R_2 are replaced. The battery and the bulb remain the same.

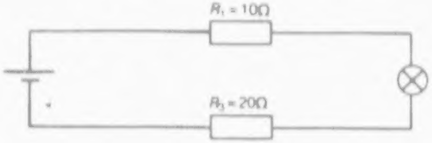
First change: The resistor R_1 is replaced by the resistor $R_3 = 20\Omega$:



Compare the current in the second circuit with the current in the first circuit. Put a tick in the box with the correct answer.

		true
1.	The current in the bulb is now less than 0.4A.	☐
2.	The current in the bulb is now 0.4A as before.	☐
3.	The current in the bulb is now greater than 0.4A.	☐

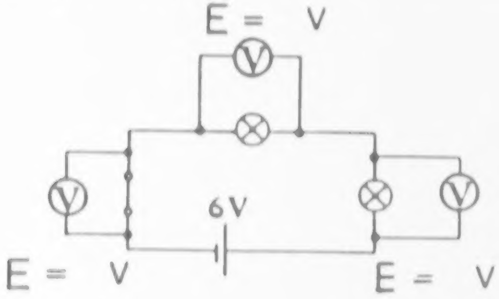
Second change: The resistor R_1 is reinserted. Then the resistor R_2 is replaced by the resistor $R_3 = 20\Omega$:



Compare the current in the third circuit with the current in the first circuit. Put a tick in the box with the correct answer.

		true
1.	The current in the bulb is now less than 0.4A.	☐
2.	The current in the bulb is 0.4A as before.	☐
3.	The current in the bulb is now greater than 0.4A.	☐

(3a) In the circuit drawn below the switch is closed. Complete the readings of the measuring instruments:



(3b) In the circuit drawn below the switch is open now. Complete again the readings of the measuring instruments:

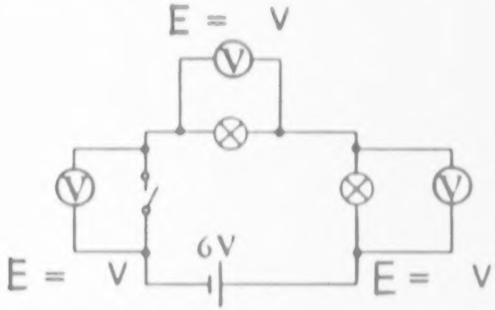
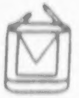


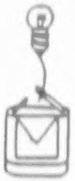
Figure 5: Tasks related to 'local' reasoning, 'sequential' reasoning and circuits with measuring instruments (test 4).

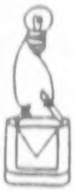
The third item of test 4 shows whether or not the student may disregard the measuring instruments in a wiring diagram. Even for students at the university level this task is difficult (McDermott, 1985). In our group item 3.a was solved correctly only by two students and item 3.b by four students. The respective interviews showed a broad spectrum from very primitive to rather acceptable argumentations.

Test 5: Semantic structures. Fig. 6 shows two items which are combined in test 5. The first item checks the ability to differentiate between voltage and current. Four students do not distinguish between these two concepts in the first item and only half of the students solve this item completely correctly. The second item of test 5 is related to the differentiation between current and voltage, too. Six students rightly ticked off the correct statement 2.2. From among these students five students solved the first item correctly.

① Look at the four drawings A, B, C and D which contain usable batteries and bulbs.


A


B


C

D

Read each of the sentences below. One sentence may be true for several drawings. Put a tick in the box if you think the sentence is true.

		A	B	C	D	I don't know
1.	The bulb will light up.		☐	☐	☐	☐
2.	There is an electric current here.	☐	☐	☐	☐	☐
3.	There is an electric voltage here.	☐	☐	☐	☐	☐

② Here you find three sentences about the electric current and the electric voltage. Read each of the sentences and tick off the correct box.

		true	false	I don't know
1.	The electric voltage and the electric current always occur together.	☐	☐	☐
2.	The electric voltage may occur without an electric current.	☐	☐	☐
3.	The electric current may occur without an electric voltage.	☐	☐	☐

Figure 6: Tasks to assess semantic structures (test 5).

Test 6: 'Local' reasoning in more complicated circuits. The last of the tests presented here recapitulates the problem of the first test item of test 4 (see Fig. 5) in a more complicated manner with different resistors in parallel: $R_1 = 10\Omega$, $R_2 = 20\Omega$ and $R_3 = 10\Omega$. Six students solved the more complicated test items 1.a/b correctly and eight students the test items 1. c/d. These results show that the students may be led by teaching, testing and discussion to better results in more and more complicated problems.

Tests 1 to 6 check the qualitative reasoning of the students and their conceptions of current, voltage and resistance. More formal tasks were presented to the students, too, but these tasks are not discussed in this paper. All the tests were submitted to the students without a preliminary announcement before the weekly lessons.

A more careful evaluation of the test and interview results shows that after some weeks the group of ten students split up into two sub-groups. One half of the students correctly differentiated between the concepts of voltage and current and formed a correct physical representation. The other half had difficulties with the physical concepts and rules in nearly every problem presented and developed an integrated representation which included many learning difficulties. For a better understanding of the problem-solving strategies used a two-dimensional graph was utilized. This graph will be described and used in the next section.

2.b The mapping of the different representations in a double hierarchy

For a better analysis of the test and interview results and the problem-solving strategies used a double hierarchy is employed which is similar to a double hierarchy proposed by Dörner, 1976. The dimensions of the double hierarchy are 'formation of theory' and 'complexity' (Fig. 7). The vertical dimension describes the 'formation of theory' at different levels. The concepts at the first level are subject to rules at the second level. A third level with Maxwell's equations could be added and again the rules and laws of the second level are subject to Maxwell's equations at the third level. The connections between the different levels are given by straight lines and each straight line may be expressed as 'is subject to'. The horizontal dimension describes the complexity of the system in attribute-relations and whole-part-relations. For example, the current may be given with a certain value or may be described by a certain verbal definition. Here the connections are given by 'is'. In Fig. 7 all of these connections are delineated for two resistors connected in series with all the required laws and rule-based constraints.

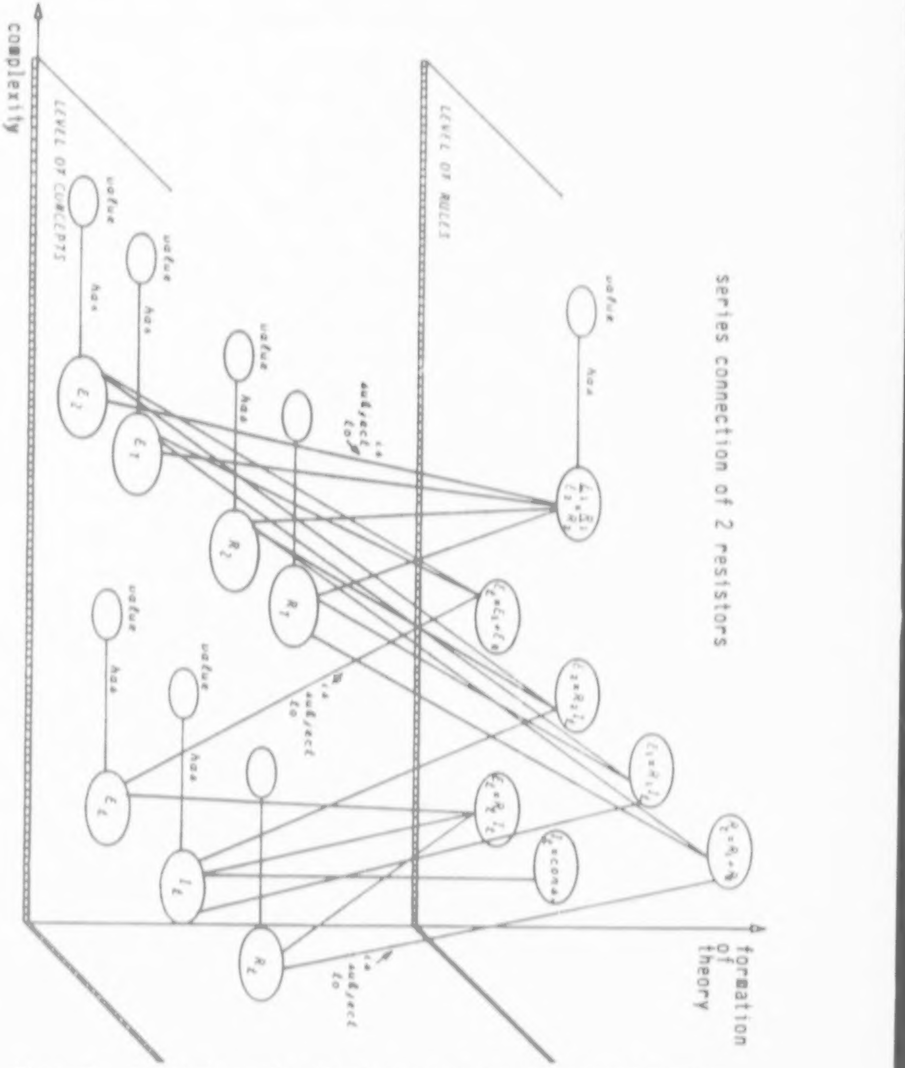


Figure 7: Double hierarchy with the concepts and rules for two resistors connected in series.

In Fig. 8 the dotted lines show the connections and inferences which are necessary for the test item 3.a of Fig. 5 to be solved. Of course the

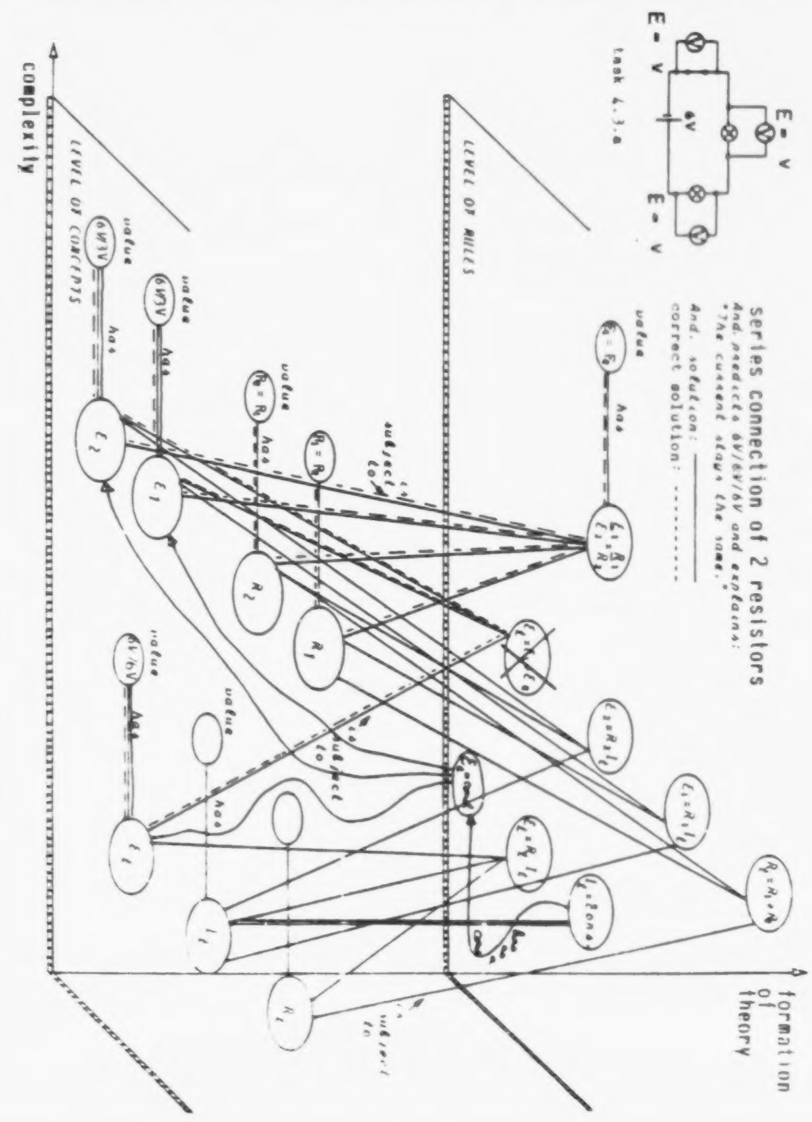


Figure 8: Correct solution of task 3.a of Fig. 5 and incorrect approach of one student.

student's solution may not incorporate these connections. The additional lines in Fig. 8 describe the incorrect solution of a student who predicted 6V across every bulb and explained that 'the current stays the same'. Obviously the rule $E_t = E_1 + E_2$ is violated and the constraint I_t = constant leads directly to the incorrect constraint E_t = constant.

It is a typical feature of this student to directly infer some rules about the voltage after stating rules about the current. Other elements of the circuit like resistors were not mentioned, i.e. Ohm's law was left out of consideration and the voltage was seen as a kind of attribute of the electric current. The connection between current and voltage may be verbalized as 'the rule for the current has as a consequence an equivalent rule for the voltage' (see Fig. 8). This problem-solving strategy is used by the same student in different test items (Fig. 9). In four cases this student stated rules for the current which had as a consequence an equivalent rule for the voltage. For this strategy a predominant conception of current may be responsible. At the same time the student's conception of current is burdened with false ideas about the consumption of current, local reasoning and sequential reasoning. All these conceptions cannot be integrated in the double hierarchy of Fig. 7. A better description is given in Fig. 9 where all of these conceptions and the additional concepts and rules are displayed. Similar representations were found amongst other students of this sub-group.

A survey of the correct representation on the one hand and the incorrect representation of a sub-group of students on the other is given in Fig. 10. The core of the latter representation is the idea that current is consumed which is already present before instruction (Rhöneck, 1983).

During instruction local reasoning, sequential reasoning and other aspects of the student's representation develop. For example, when the direction of the current is introduced sequential reasoning is inevitable.

A popularized description of the integrated representation of students is illustrated by the cartoon in Fig. 11. Current consumption, local reasoning, sequential reasoning, compensation and the defective differentiation between voltage and current are all consistent with the picture of Fig. 11.

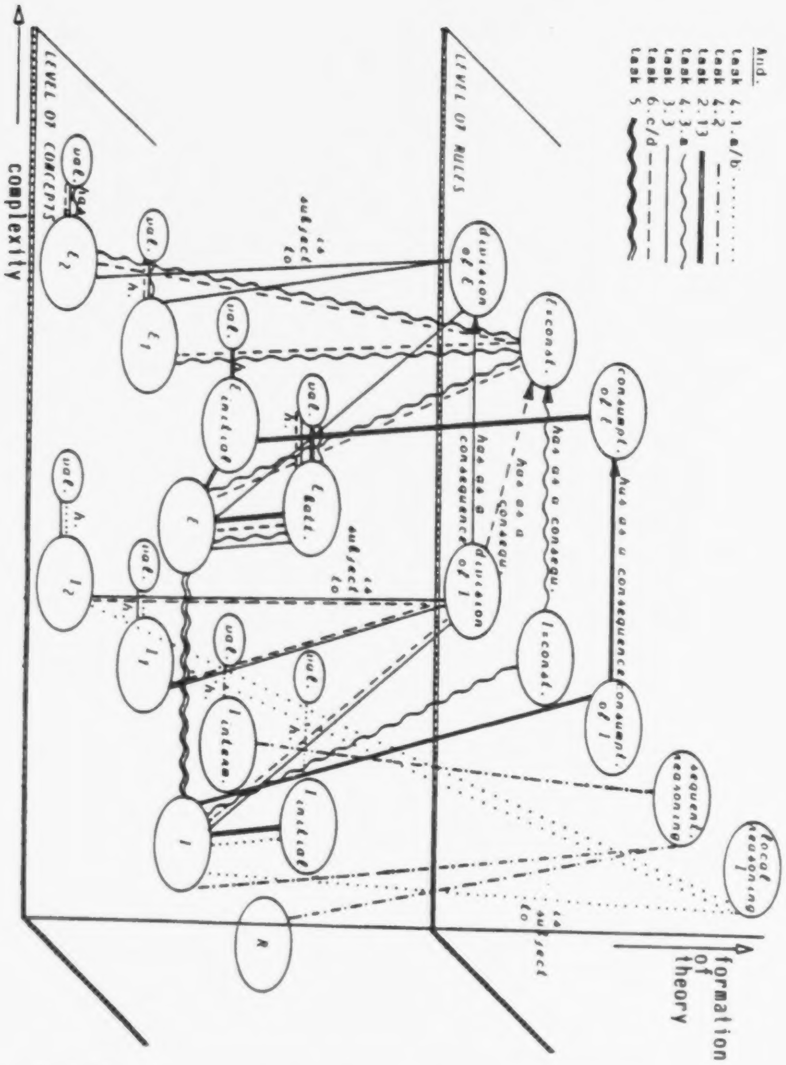


Figure 9: Representation of incorrect solutions of one student.

A more generalized description of the student's representation may be given in different ways. It is possible to point out the lack of differentiation between voltage, current and energy (Rhöneck, 1982). Another view may emphasize the local reasoning which is related to sequential reasoning

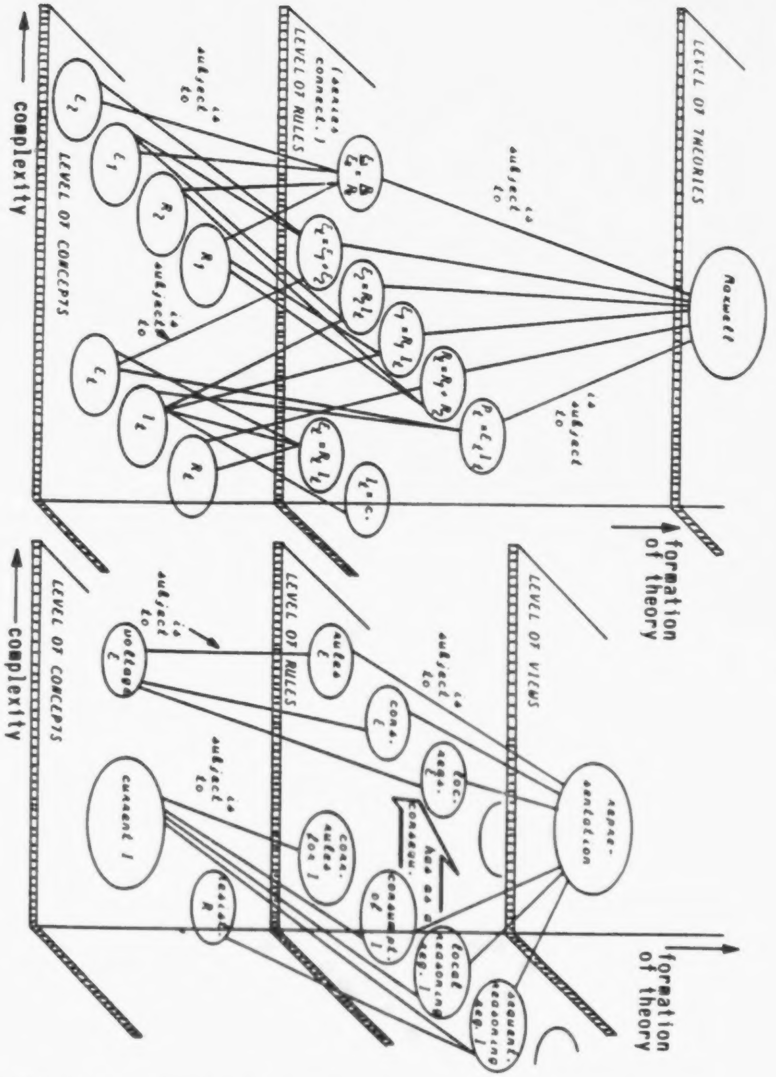


Figure 10: Comparison of the correct representation and the student's representation.

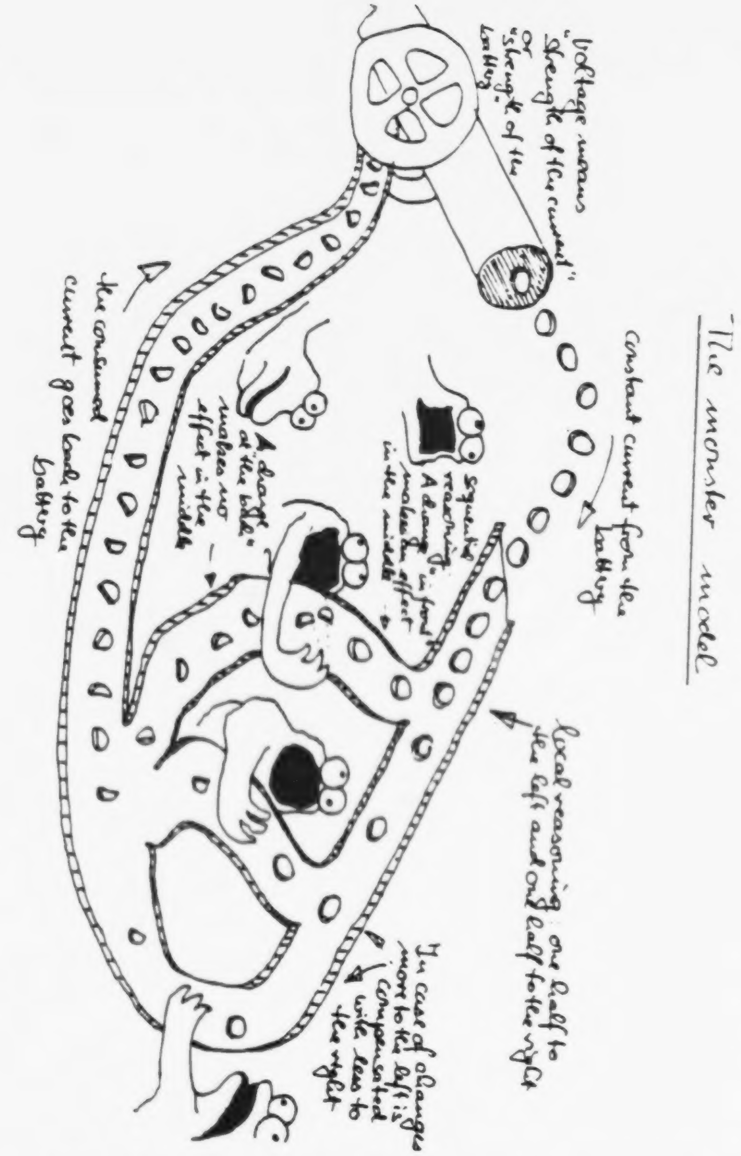

The monster model
Voltage means "strength of the current" or "strength of the battery"
constant current from the battery
sequential reasoning: A change in front makes the effect in the middle
A change at the back makes no effect in the middle
local reasoning: one half to the left and one half to the right
In case of changes more to the left is compensated with less to the right
the consumed current goes back to the battery

Figure 11: The monster model which illustrates the student's representation.

(Shipstone, 1984). A change at one point in the circuit leads to effects at that point or at a point 'after'. A third interpretation may describe the student's representation in terms of causality. Tiberghien and Delacôte, 1976, point to a linear causal effect between batteries and bulbs where the battery acts on the bulb by means of electricity or current. Andersson, 1986, emphasizes the causality gestalt. Gestalt means that the student's representation is more than its parts, and indeed, this representation in one sense does make sense.

As a final remark on students' representations, the following fact should be added. The students' representations disintegrated at the end of the teaching phase. In the last test we only sometimes found typical conceptions. But even after the disintegration of the students' representations those students who had not spontaneously developed physical representation had great difficulty in learning the correct concepts and rules. Their difficulties are brought out by the statistics. Tab. 1 shows the correlations between the results of the following tests: I.Q. test, the first six tests, the final class test, a retention test and a Piaget test on formal operations. There are considerably high correlations between the results of the first tests, the class test and the retention test. That means the score of the first tests which show the ability of the student to learn spontaneously is a good predictor of successful learning.

	I.Q.	first tests	class test	retent. test	Piaget test
I.Q.	1.00	0.32	0.17	0.51	0.44
first tests	0.32	1.00	0.90*	0.83*	0.62
class test	0.17	0.90*	1.00	0.70	0.57
retention test	0.51	0.83*	0.70	1.00	0.58
Piaget test	0.44	0.62	0.57	0.58	1.00

Table 1: Correlation matrix (grade 9, N=10, gifted students). Significance: * 0.01

3. The results of a teaching sequence in a normal class situation

In a next step we wanted to analyze the learning processes in a 'normal' class situation. The class consisted of 21 students who had chosen

French as their second foreign language (Realschule [a kind of technical secondary school], grade 8, three boys and 18 girls). This group of students was taught four hours per week over a period of six months on the same principles as the group of gifted students.

A general impression of the results is that the learning processes and the development of an integrated representation were less stable. There may be different reasons for this: The stress on performance in the first and second class test which disturbed the spontaneous learning processes, the composition of the group with students who are primarily interested in learning languages and not physics, and the fact that the students were on average less intelligent and one year younger than the group of gifted students. For this reason it was necessary to reduce the subject-matter to very simple parallel-series connections.

In the normal class the search for predictors of successful learning was extended to additional psychological variables. Data were collected on the following:

- I.Q. (I-S-T, 1971).
- interest in electricity (Häußler, 1987). In this test the students indicate their interest on a multiple choice scale between very high and very low. Only those parts of the test results were evaluated which are related to electricity.
- mutual assessment of interest in electricity. Here the students indicate the interest in electricity of their class-mates.
- the development of formal operations [the Piaget test] (Lawson). This test presents 15 demonstration items that illustrate physical problems. Finding solutions to the problems involves problem-solving strategies which are interpreted in the context of Piaget's theory on cognitive development.
- the students' spontaneous grasp of information about physical concepts and rules [the first tests].
- achievement motivation [in form of a so-called grid test] (Heckhausen et al., 1985). This test measures various components of the students' achievement motivation separately. It consists of 18 picture situations in which achievement could play a role.

Also included in our considerations were the following attainments in physics:

- the results of the first class test. This test combines test items related to the concepts of current, voltage, resistance, parallel connections and series connections.
- the results of the second class test. This test is concerned with simple parallel-series connections.
- the results of a retention test. This test recapitulates the problems of the first class test with similar items two months after the first class test.

Finally data in the form of school marks were collected on the students' ability

- in mathematics, German and biology in the last grade.

The correlation matrix for the ten variables is given in Tab. 2. It indicates that again the results of the first tests correlate with the results of the achievement tests, but are less dominant than in the gifted student group (see Tab. 1). A strong correlation exists between the way students assessed their class-mates' interest in electricity (mutual assessment test) and the results of the spontaneous learning tests (first tests), the achievement tests and the retention test. The different achievement tests also correlate with each other. Nearly all other correlations are surprisingly small.

To understand in particular the unexpectedly low correlation between I.Q. and the results of the class tests, found in the gifted student group as well (see Tab. 1), we compared the data on these variables for each student. Fig. 12 shows the interrelations. The division of the group of gifted students into two sub-groups, as described in the foregoing section, can be easily seen here. The results of the normal class mirror the relationships found in the gifted student group. As in this group I.Q. and the ability to construct a physical representation in the normal class hardly correlate. Jan, for example, with a relatively high I.Q., did not develop a physical representation but showed different aspects of a student's integrated representation such as consumption, sequential reasoning and compensation. Pet, however, with a much lower I.Q., was able to construct the correct representation. He gradually adopted the roll of the class expert. Similar effects were noticed in the group of gifted students (see Tho and Mat).

With the collected data for the ten variables a factor analysis was carried out. The basic assumption of such an analysis is that the observed correlations between the variables result from their shared underlying dimensions, called factors. Conversely, these not-directly-observable factors

	marks	I.Q.	interest	first tests	class test 1	Piaget test	class test 2	achievement motivation	retention test	mutual assessment
marks	1.0000	.4236	.3168	.0022	.1757	.2719	.0833	.4437	.1033	-.0521
I.Q.	.4236	1.0000	-.0637	.1465	.1562	.2518	.0100	-.0234	.1266	.0637
interest	.3168	-.0637	1.0000	.4066	.3267	.2365	.1280	-.0118	.2994	.4810
first tests	.0022	.1465	.4066	1.0000	.5163	-.1166	.3937	-.2000	.3200	.8391**
class test 1	.1757	.1562	.3267	.5163	1.0000	.3709	.5748*	-.2271	.1894	.6377*
Piaget test	2719	.2518	.2365	-.1166	.3709	1.0000	.2971	-.0600	.5066	.2436
class test 2	0833	.0100	.1280	.3937	.5748*	.2971	1.0000	-.3069	.5691*	.5799*
achieve. mot.	.4437	-.0234	-.0118	-.2000	-.2271	-.0600	-.3069	1.0000	-.3925	-.3553
retent. test	.1033	.1266	.2994	.3200	.1894	.5066	.5691*	-.3925	1.0000	.5479*
mut. assess.	-.0521	.0637	.4810	.8391**	.6377*	.2436	.5799*	-.3553	.5479*	1.0000

Table 2: Correlation matrix (grade 8, N = 21, normal class situation). Significance: * 0.01, ** 0.001

are defined by groups of the variables. One goal of factor analysis is to represent relationships between variables using as few factors as possible. In the present case a four-factor model seems to be adequate to represent the

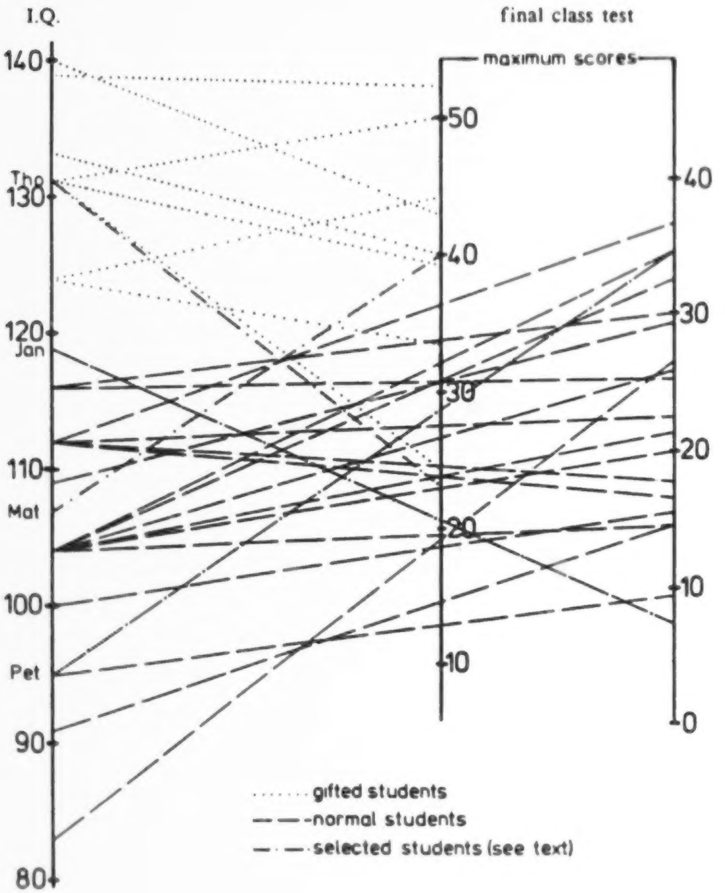


Figure 12: Comparison of the students' I.Q.s and the results of the final class tests.

data. The different techniques for factor extraction of the SPSS/PC+ programm used (principal components analysis, principal axis factoring, maximum-likelihood method, alpha factoring, image factoring, unweighted least-squares method, generalized least-squares method) yielded nearly the same groups of variables for each of the first three factors. For the fourth factor only four of the seven methods led to the same configuration of variables. Nevertheless, the result as a whole is consistent and interpretable.

The four factors explain about 67% of the total variance. The first factor is closely related to the first tests, the mutual assessment and the first class test. This factor can be interpreted as the willingness and ability of the students to accept more or less spontaneously the new concepts and rules which lead to a new frame for understanding the physical world. Whereas the first factor describes the entry into the new frame, the second factor combines the long-term effects which structure this view. The Piaget test, the retention test and the second class test form this factor. The third factor based on the variables of motivation and subject attainment is an expression of the students' hope of success. The fourth factor may be related to general intelligence. The first factor explains approximately 33% of the total variance, the other factors 15%, 10%, and 9%, respectively.

The SPSS/PC+ programm used allows us to compute factor scores for each student. This has been done for the factors extracted with the principal axis method. The standardized factor score values for the two students Pet and Jan are given in Tab. 3.

	Factor 1	Factor 2	Factor 3	Factor 4
Jan	-.99	1.37	1.11	.96
Pet	2.28	.53	-.84	-1.14

Table 3: Factor scores for two selected students.

The values of Tab. 3 exemplify the results discussed above. Jan, who has a relatively high I.Q., is motivated and able to structure knowledge in a formal way, did not develop a correct representation owing to an inability to accept spontaneously the new concepts and rules. Pet, on the other hand, who has a low I.Q., is unmotivated and only to some extent able to structure knowledge, constructed the physical representation because of his ability to follow instruction spontaneously. Similarly, but less dramatically, the factor score values of other students confirm these results.

To broaden the basis of our research, we intend in the near future to investigate approximately 100 students from grade 8 in the same way .

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TOWARD A COGNITIVE PHYSICS COURSE

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Introduction

The "new crisis" of science illiteracy has focused attention on major problems in the structure and teaching of traditional physics courses offered in the high schools and universities. Research in science education and cognitive science has led to a number of results which directly impact on how students learn, or in many cases do not learn. Often these results illuminate some of the frustrating situations we encounter in the physics classroom. Students should be learning to think, apply general principles and solve problems in physics courses; but this apparently is not happening. New, or reorganized courses and textbooks seem to be needed to address these considerations. In this paper I will report on one such course.

Problems With Traditional Courses

The typical introductory college physics sequence course has changed little in the past fifty years. Other than in details, physics texts used today are very similar in structure to those used in the 1930's. However, there is mounting research evidence that students are not "learning physics". Over the past several years a great deal of time has been spent at professional meetings, such as those of the American Association of Physics Teachers, on consideration of problems involved in the teaching of introductory physics. Among the concerns raised at these meetings are the following:

a. Most students come to our classes with a fairly well developed, if somewhat incoherent, conceptual system relating to the physical world. We may call these views misconceptions, alternative frameworks or simply wrong, but they cannot be ignored in our instruction. Research and reflection on our own experiences demonstrate that these student views are highly resistant to change.

b. Students try to learn physics completely or partially by rote. The traditional organization of physics texts and courses is logical and follows the historical development of the discipline, but it does not "work" psychologically. Major concepts, such as fields or waves, are usually discussed in fragments separated by several hundred pages in the text and, perhaps, months of class time. Cognitive scientists have investigated how information is organized and processed. People think and learn with concepts; therefore meaningful learning must focus on concepts and their relations. Concepts are organized, in our memories, into a hierarchical cognitive structure. New concepts are most efficiently learned and retained when they are linked to existing general concepts already present in the cognitive structure of the learner.

c. Students do not learn the conceptual structure of physics implicitly, rather they must have explicit instruction about the structure of knowledge. Rote learning does not encourage students to develop or appreciate the conceptual structure of physics.

d. Since the introductory course is the only exposure to physics for many of the students, it is imperative that some modern physics be included and that they get some appreciation for the differences between the "new" physics and that which takes up most of a typical textbook.

The Reorganized Course

Over the past year I have reorganized the introductory, three quarter, physics sequence that I have taught for several years, with the above concerns and my own experiences in mind. This course is the noncalculus sequence which is taken by a wide variety of students--for example life science majors, preprofessionals, energy management majors, and some industrial technology students. As one would expect, their backgrounds are extremely varied, as are their abilities. The class enrollment, at its start, is usually 40-60.

My approach was to structure each quarter around a few general concepts. The concepts and the major topics covered are summarized separately below.

Physics 111

Concepts: 1. Matter exists in the form of units which can be classified into hierarchies of organizational levels.

2. Units of matter interact. Most changes we see about us; e.g., motion, are due to interactions between units of matter. At present it is believed that all ordinary interactions are electromagnetic, gravitational and nuclear. These interactions can be described in terms of fields.

Topics:
1. Nature of matter
2. Motion-kinematics
3. Interactions-momentum
4. Interactions-force
5. Fundamental interactions

Physics 112

Concepts: 1. All interacting units of matter tend toward equilibrium states in which the energy content is a minimum and the energy distribution (entropy) is most random. In attaining equilibrium energy transformations, matter transformations and matter-energy transformations may occur; but the energy-matter sum remains unchanged.

2. One form of energy is the motion of units of matter. Such motion is responsible for heat and temperature as well as the three states of matter.

Topics:
1. Work and energy
2. Conservation of energy
3. Gravitation and fields
4. Electric fields and energy
5. Electric current
6. Temperature and heat
7. Kinetic theory and states of matter
8. Thermodynamics

Physics 113

Concepts: 1. Energy may be transferred by particles or waves. Wave motion permeates nature.

2. Phenomena at the macroscopic and microscopic level are fundamentally different. At the microscopic level there is a wave-particle duality, or complementarity.

Topics:
1. Oscillatory motion
2. Mechanical waves
3. Sound
4. Electromagnetic waves
5. Physical optics
6. Electron waves and atomic structure.

Such a reorganization by itself does not guarantee that the concerns outlined above will be adequately addressed. The classroom atmosphere must also change. While lecturing is an efficient mode of conveying factual information, it rarely fosters conceptual learning. Students must grapple with the new concepts through discussion and application, such as problem solving. One way of providing such interactions is through group work. Much less material is being covered and it must be covered in a different manner. As has been well documented, misconceptions die hard; forcing students to challenge their own ideas is time consuming. Part of that time must be spent in laboratory activities. For my course I had to restructure the laboratory sessions so that they would reinforce the conceptual development presented in the classroom. Previously, the main focus of the lab had been the learning of laboratory technique, rather than the development of conceptual skills. The amount of number crunching was greatly reduced, partly by collecting less, but more meaningful data, but also by more efficient use of computer analyses.

Students were encouraged to focus their attention on the structure of the knowledge they were learning in class, lab and problem solving. Two heuristic aids were used for explicitly bringing out this structure. Concept maps were recommended for organizing each unit of material. For both lab and problem solutions students were required to construct knowledge vees. Construction of a concept map visually illustrates how new concepts relate to those which have already been learned, and the knowledge vee shows, again in a visual manner, the process of studying an event, either in the lab or in a problem situation, in terms of concepts and principles needed for its analysis.

Results and Conclusions

I am convinced that this organization of material works better than the usual logical order of a typical textbook. The experimental group of students seems to have grasped the process of physics better than my past classes; misconceptions were fewer, as measured by test scores, at various stages of the course; the GPA was higher and they were able to work novel problems on final exams. For example, on the final exam for Physics 113 I gave them a series of questions/problems and asked them to explain how they would go about developing an answer. They were asked to explain in terms of concepts, principles and theories how they would approach each case, but not give a full solution. Usually they could not have been expected to provide a complete answer, as their conceptual tool kit was missing some of the pieces needed for a solution.

As mentioned, average grades were a bit higher than what I had experienced in the past, but the main indicator that something useful was occuring was the dropout rate for Physics 111. No matter who has taught the course over the last decade (myself included), this rate has been about 25%. My dropout rate last fall was only 4%, and both students withdrew from school. In the past, few students who completed the first quarter withdrew from later courses in the sequence, and this trend continued with the experimental group.

Finally, the affective level was high. Students seemed to feel good about what was happening in the course. They felt that they were active participants in the teaching-learning process. Throughout the year I kept them informed about what I was trying to do and stressed that they were responsible for their own learning.

Overall I have been very happy with this approach and will continue it, with some refinements, next year. Too much material is still being covered, even with the paring that has been done. I am convinced that some problem solving must be done in the classroom by the students, either alone or in groups. The current practice of having all problems worked away from class, or even the campus, is inefficient and leads to frustration as a result of lack of guidance and encouragement. Such classroom work will take yet more time away from material; but it will, I think, increase learning. The labs need refinement to make them more supportive of the student's conceptual development. I would also like to develop some techniques for assessing, in a quantitative way, the results of conceptual instruction.

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